Master in Economics

Thesis

The design of innovation policy for the energetic transition

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Implementation of big innovative projects is often subject to big financing problems. On the one hand, the possibly profitable project can require an outstanding investments at the beginning, which can be not feasible for the firm to acquire on its own, so the investor is needed. On the other hand, such innovative projects have usually the high level of uncertainty, so the uncertain revenue can seems not high enough reward for the investments, which need to be done initially. However, it can be the case that the project, if successful, would have a great social value (for example, innovative green projects), and thus the government (or government agency) can be interested in stimulating the firm to invest into such project. However, intuitively, the firm can acquire more valid information about the quality and expected payoffs of the project. Thus, the government needs to stimulate the firm, while reducing the costs of information asymmetry.

This paper studies the question of an optimal mechanism of supporting a firm willing to undertake some green innovative project. We concentrate on the projects, which require the development of technology at the beginning and then, in case of success can be commercialized. The firms differ in the "quality" of the technology. For the projects of such type we show that among the deterministic mechanisms there is no opportunity for the governmental agency to do better than propose a single mechanism to all the firms, which means that discrimination is not possible in this conditions. After derivation of the optimal mechanism, we add the possibility of moral hazard for the firms, given by the possibility to hide the success of technology. From the comparison of optimal mechanisms in the case of pure adverse selection and with addition of moral hazard we determine the value of information on the success of technology development for the agency.

The problem of the optimal mechanism to support such innovative green projects has a high practical value since there exists a number of governmental agencies almost in each country the aim of which is to support financially the firms, that are able to undertake the innovative project. In France one example of such agency is Ademe, in collaboration with which the idea of this research was drawn. It specializes on the financing of the innovations in energetic transition. Thus. the structure of the project presented in the main model is based on the real life projects, which increases the practical value of the current research.

On the theoretical side the model represents principal agent framework, which includes simultaneously adverse selection and moral hazard. On top of this the project generates separate benefit for both the agent and the principal and the outside option for some types also has some value for the principal. All those features are not usually presented in the literature in the same model, so the paper contributes also to the theory of principal-agent framework.

1 Related literature

There are a multiple branches of literature on principal-agent framework (see for example Laffont & Martimort (2002)) and mechanism design to which this research can be classified. The first big part is the literature on financing innovations. This field is very broad and currently fast expending. It includes very different kind of research questions and techniques used to tackle them, but for all of them the main question at stake is the optimal financing of technological development. The part of such papers concentrates on experimentation under adverse selection. Even though they study mostly only technological development and the important part of their mechanism is the stopping time for experimentation, which is not one of the concerns of our model (we work in deterministic time), the results of some of these researches are still closely related to ours. A good example is the paper Gomes et al. (2016). They present the multiperiod model, in which the agent can choose each period to undertake the risky project of uncertain quality, but which generates some positive payoff or to proceed with the project generating known payoffs of value 0. The situation is analysed as a two-arm bandit problem embedded into principal-agent framework. Under assumptions of impossibility for the agent to start the project without the financial help of the principal, the result achieved shows that the first-best action plan is achievable under the asymmetric information about the quality (expected payoff of the risky project) by profit-sharing payment rule together with the repayment of all the opportunity cost of choosing risky project over the safe one. However, the first-best is no longer achievable if we consider uncertainty on the opportunity cost of undertaking risky projects instead of the safe one. The limited-liability issues were not considered, however, but it is stated that this can affect the result for the case of uncertainty on probability of the risky project to be good. The result on the possibility to achieve first-best goes in line with the corresponding result of the simplified version of the current model with technological stage only, which is presented in Meunier & Ponssard (2017). Also the optimal mechanism is related to our solution since it consists of rewarding failure in the presence of adverse selection.

Similar framework as in Gomes et al. (2016), but with addition of commercialization stage is considered by Khalil et al. (2017). This addition changes the result dramatically, leading to such a non-classic feature of equilibria as over-experimentation under some parameter settings. Also it is stressed out that the presence of adverse selection makes it optimal to reward failure, which goes in line with our result. As an extension Khalil et al (2017). consider an addition of moral hazard in the form of possibility for an agent to hide success in experimentation or to reveal it later than it has occurred. Then it is shown that the presence of such opportunity requires rewarding success at each stage and affects the reward of failure at the equilibrium, since the cost of failure reward increases. This result has the similar origin with the result presented in Section 5 of current paper.

A lot of literature concentrates precisely on ex-post participation constraints and limited liabilities, i.e. the constraints on agent's expected utility after the type is realized. Sappington (1983) shows that in the presence of such constraints it is not optimal to induce socially efficient outcome (first best), but the best strategy is to induce the efficient outcome from the most productive type. This classic result remains true in our solution. Ollier & Thomas (2013) consider the model with both adverse selection and moral hazard, which results in binary output: success or failure, so the agent's type is directly related to probability of success and an effort may increase this probability. The contract proposed should satisfy the agent's ex-post participation constraint for each type, since the agent has an infinitely negative utility in case of negative ex-post payoff. The optimal mechanism is a composition of fixed payment and positive bonus in case of success and represents pooling solution. The information rent is also decreasing and set up to 0 for the highest type, which coincides with our findings. The presence of success bonus is caused by the presence of moral hazard on top of the adverse selection.

Bergemann, Castro & Weintraub (2017) concentrate on the two-stage model of adverse selection with sequential revelation of the type (valuation) for the agent and only two types. Before contracting the agent learns its ex-ante type, which is the distribution of possible valuation. The exact valuation is learnt after contracting, but the agent will not fulfil contracting responsibilities if her ex-post utility (given exact valuation) is negative. The possible screening is related to ex-ante type, and it is shown in the paper that ex-ante screening (i.e. dynamic contract in terms of Bergemann et al.

(2017) is optimal if ex-ante types are far enough. The model of Bergemann et al. (2017) has one additional incentive constraint compared to our model of technical stage, which rises from the unknown ex-ante type (distribution for the principal), which gives a possibility to ex-ante screening. However, this benchmark can be extremely useful for the logical extension of the model, which would allow the agent to have also some superior information on market outcome ex-ante.

Another part of literature concentrates on the questions of type-dependent reservation utility for the agent, which is also the case in our model. An important benchmark here is the model of Bruno Jullien (Jullien(2000)). He considers type-dependent reservation utility with different properties and shows the conditions needed for the optimal solution to be separating or instead represent bunching. However, in the model he assumes reservation utility to be non-monotonic, which is more difficult case than the one considered in the current paper. Thus, the majority of the difficulties present in Jullien (2000) are not applicable to our model, since we presume monotonicity and can use monotonic information rents.

There exist also an interesting relation of the model which we use with the models used to study an optimal licensing contract especially for pharmaceutical industry. Crama et al. (2014) concentrate on the R&D project, which consists of technical and commercialization stages with technical stage divided also in different sub-steps. During the technical phase, each research step is characterize by a probability of technical success, which is privately known by the agent. An information about costs and sales estimates is shared by the two parties. However, the commercialization stage includes also an effort from the agent, which gives a rise to a moral hazard problem, but of the other sort than the one we are interested in. However, the model is indeed close to the one presented here, except the optimal mechanism should include payments in other direction (from agent to the principal), which changes the nature of solution.

The rest of the paper will be organized as following: Section 2 will give the description of the model; Section 3 will present briefly the underlying one-stage model of adverse selection; section 4 will give the optimal solution for the main problem without moral hazard, Section 5 will do this in the presence of moral hazard; section 5 will present a comparison of the cases and derive the value of information for the principal, section 6 presents some numerical study of the theoretical result, section 7 concludes.

2 The model

Assume there exists an agent, which is willing to implement some innovative project, which is risky and requires some initial investments. More precisely, at the beginning the agent can invest some fixed amount into the project (sunk cost) and start the development of the technology. If the technology is successfully developed the outcome of it can be commercialized (for example, it can be a new tool to replace less effective one). The commercialized project generates a profit for the agent, but also the "social benefit", in which the principal is interested. Note, that the tool can be commercialized if technology is successfully developed, however, the firm can take an opposite decision.

We assume that the agent is able to invest the required amount into the project and does not need the help of the principal to finance the initial step. However, the expected profit from the project can be not enough for the agent to be interested in undertaking it. At the same time the expected social benefit can be positive and high enough for the principal to be interested in providing some financial incentives to the agent.

As it was mentioned the project is risky. More precisely we model the riskiness of the project through the two pieces of information, which can be unknown for the agent or for the principal. The first one is the quality of the technology, which is determined by the probability of technical success, i.e. by the probability that the development of the technology will be successful and the product will be ready for commercialization. The second source of riskiness of the project comes from the final revenue (social benefit), which in general depends on future market conditions. It can be not known by the agent or the principal and can take the value from the interval.

Usually the firms are more aware about the quality of the project and have better predictions of the future market than the governmental agencies. We keep this feature and introduce the information asymmetry into the model. Also, given that there can be sufficient time difference between the beginning of the project and the end of the technical phase we introduce some information dynamics through the possibility to learn additional information on the future market revenue. This corresponds to the reality since indeed the information on the future market conditions improves with time.

Given what we have said there are different possibilities of information structure. However, only part of them are practically interesting. So we assume that the "quality" of the project is a private information of the agent,

	Principal	Agent
Ex-ante (t=0)	-	p
Interim stage (t=1)	Technical output	Technical output & R
Ex-post (t=2)	Final output	Final output

Table 1: Information structure with pure adverse selection

	Principal	Agent
Ex-ante (t=0)	-	p
Interim stage (t=1)	-	Technical output & R
Ex-post (t=2)	Final output	Final output

Table 2: Information structure with adverse selection and moral hazard

and that she learns exact market outcome immediately after technology is successfully developed. In this paper we concentrate only on the loss in the principal's revenue with the addition of the moral hazard, which is generated by the asymmetric information on the technical outcome (which is the ex-post Tech. clock of the tables). So we restrict ourselves to consider the information structures, in which market revenue is ex-post observed by both participants. To add asymmetric information on this (through non-observability of the revenue ex-post for the principal) would lead to additional problem of multidimensional adverse selection with two consecutive adverse selection problems, which could be an interesting extension by is out of the scope of this paper. The two information structures under consideration are presented in the Tables 1 and 2. Now we can go to the formal statement of the two problems.

2.1 Notations

We will use the following notations for the variables. Those notations with bar will denote the case of pure adverse selection, the notations with tilde will denote the joint case of adverse selection and moral hazard.

- F - the amount of the initial sunk investments needed to start the project;
- p - the probability of successful development of technology (agent's type);

- $\Psi(x)$ - ex-ante distribution of the types;
- R - the final (commercial) outcome of the project;
- $G(x)$ - ex-ante distribution of the possible revenues;
- $\phi(R)$ - the function, which relates the agent's revenue and a principal's revenue (social benefit);
- $M(x, R) = (H(x), s_1(x), \gamma(x, R), s_2(x, R), t(x, R))$ - the mechanism proposed by the principal to the agent of type x,

  where $H(x) \in \{0, 1\}$ - the allowance to the agent to invest into the project;

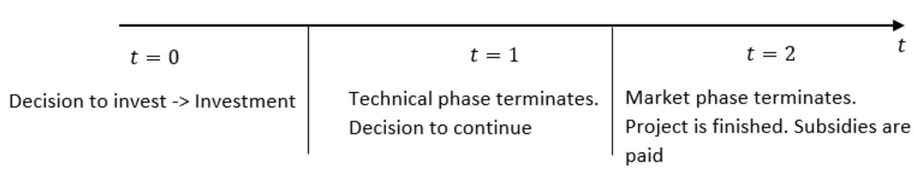
  $s_1(x)$ - the transfer from principal to the agent in case of failure in technological development;

  $\gamma(x, R) \in \{0, 1\}$ - the allowance to the agent to commercialize the product after the successful development of the technology;

  $s_2(x, R)$ - the transfer from principal to the agent if the product was commercialized;

  $t(x, R)$ - the transfer from principal to the agent if the product was not commercialized.

As we mentioned before we are looking to the class of deterministic mechanisms, which explains the fact that we restrict $H(x)$ and $\gamma(x)$ to be either 0 or 1. The variables in the contract, which are related to the development of technology depend only on the agent's type, while those related to the commercialization of the product may depend also on the generated revenue, which is observed ex-post. To better understand the logic of this, we introduce below the timing of the two considered problems.



In both cases the agent learns the future revenue only, when the future revenue is realized. Thus, there is no way to condition the initial recommendation to invest and compensation paid in case of technical failure on the market outcome. However, the other variables can be conditioned, so we write as a function of two variables.

The function $\phi(R)$, which relates agent's and principal's revenues is deterministic. This means, that the principal by observing its own benefit observes exactly the revenue of the agent (otherwise we would be in the situation of two dimensional adverse selection, which is outside the scope of this paper).

We should introduce also some assumptions that are crucial for the model:

- The mechanism is proposed by the principal after the type (the "quality" of the project) is known by the agent. The full commitment is present.
- $\int_{R-}^{R+} Rd\Psi(R) - F > 0$ - this formalises the idea that at least some high types are willing to undertake the project on its own.
- $R + \phi(R)$ is increasing, which means that at least total benefit is increasing with the private profit of the agent.

2.2 Two problems

In this section the two problems (without and with moral hazard) are presented. The first we will consider pure adverse selection problem. Let's firstly add some more notations to simplify the formulation of the problem. Denote as

$$\bar{R^T}(p,\hat{p}) = H(\hat{p})\Bigg\{p\int_{R^-}^{R^+}\Bigg(\gamma(\hat{p},R)[R+s_2(\hat{p},R)] + (1-\gamma(\hat{p},R))t(\hat{p},R)\Bigg)dG(R)$$
$$-(F-s_1(\hat{p}))(1-p) \geq 0$$

which is the total expected revenue (given the subsidies), which receives the agent of type p and report the type to be $\hat{p}$.

In the formulation of the problem we already use revelation principle, and so we consider only direct truthful mechanisms.

Denote as $\bar{R^T}(p) = \bar{R^T}(p,\hat{p})$, which is the revenue of the agents, which reveals its type truthfully.

$$\max_{M(x,R)}\int_0^1 H(x)\Bigg\{x\int_{R^-}^{R^+}\Bigg(\gamma(x,R)[\phi(R)-s_2(x,R)]-(1-\gamma(x,R))t(x,R)\Bigg)dG(R)$$
$$- s_1(x)(1-x)\Bigg\}d\Psi(x) \quad (1)$$

subject to

$$\bar{R^T}(p) \geq \max\{0, p\int_{R^-}^{R^+} RdG(R) - F\} \qquad \forall p \in [0,1] \tag{2}$$

$$\gamma(R)[R + s_2(R)] + (1-\gamma(x,R))t(x,R) \geq 0 \tag{3}$$

$$\bar{R^T}(p) \geq \bar{R^T}(p,\hat{p}) \tag{4}$$

$$s_1(x) \geq 0 \tag{5}$$

$$s_2(x,R) \geq 0 \tag{6}$$

$$t(x,R) \geq 0 \tag{7}$$

$$H(x) \in \{0,1\} \tag{8}$$

$$\gamma(x,R) \in \{0,1\} \tag{9}$$

Constraint (2) is ex-ante Individual Rationality constraint. Indeed, the agent will not be willing to start the project if the expected revenue is negative and will not take the proposed contract if it's worse for the agent than Business as Usual situation. Constraint (3) is interim individual rationality constraint, which means that the agent will not commercialize the project, if she expects negative payoff.

Constraint (4) is an incentive compatibility constraint, which says that the agent should prefer the contract related to his true type than to any other type (remember, we are looking for direct and revealing mechanism).

Constraints 5-7 are non-negativity constraints on transfers, which we explained earlier. Constraints 8 and 9 restrict the set of possible mechanisms to only deterministic ones, i.e. those in which the contract proposed by the principal includes actions only of type {invest, not invest} and {continue, stop}, but does not include mixed actions of type "invest with probability x".

If we introduce moral hazard the objective function as well as the majority of constraints remain the same. The only difference in the formulation appears in the interim individual rationality constraint. Indeed, given that the success of technology is not any more observed by the principal, the agent,observing negative revenue in the future may have incentives to report failure to the principal and receive compensation $s_1(x)$. That means that the principal need to give more to the agent during the market stage of the project to stimulate the truthful revelation of the outcome of technological development. The new constraint is the following:

$$\gamma(R)[R+s_2(R)]+(1-\gamma(x,R))t(x,R) \geq s_1(x) \qquad \forall R \in [R^-, R^+] \quad \text{(3bis)}$$

Thus, the pure adverse selection problem is characterized by equation (1) subject to constraints (2)-(9). The problem with additional moral hazard resulting from non-observability is summarized by equation (1) subject to constraints (2),(3bis), (4)-(9).

Before going to the solution of those two problems, we would like to introduce the solution to the simpler one-stage problem, in which only the technology is developed and the outcome is realized fully after and is known from the beginning. We will see later that the solution of this one-stage problem will be the part of the more complicated two-stage problem, which is considered in this paper. So the next section will discuss this question.

3 One-stage model

The solution to the one-stage model is presented in Meunier & Ponssard (2019), where the model with technical phase only is analysed. The payoffs are assumed to be deterministic, publicly known from the beginning. Let's denote them R and V in this section for simplicity. They show firstly that any strategy will consists of the $H(x)$, which determines the threshold, i.e. it is equal to 1 for all the projects with success probability higher than some identified value. Then, the authors divide the problem in two stages: firstly, for any given threshold the cost-minimizing subsidy scheme is defined, and then, given that the optimal threshold (which becomes the only variable left) is determined through the maximization of the net principal's payoff. The paper provides two important results concerning the optimal subsidizing mechanism, which are relevant to the models considered in the current paper. The first one states the optimal pair of subsidies:

Proposition 1 *Whatever the targeted threshold type p, the sceme that minimizes the expected cost of subsidies is to set $s_2 = 0$ and $s_1 = (F - pR)/(1 - p) > 0$.*

Note that this result holds only under the positivity constraints on subsidies, which we assumed at the beginning. Without those constraints the paper shows that it is possible to reach first-best with another pair of subsidies. The second important result describes the optimal outcome of the second step, i.e. the determination of optimal probability threshold;

Proposition 2 *The first best is not achieved and the optimal threshold is as following:*

- *If*
$$b \le \frac{R^3}{F(R-F)} \int_{F/R}^{1} (1-x) dG(x)$$
then $p^{SB} = p^{FB}$ *and* $s_1 = 0$

- *Otherwise* $s_2 > 0$ *as stated in the previous result,* $p^{FB} \le p^{SB} \le p^{BAU}$ *and is determined from the following equation:*
$$p^{SB} = p^{FB} + \frac{1}{g(p^{SB})} \frac{R-F}{b+R} \int_{p^{SB}}^{1} \frac{1-\theta}{(1-\theta^{SB})^2} dF$$

4 Solution of two-stage problem without moral-hazard

From the previous section we know the solution of the problem with only technical stage. In this section we will show that two-stage problem can be solved backwards, and the solution for the technical stage will be as if there is no market phase and payoffs are replaced by its ex-ante expectations given the optimal solution for the market stage. But, firstly, we discuss briefly the first-best solution, i.e. the best principal could achieve, if the information would be symmetric.

4.1 First-best solution

First-best solution is determined by the two functions $H^{FB}(x)$ and $\gamma^{FB}(x, R)$, which means that it shows, which projects are undertaken and which of them are commercialized in case of technological success.

The problem ex-ante can be formulated as following:

$$\max_{H^{FB}(x), \gamma^{FB}(x,R)} \int_0^1 H^{FB}(x) \left\{ x\gamma^{FB}(x,R) \int_{R^-}^{R^+} (R+\phi(R))dR - (1-x)F \right\} \quad (10)$$

Claim 1 *The first-best solution of the two-stage problem is:*

$$H^{FB}(p) = \begin{cases} 1, \textit{if} \qquad x\int_{R^-}^{R^+}(R+\phi(R))dR-(1-x)F \\ 0, \textit{otherwise.} \end{cases} \tag{11}$$

$$\gamma^{FB}(p,R) = \begin{cases} 1, \textit{if} \qquad R+\phi(R)\geq 0 \\ 0, \textit{otherwise.} \end{cases} \tag{12}$$

The solution presented in the claim 1 is very simple to interpret. It says that the principal should undertake all the project, which give non-negative expected total payoff and then at the interim stage to continue those of them, which generate non-negative real total payoff (not expected at this stage).

4.2 Second-best solution

Now, having in mind the one-stage model and the first-best solution presented above, we can turn to the second-best solution of our initial problem. Our tactic is to solve the problem backwards: start with interim stage and determine the optimal mechanism for the market phase, and then apply the one-stage model to complete the mechanism with the technical phase transfers. After that we will proof that this mechanism is also ex-ante optimal, so it is time-consistent, and thus, it is the best mechanism from the class of deterministic mechanisms. First of all, notice that the principal's objective can be rewritten in the following way:

$$\int_0^1 H(x)\left\{x\int_{R^-}^{R^+}\gamma(x,R)[R+\phi(R)]dR-F-\bar{R}^T(x)\right\}dx \tag{13}$$

Thus, the principal would like to make the profit of each type agent as small as possible given that constraints are satisfied. From this formulation, by comparing maximization problems (10) and (13) it is easy to see that first-best is not achievable. Indeed, if $\int_{R^-}^{R^+} RdR \geq F$, which is true by assumption, then some high types will receive at least their business-as-usual profits, by constraint (2), which is non-negative. So, the benefit of the principal will be lower.

Now we concentrate on the interim stage and assume that principal maximizes there its net benefit from the market stage of the project. That means that at the interim stage the principal solves the following problem.

$$\max_{\gamma(x,R),s_2(x,R),t(x,R)} \int_{R^-}^{R^+} \left(\gamma(x,R)[\phi(R) - s_2(x,R)] - (1-\gamma(x,R))t(x,R)\right) dR \tag{14}$$

subject to constraints (3), (5), (6) and (9). The solution to this problem is quite simple. Evidently there is no sense in setting $t(x) > 0$, so in the optimum $t(x) = 0$. Then, we choose the minimum $s_2(x)$ possible for the agents, for which $\gamma(x,R) = 1$. Notice also that at the interim stage there is no sense to condition parameters of the mechanism on the type of the agent, which is not relevant already. The minimum $s_2(x)$, which should satisfy constraint (3,) is $-\min\{R, 0\}$. And the projects, which should be continued are those, for which $R + \phi(R) \geq 0$, which is the same threshold as in the first-best solution.

To recover the full solution we now need to apply the one-stage model to the average market phase payoffs given $(\gamma(x,R),\ s_2(x),\ t(x))$ described above. The corresponding values are:

$$\bar{R} = \int_0^{R^+} Rg(R)dR \tag{15}$$

$$\bar{V} = \int_{R_{FB}}^0 R + \phi(R)g(R)dR + \int_0^{R^+} \phi(R)g(R)dR \tag{16}$$

where R_{FB} is the threshold value for R such that $R + \phi(R) = 0$, which is the fixed point of function $-phi(x)$. Now we can formulate the optimal subsidy for two-stage model in the case of pure adverse selection.

Proposition 3 *Under pure adverse selection the optimal mechanism for the principal is which maximizes its net benefit, for any targeted value of* p_{SB} *is:*

$$\bar{\gamma}(x,R) = \begin{cases} 1, & \textit{if } R \geq R_{FB} \\ 0, & \textit{oterwise} \end{cases} \tag{17}$$

$$\bar{s_2}(x,R) = \begin{cases} \max\{0, -R\}, & \textit{if } R \geq R_{FB} \\ 0, & \textit{oterwise} \end{cases} \tag{18}$$

$\bar{s_1}(x) = \bar{s_1}$ *for all* $x \in [0,1]$ *and is equal to:*

$$\bar{s_1} = (F - \bar{p}_{SB}\bar{R})/(1 - \bar{p}_{SB}) \tag{19}$$

$$\bar{H}(x) = \begin{cases} 1, & \textit{if } x \geq \bar{p}_{SB} \\ 0, & \textit{otherwise} \end{cases} \tag{20}$$

and $\bar{p}_{SB}$ *defined by equation (4) putting* $\bar{R}$ *and* $\bar{V}$ *instead of* R *and* V *respectively.*

Proof. We have shown already that this mechanism can be obtained by backward induction. However, we need to check that this mechanism is time-consistent.

Let's consider the possible form of the expected total revenue of the agent ($R^T(x)$), i.e. the feasible set based on incentive constraints. Denote by $R(x)$ the expected market profit of the agent. Consider two agents of types x and y such that $H(x) = H(y) = 1$. Then the incentive constraint (3) can be rewritten in the following way:

$$xR(x) + (1-x)s_1(x) \geq xR(y) + (1-x)s(y)$$

or equivalently as

$$R^T(x) \geq R^T(y) + (x-y)R(y) + (y-x)s(y) \tag{21}$$

Evidently the same is true for y. Then summarizing constraints for x and y we get:

$$R(x) - s(x) \geq R(y) - s(y) \qquad \forall x > y \tag{22}$$

Thus $R(x) - s(x)$ should be increasing. This result is very intuitive. Indeed, the higher types value more the revenue from commercialization, since they have higher probability of successful technology. On the contrary, lower types prefer to have higher reward in case of technological failure since for them this event is more likely.

From the equation (21) we can get

$$\frac{R^T(x) - R^T(y)}{x-y} \geq R(y) - s(y)$$

And from the symmetric equation for y we get:

$$\frac{R^T(x) - R^T(y)}{x-y} \leq R(x) - s(x)$$

We can be sure that the profit function does not have any jumps in it. Indeed, if there is a jump, than we can just decrease all the profits on the right by the amount of the jump and constraints will still be satisfied. Then the only non-continuities possible in the profit function are the kinks. Thus, we can say that, by pushing $y \to x$ in every point of differentiability it should be true that:

$$R^{T'}(x) = R(x) - s(x)$$

This means that the profit function of the firm is not concave (this follows from the equation (22)). TO see the intuition of why this is true, let's look on the graphical representation of the total expected profit function.

Figure 1 demonstrates 2 possible forms of the agent's total expected profit function: the left part demonstrates the concave case and the right side the convex case. Each point on the profit line can be represented in many possible ways by an item $(s_1(x), R(x))$, which are represented graphically by the straight line intersecting the profit curve at the point $p = x$ (for any x). Naturally there are a lot of ways to chose that line (which means to chose the subsidies to reach this particular point on the profit curve), however not all of them are incentive compatible. Such, the line demonstrated on the left-hand side of the Figure 1 is not achieved by incentive compatible subsidies, since type y then also prefers the item of x (gives strictly higher profit for him than $R^T(y)$ given by the profit curve). Then, it is clear that for the profit curve to be able to be completed by an incentive compatible menu, it should be possible to build a straight line at each point of the curve, shich is below the curve. Clearly it is possible only in two cases. Firstly, it is possible, when the profit curve is convex as on the right-and side of the graph. Then, incentive compatible menu is represented by the tangents to the profit curve. The seconf possibility is pooling solution, when $R(x)$ and $s_1(x)$ are constant for all types x. Then the profit curve is a straight line, and such mechanism is also incentive compatible.

The discussion above lead us to the set of incentive compatible mechanisms. However, we should restrict it also using individual rationality constraints, more precisely the $s_1 \geq 0$ and $R^T(x) \geq x\bar{R} - F$. Now we are going to show that pooling mechanism is the one (among incentive compatible and satisfying IR's) that generates the highest net benefit for the principal. To do that we will start with the case, when there is minimum $\bar{R}$ fixed. In our two-stage settings it is not true, since we can reach any value of R by changing $\gamma(x, R)$ even keeping the transfers $s_2(x)$ and $t(x)$ positive. Indeed, we can set the threshold to be greater than 0, which will reduce the market stage profit of the agent (but also it reduces the market stage benefit of the principal).

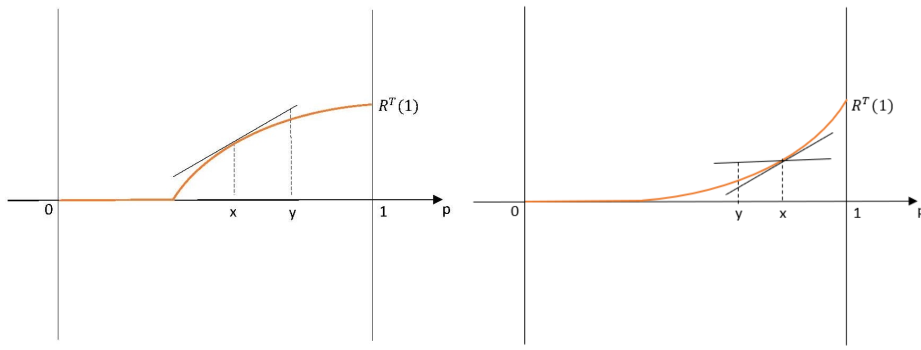


Figure 1: Concave and convex total expected profit of the agent

However, if there is some fixed minimum for $\bar{R}$ than we can not set it lower because of positivity constraint for transfers. This case is equivalent to the case of one-stage model, however, we think it is crucial to consider it there to make a logical step to the more complicated case.

Assume any chosen threshold of probabilities given by $H(x)$ and denote it by $\bar{p}$. To find the optimal subsidy, we need to find the optimal agent's total expected curve among incentive compatible, such that the menu induced by this curve, maximizes the principal's revenue for given $\bar{p}$.

Figure 2 shows different expected overall profit curves we could possibly consider. The red curve represents the best for the principal pulling scheme, which rewards only failure. The blue curve represents the schemes, which generate the strictly lower profit for at least some types of the agents than in the case of best pulling algorithm (on the picture it is for all of them, but it could intersect the straight line also). However, than those types would necessarily receive negative subsidy in case of success, tangent will be lower than $\bar{R} - F$, thus those mechanisms does not satisfy positivity constraints on transfers.

The green curve on the Figure 2 represents the profit curves obtained from the mechanisms that satisfy all the constraints, and so they lie higher than the profit curve of pooling mechanism. Since the overall payoff with fixed $\bar{R}$ is fixed for each x, giving more profit to the firms generate less benefit for the principal, and thus pooling mechanism is strictly preferred by principal to any other menu for any given $\bar{p}$. Thus, the problem reduces to the choice of $\bar{p}$ and then pooling mechanism producing this $\bar{p}$ is the optimal solution.

Note that we concentrated on the differentiable convex curves in the il-

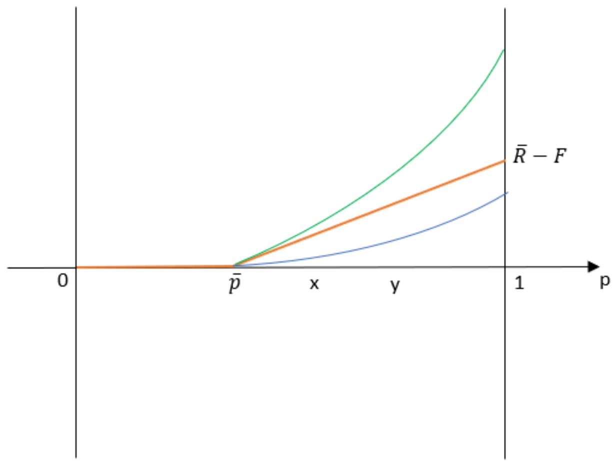


Figure 2: Optimal menu, fixed $\bar{R}$

lustrations. However, clearly the same logic applies to the piecewise straight lines with kinks.

Evidently the solution presented above with $\bar{R}$ given by equation (15) is also the only possibility, if there is a commitment problem from the principal or the agent, i.e. they can commit at the beginning to stick to the chosen plan after the technical stage outcome is realized. Indeed, we can decrease expected market stage profit of the agent below $\bar{R}$ only using $\gamma(x, R)$ by making some firms with positive R to stop the project. Such schemes are possible only under strong commitment, since for both, principal and agent, when they face technical success, they prefer to switch to γ given by equation (17).

Now we are ready to discuss the more general case of two-stage adverse selection model with full commitment. Now the mechanisms corresponding to the green line on the Figure 2 can not be excluded as non-satisfying constraints. However, according to the discussion above R can be set below $\bar{R}$ given by equation (15) only by increasing the threshold (represented by $\gamma(x, R)$). Evidently, such increase in $\gamma x, R$ reduces also the market stage benefit of the principal. Also, note that if by changing threshold of R we reduce the agent's market profit steadily, there is a drop in the market benefit of principal, which is equal to the principal's benefit of the projects between R^{FB} and 0. It could be compensated by lower average failure transfer $s_1(x)$.

Figure 3 shows the possible mechanism, which may (or may not) outperform the best pooling solution, which is represented by the blue line. The dotted line on the right represents the line $\bar{R}x - F$, which is business as usual profit, which means that blue light should have the slope not exceeding $\bar{R}$ (otherwise constraint (2) is not satisfied). On the figure we put the same threshold $\bar{p}$ for the "blue" mechanism as for the pooling mechanism, and we put the profit of the firm of type 1 to be $\bar{R} - F$, however, we should check all the set of convex mechanisms to be sure that there is none of them can outperform the pooling one. Note that we can not use the same tactics as before by trying to show that for any chosen threshold p polling mechanism is the best. In general it can be not true depending on the distribution of p. However, we should employ there the fact that the "red" mechanism is the principal's benefit maximizer among all pooling mechanisms.

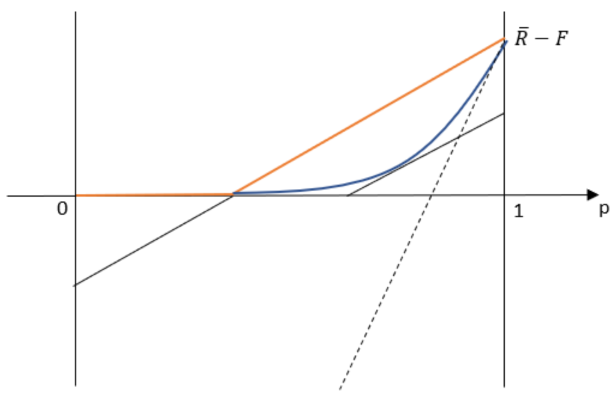


Figure 3: Minimal profit not fixed

Let's start by considering the class of mechanisms generating the profit curve consisting of two straight parts with a kink as presented on the Figure 4. We would like to show that the pooling mechanism is always preferred to such two-part mechanism.

Assume the random mechanism, $(y, \bar{R} - F)$, which gives a threshold probability y and $R(1) = \bar{R}$ (for simplicity we will call this mechanism y-mechanism). So the principal's market benefit is $\bar{V}$. Denote corresponding failure subsidy as s_y. We would like to know, with which another mechanism it is better to combine the y-mechanism to maximize principal's overall net benefit. Denote the resulting threshold as a and the subsidy of additional part as s_a. The two examples are given on the Figure 4. We see that for additional left part the market revenue can be lower than $\bar{R}$. Denote it as

R_a and corresponding market benefit as V_a. Then, in order to find the best additional part we should solve

$$\max_{a,b} \int_y^1 [\bar{V}x - (1-x)s_y]dG(x) + \int_a^y [V_a x - (1-x)s_a]dG(x)$$

$$+ \int_y^b [(V_a - \bar{V})x - (1-x)s_a + (1-x)s_y]dG(x)$$

Fix a and denote the best b for each a. With the increase in b the subsidy s_a evidently decreases, as well as decreases the difference between V_a and $\hat{V}$. Then, clearly the net benefit is maximized by setting $b = 1$ for each a. So, we got that the pooling solution is better than any two-part solution with the same threshold probability. And since the best pooling solution maximizes among all of them, it is also better than any two-part mechanism.

Above we automatically chosen as the random mechanism the one which assigns exactly $\bar{R} - F$ to type 1. However, the result is without loss of generality. Clearly, if $R(1) < \bar{R}$, we can always improve it by replacing this part by the one, which assigns $\bar{R}$ to type 1 and has the same probability threshold, since it generates higher principal's market benefit and requires smaller subsidy. If we chose the mechanism with $R(1) > \bar{R}$ it is also worse then the mechanism with the same threshold, but $R(1) = \bar{R}$ since it has greater share of agent's profit in the fixed welfare.

Thus, we have shown above that the pooling mechanism is better than any two-part mechanism. Importantly, we have shown that this is true even for the random threshold, not necessarily the one generated by the best pooling mechanism.

Consider now the three-part mechanism as illustrated on the Figure 5. Consider two left parts. Assume that we maximize this left part as we did before. Now take instead of $\bar{R}$ and $R_a(1)$ the value of expected profit at point y, which is $R(y)$. Then, by similar calculations as earlier we get that left part is maximized, if there is no kink. So three-part mechanism converges to two-part mechanism and we now that they all are inferior to the best pooling mechanism.

We can increase the number of kinks and, by applying the same logic, we will get that the pooling mechanism is always better. But any convex continuous profit curve can be approximated by the partwiese straight curve with kinks. Thus, the best pooling mechanism is always preferred by the principal, which finalizes the proof. ■

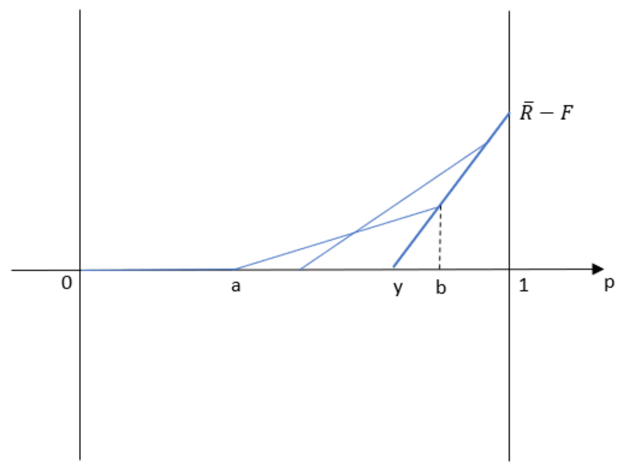


Figure 4: Two-part mechanisms

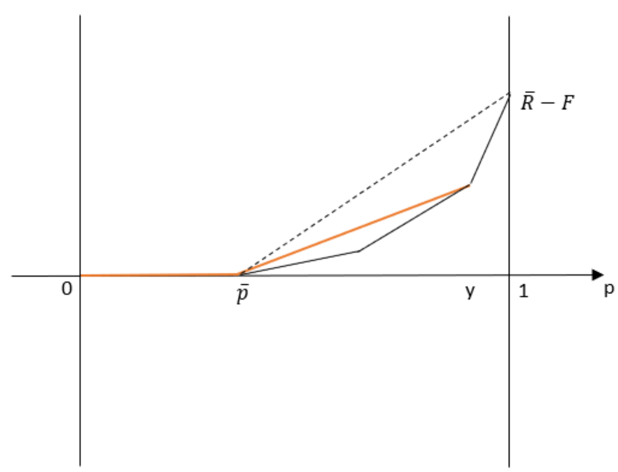


Figure 5: Two-part mechanisms

4.3 Small private payoff

One of the main assumption is that at least some high types are willing to undertake the project even the help of the principal, which was summarized by condition $\int_{R^-}^{R^+} RdG(R) - F \geq 0$. However, we can be interested also in the case, when costs are high enough to make the project too expensive for the agent alone. So assume in this section that $\int_{R^-}^{R^+} RdG(R) - F < 0$ and $\int_{R^-}^{R^+} R + \phi(R)dG(R) - F < 0$. The structure of first-bes solution is not changed then and, we can show that in that case, the first-best solution is achievable by the principal.

Proposition 4 *If the project is too expensive for the agent of any type to proceed on its own, then the first-best solution is achievable for the principal with the following mechanism:*

$$\gamma(x, R) = \begin{cases} 1, & \textit{if } R \geq R_{FB} \\ 0, & \textit{oterwise} \end{cases} \tag{23}$$

$$s_2 = \begin{cases} F - \bar{R} + \max\{0, -R\}, & \textit{if } R \geq R_{FB} \\ F - \bar{R}, & \textit{oterwise} \end{cases} \tag{24}$$

$$t(x) = F - \bar{R} \tag{25}$$

$$s_1 = F \tag{26}$$

$$H(x) = \begin{cases} 1, & \textit{if } x \geq p_{FB} \\ 0, & \textit{otherwise} \end{cases} \tag{27}$$

and $p_{SB} = p_{FB}$

It is easy to check that such mechanism leads exactly to the first-best solution and, thus, is the optimal one for this problem.

5 Adverse selection and moral hazard combined

Now we will study the second problem, in which moral hazard appears in the form of possibility to hide the technical success. We can tackle the problem backward in the same way as in the previous section. However, we should emphasize the crucial difference, which arises from the moral hazard. Now the part of mechanism related to the technological development affects directly the incentives of the agent during the market stage through constraint (3), where $s_1(x)$ appears now at the right-hand side.

Then, considering the interim stage, the minimum subsidy, which is needed to be paid to the agent for continue is now $\max\{s_1(x) - R, 0\}$, which increase the subsidy needed for each R. However, $s_2(x)$ is not enough, since the constraint should be satisfied also for the projects, which are not continued. So we need to set also $t(x) > 0$ and the minimum possible value is $t(x) = s_1(x)$. Then solving problem (16) we can see that equation for $\gamma(x, R)$, which determines the threshold should not change. Then, the expected market profit of the agent and principal's benefit are now:

$$\tilde{R}(x) = \bar{R} + \delta(s_1(x)) \tag{28}$$

$$\tilde{V}(x) = \bar{V} - \delta(s_1(x)) \tag{29}$$

where

$$\delta(s_1(x)) = \int_{R^-}^{0} s_1(x) d\Psi(R) + \int_{0}^{s_1(x)} s_1(x) - R d\Psi(R) \tag{30}$$

is the additional gains (losses) caused for the agent (principal) by the presence of moral hazard. However, the usage of one-stage model is not possible for the same reason of dependence of agent's incentives and payoffs of the market stage on the technical failure compensation. However, we can use the market phase part of mechanism to reduce the number of variables in maximization problem.

Proposition 5 *Under joint adverse selection and moral hazard the optimal mechanism for the principal is which maximizes its net benefit is:*

$$ga\tilde{m}ma(x, R) = \begin{cases} 1, \textit{if} \qquad R \geq R_{FB} \\ 0, \textit{otherwise} \end{cases} \tag{31}$$

$$\tilde{s_2}(x, R) = \begin{cases} \max\{0, \tilde{s_1} - R\}, \textit{if} \qquad R \geq R_{FB} \\ 0, \textit{otherwise} \end{cases} \tag{32}$$

$$\tilde{t}(x) = \tilde{s_1} \tag{33}$$

$$\tilde{H}(x) = \begin{cases} 0, \textit{if} \qquad x \geq p_{\tilde{SB}} \\ 1, \textit{otherwise} \end{cases} \tag{34}$$

and $s_1(x) = \tilde{s_1}$ *is equal for all* x *and is determined as following:*

$$\tilde{s_1} = arg\max_{s_1} \int_{p_{\tilde{S}B}}^{1} \left[x(\bar{V} + \delta(s_1)) - (1-x)s_1 \right] dG(x) \tag{35}$$

where

$$p_{\tilde{S}B} = \frac{F - \tilde{s_1}}{\tilde{R} - \tilde{s_1}} \tag{36}$$

Proof. As in the case of pure adverse selection, the mechanism obtained by backward induction is the best if there is no commitment in the model. However, otherwise we should check that this solution is ex-ante optimal.

We can apply the same logic as in the proof of Proposition 4 to show that the optimal pooling mechanism should be preferred to the convex mechanism. Indeed, we can reach the analogical equation as (23) and have the similar result. However, there is still question, if the pooling mechanism, which gives the minimal possible transfer during the market stage, is the optimal one. Figure 6 demonstrates the profit lines generated by different possible pooling mechanisms, which lead to the same threshold $p_{\tilde{S}B}$. The red line corresponds to the pooling mechanism described by the Proposition 5. The green line is a mechanism, giving higher failure subsidy, which is compensated by reduction in market transfers (through the change in $\gamma(x, R)$). The blue line shows a mechanism, which proposes smaller failure compensation, but also an additional transfer during market stage on top of the minimal transfer required. First of all, it is clear that the transfer represented by green line are not optimal, since they results in both, higher failure subsidy and, a reduction in the market benefit. So, the only possibility to improve the proposed pooling mechanism is by considering mechanisms with lower failure subsidy (as one represented by the blue line). To show this let's write the gains, which we get from "blue" mechanism compare to the "red" mechanism. Denote by δ_b and δ_r the change in principal's benefit caused by "blue" and "red" mechanisms correspondingly as defined by equation (29). Denote by s_{1b} and s_{1r} corresponding failure transfers. Then the gain from "blue" mechanism is:

$$\int_{\tilde{p}}^{1} [x(\delta_r - \delta_r) - (1-x)(s_{1b} - s_{1a})] dG(x)$$

From the definition of the $\tilde{p}$ we know that:

$$\delta_r = \frac{F - (1-\tilde{p})s_{1r} - \bar{R}\tilde{p}}{\tilde{p}}$$

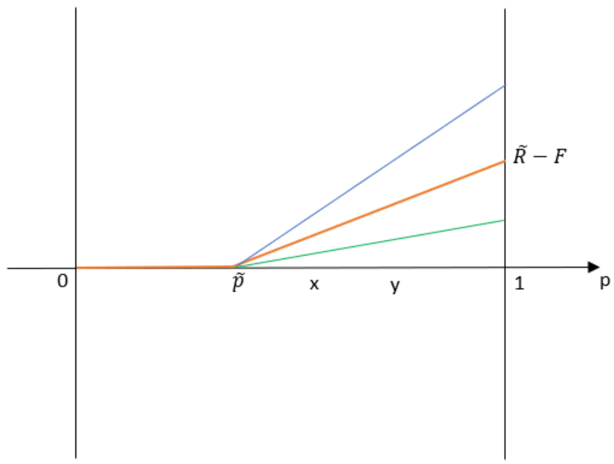


Figure 6: Different mechanisms resulting in the same threshold $\tilde{p}$

and

$$\delta_b = \frac{F - (1-\tilde{p})s_{1b} - \bar{R}\tilde{p}}{\tilde{p}}$$

Putting this into initial equation we get:

$$\int_{\tilde{p}}^{1} (s_{1b} - s_{1r})((1-\tilde{p})\frac{x}{\tilde{p}} - (1-x))dG(x)$$

Clearly, $s_{1b} < s_{1r}$. Since $x \geq \tilde{p}$ for all x, it is true that $1 - \tilde{p} \geq 1 - x$ and $\frac{x}{\tilde{p}} > 1$. Then the equation under integration is negative for all x. Then the total gain is also negative. That means, that any "blue" mechanism is worse than "red" mechanism, which finalizes our proof. ■

5.1 Low private payoff with moral hazard

This section is analogous to the section 4.3. We would like to relax the 2nd assumption and would like to assume that the project can be not beneficial even for the highest type. The condition for this to become important should be corrected using $\gamma(x, R)$, $s_2(x)$ and $t(x)$ specified before. So in this section we call the low payoff case, the case, in which $\tilde{R} - F < 0$. In that case the mechanism presented in Proposition 5 is not enough to stimulate the agent to undertake the project. However, compare to the pure adverse selection case, or one-stage model, in that case it is not possible to reach first-best solution. Indeed, the strategy of the previous sections included setting s_1 as close is possible to F, while keeping the fixed part of s_2 close to $-\bar{R}$. Now it is not possible, since $\bar{R}$ is bounded below by $s_1(x)$, so it is not possible to make the firm to have zero profit.

6 Comparison of the solutions

In this section we will discuss, how the outcome is changed after introducing moral hazard. The first step is to analyse how the welfare, agent's total revenue and principal's total revenue change when going from one information structure to another.

The simplest question to answer here is the change in the principal's total revenue V^T. The adverse selection problem solves equation (1) (or (13)) subject to constraints (2)-(9), while the problem with moral hazard is summarized by equation (1) ((13)) subject to constraints (2),(3bis), (4)-(9). So, the problems are the same except the difference between the constraints (3) and (3bis). Clearly, constraint (3bis) is strengthening the constraint (3) in all the cases except if optimal $s_1(x)$ in the pure adverse selection problem is 0. In all other cases evidently it is not possible to duplicate the solution of pure adverse selection problem in case of moral hazard. Thus, the value of the optimization function is always lower in the latter case.

Now, to make an exact statement we need to find, if $s_1(x) = 0$ can be an optimal solution for the initial problem in some cases. It is optimal only if the optimal probability threshold p_{SB} is such that $p_{SB}\bar{R} - F \geq 0$, which is Business as Usual threshold or higher. Clearly, it is not possible case, that the principal would like in the optimum to set the threshold higher than business as usual situation. Thus, the only case we need to check is exactly the case, in which $p_{SB} = p_{BAU}$. For this we can go back to the one-stage and see that, for low enough $\bar{V}$, i.e. the principal's payoff from the project, it is better not to intervien during the technical phase. However, the part of mechanism related to the commercialization will stay the same. Thus, for the low enough principal's payoff there is no difference between the two cases.

Proposition 6 *Given the optimality of solutions of both problems it is always true that:*

- $\tilde{V} \leq \bar{V}$ - *the value of information on technical outcome is always non negative and is strictly positive or $\bar{V}$ high enough;*

- $\tilde{V} \leq \bar{V}$ - *giving information on the technical outcome to the principal is welfare improving;*

- $\tilde{s_1} \leq \bar{s_1}$

7 Numerical illustration

In this section we would like to provide some illustration of the results above. We would like to see the percentage losses caused by presence of moral hazard and to check some hypotesis about how the parameters affect this difference presented in the section 6.

The main hypothesis is that what matters for the percentage of losses in principal's benefit and welfare is not the exact values of the parameters, but the relation between $\bar{b}$ and $\bar{R}$. This relation can be deduced from the inequality from Proposition 2. However, this relationship is difficult to work with. However, we can study the dependence of benefits losses on the percentage of $\bar{b}$ in total market payoff. To do this, we fix the market payoff ($\bar{R} + \bar{b}$) and increase the percentage of principal's benefit in total payoff. We proceed in two ways: firstly, by changing R^-, and then by changing R^+. Our hypothesis suggests that the structural relation between two model should depend only on the shares in total market payoff, but not on the value of R^- or R^+.

Figure 7 demonstrates the case of changes in R^-. On all the graphs the horizontal axis represents the percentage of $\bar{V}$ in total market payoff. Intuitively, the share of principal's total benefit in total payoff increases with the increase of corresponding market share in both models (graphs a) and b)). However, the percentage losses from moral hazard increase with the relative growth of $barV$. The both subsidies increase and, exactly as was shown in previous section, the transfer in case of the presence of moral hazard is lower. Finally, we see all thre threshold probabilities (including first best). The dynamics of two cases is the same. The first increase is driven by the decrease of $\bar{R}$ given that $\bar{V}$ is small enough for 0 failure transfer to be optimal. However, after the transfer becomes positive, the threshold decreases and converges to first-best as the share of principal's benefit increases. Figure 8 demonstrates the same results, but for the change in R^+. We can see that the results are very similar, which confirms our hypothesis that the

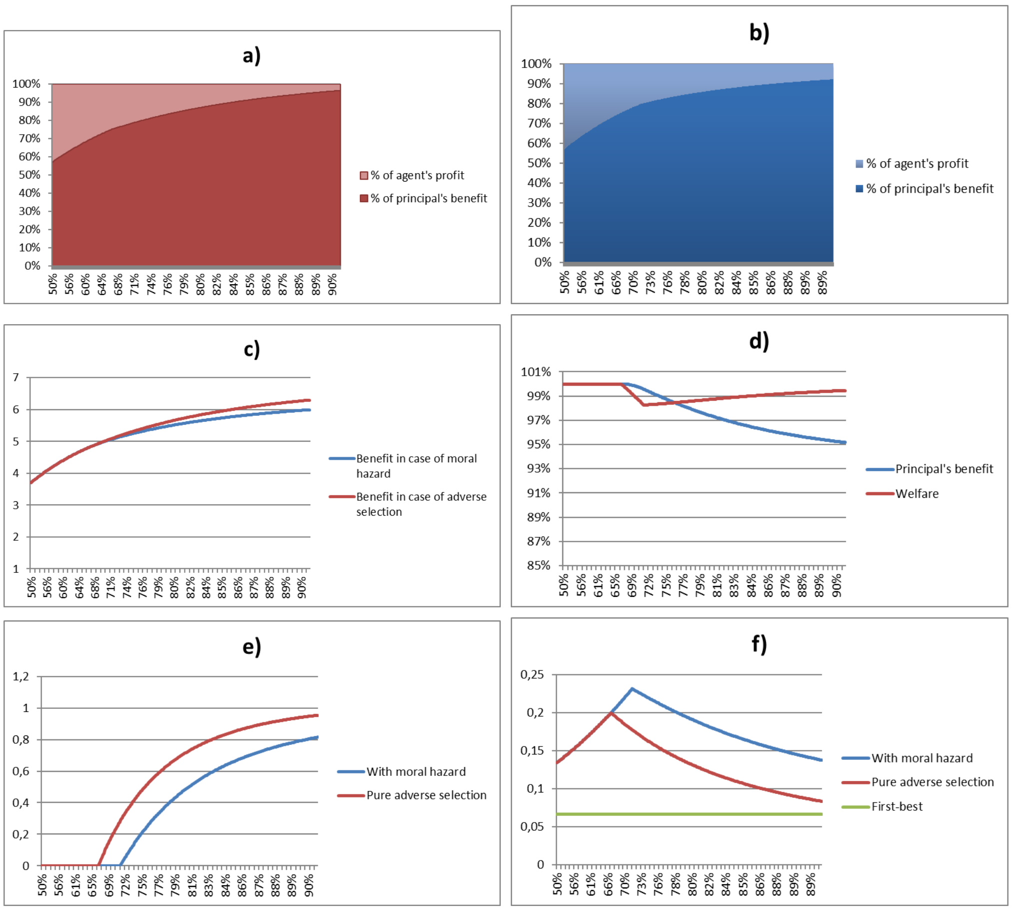


Figure 7: Change in R^{-}: a) Total payoff division - adverse selection; b) Total payoff division - moral hazard; c) total principal's benefit; d) Moral hazard payoffs as percentage of adverse selection case; e) Failure transfers; f) Threshold probability

expectations of market payoff matters for the difference of solution, but not the way those expectations are achieved. However,the ratio of agent's and principal's benefit as we discussed before does not perfectly fit to qualify the solution structure. To show this we compared the solutions of models with different parameters, but with the share of principal's benefit in the total market payoff constant. The results are presented on Figure9. We see that solution structure changes as the total market payoff (horizontal axis) increases. As was already mentioned above the perfect criteria for the solution would be the relation between $\bar{V}$ and $\bar{R}$ induced from Proposition 2.

8 Conclusion

We have modelled the two typical information structures, which a government can face in providing incentives for innovations. While the one model represented pure adverse selection problem, which proceeds in two stages, the other one added the moral hazard possibility between the two stages.

We have developed the optimal financing mechanism for the principal (governmental agency) to stimulate innovation under those two possible information structures. We have shown that the mechanism, which maximizes net social benefit for the principal among all deterministic mechanisms, is a single set of transfers - the same for all types, so the optimal solution is pooling.

We have shown also that in the presence of moral hazard the optimal mechanism always generates the benefit for the principal, which is at most as high as the benefit in the pure adverse selection case. The difference between the two benefits corresponds to the value for the principal of the technical outcome information and,thus, is the price the principal would be ready to pay for this information.

We have shown numerically that the relation between two models depends strongly on the relation between principal's market benefit and agent's market profit. Relatively low principal's benefit makes the two situations completely equivalent. However, the large principal's benefit may lead to large percentage difference in the quality of solutions (from the principal's point of view).

The problem studied is of high practical value, since it represents the question of the optimal way of financing green innovations, which is usually

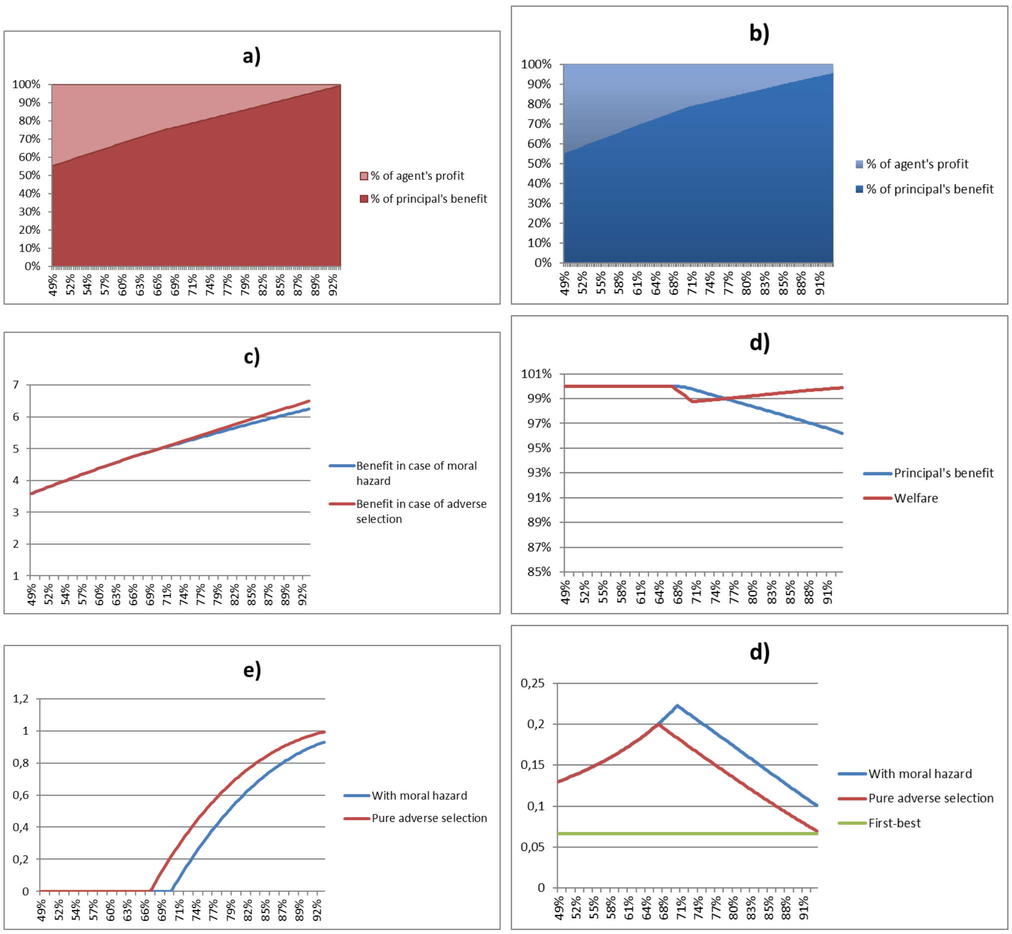


Figure 8: Change in R^+: a) Total payoff division - adverse selection; b) Total payoff division - moral hazard; c) total principal's benefit; d) Moral hazard payoffs as percentage of adverse selection case; e) Failure transfers; f) Threshold probability

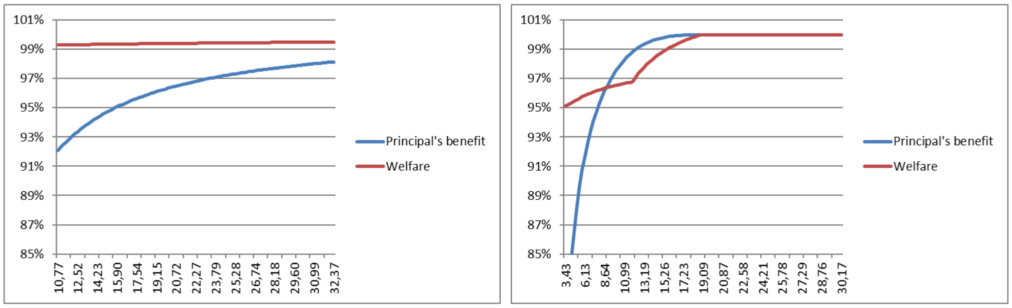


Figure 9: Moral hazard payoff as percentage of pure adverse selection case: left side - $\bar{V}/(\bar{V} + \bar{R} = 0,9$, right side - $\bar{V}/(\bar{V} + \bar{R} = 0,7$

different governmental structures. The solution provided represents the valuable benchmark, which can be used to optimize the currently-used financing practices. However, there are still some practical and theoretical issues we can address by exploring the model. The one of possible directions is to study the value of ex-post information on market outcome, which can be of practical interest in the case of costly auditing.

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Appendix A

A.1 Proof of Claim 1

To see that this solution is indeed first-best it's sufficient to show that any change in $H(x)$ or $\gamma(x,R)$ will lead to the decrease in the principal's revenue.

Note from equation (10) that for any chosen $\gamma(x,R)$ $H^F B(x)$ should be 1 if , and 0 otherwise. Then, the question is to maximize $\int_{R^-}^{R^+}(R+\phi(R)dG(R)$. First of all, note that, independent of the threshold for revenue given by $\gamma(x,R)$, if the threshold is on smaller than 0, the revenue of the agent remains unchanged. Indeed, if $R+\phi(R)>0$ and $R<0$ than the agent will be compensated up to 0 (because of participation constraint (3). If $R>0$ than the agent will just receive its payoff. If $\gamma(x,R)=0$ agent receives just 0. This can be graphically illustrated. Clearly, the joint payoff of the market phase is maximized with the threshold given by equation (12).

A.2 Proof of Proposition 6

Consider the two reduced optimization functions corresponding to the two models (given the optimal market transfers and the fact that $H(x)$ defines threshold). Denote the optimization function of pure adverse selection model as following:

$$V(s_1)=\int_{\bar{p}_SB}^{1}[x\bar{V}-(1-x)s_1]dG(x) \tag{37}$$

Note that $\bar{p}_{SB}$ is itself a function of s_1. Then the corresponding optimization function for the problem with moral hazard is:

$$V_{mh}(s_1)=\int_{\tilde{p}_{SB}}^{1}[x(\bar{V}-\delta(s_1))-(1-x)s_1]dG(x) \tag{38}$$

Assume that $\tilde{p}_{SB}<\bar{p}_{SB}$: Then the function $V_{mh}(s_1)$ can be rewritten as follows:

$$V_{mh}(s_1)=\int_{\tilde{p}_{SB}}^{\bar{p}_{SB}}[x(\bar{V}-\delta(s_1)-(1-x)s_1]dG(x)-\int_{\bar{p}_{SB}}^{1}x\delta(s_1)dG(x)+ \\ \int_{\bar{p}_SB}^{1}[x\bar{V}-(1-x)s_1]dG(x) \tag{39}$$

$$V_{mh}(s_1)=\int_{\tilde{p}_{SB}}^{1}[x(\bar{V}-(1-x)s_1]dG(x)-\int_{\tilde{p}_{SB}}^{1}x\delta(s_1)dG(x) \tag{40}$$

In this equation the second term is decreasing in s_1. Thus, only the first term can be increasing in s_1. Clearly for the same s_1 the effect of a change in s_1 on the first term of V_{mh} is smaller then the effect on $V(s_1)$ (equation 37). Indeed the decrease in $\bar{p}_{SB}$ is larger than in $\tilde{p}_{SB}$. Then, given that in optimal s_1 for adverse selection case, the derivative of $V(s_1)$ is 0, the derivative of $V_{mh}(s_1)$ can never be 0 for the same value of s_1. Moreover, it is for sure negative. So, the optimal failure transfer in case of moral hazard is lower than in pure adverse selection model.