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Output-based allocations in pollution markets with uncertainty and self-selection

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Abstract

We study pollution permit markets in which a fraction of permits are allocated to firms based on their output. Output-based allocations, which are receiving increasing attention in the design of carbon markets around the world (e.g., Europe, California, New Zealand), are shown to be optimal under demand and supply volatility despite the output distortions they may create. In a market that covers multiple sectors, the optimal design combines auctioned permits with output-based allocations that are specific to each sector and increasing in its volatility. When firms are better informed about the latter or must self select, the regulator resorts to some free (i.e., lump-sum) allocations to sort firms out.

JEL Classification: D24, L13, H23, L74

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1 Introduction

The implementation of emission trading must usually confront two contentious issues: how to initially allocate permits and how to introduce some flexibility in the system in response to market fluctuations. In this article we study output-based allocations (OBAs) as a way to introduce such flexibility. Under an OBA scheme a fraction of permits are allocated to firms based on their output while the remaining fraction is allocated through either auctioning or grandfathering. OBA schemes have been introduced and considered in a number of existing and proposed permit markets, most notably, the carbon markets in the EU, California, and New Zealand.[1] Because OBA schemes have the ability to directly affect firms' output decisions, they have received considerable attention to deal with leakage and market power problems. Our focus in this paper is different: to what extent can an OBA scheme handle market shocks better than a fixed allocation, that is, keep permit prices closer to (expected) marginal pollution damages?

We consider a permit-market design in which a fixed number of permits are auctioned off (or freely distributed in a lump-sum fashion to firms) and in addition a variable number of permits are issued proportionally to firms' output. At the time of market design —when the regulator must decide on both the number of permits to be auctioned off and the OBA rate— there is uncertainty about future market conditions (e.g., output demand). Such OBA market design helps introduce flexibility by indexing the total number of permits in the market to market conditions (i.e., firms's output) but at the cost of distorting output away from the socially optimal level.

An OBA scheme will inevitable present the regulator with this trade-off because the adjustment in the number of permits always comes at the cost of subsidizing production. We analyze this trade-off with two different settings, first in a single-sector setting where firms decide on output and abatement and second, in a multi-sector setting where firms only decide on output (emissions are equal to output). These two settings allow us to illustrate the two main misallocations that operate in an OBA scheme: the misallocation between output and abatement in a particular sector, and the output misallocation that may occur across sectors.

The cost and benefit associated to these misallocations are studied in detail in different sections of the article. We start in Section 2 by asking whether an OBA scheme would

[1] In Sweden, a refunded emission payments program based on firms' output was introduced in 1992 to control NOx emissions. It was made output-based to facilitate the industry acceptance to the regulation. For an evaluation of this scheme see Sterner and Isaksson (2006) and for a theoretical analysis of optimal refunding scheme with imperfect competition see Gersbach and Requate (2004).

ever be optimal for an industrial sector with a large number of firms. We establish that an OBA scheme should be considered whenever we observe a positive covariance between permit prices and output under a fixed permit allocation (Proposition 1). This is true even if the market design already considers a price stability mechanism such as price thresholds at which the regulator is ready to either sell or buy back permits to stabilize prices (following Roberts and Spence, 1976, and the California market). Whenever these price thresholds are optimally set, we establish (for linear demand and marginal costs) that introducing OBAs is indeed optimal (Proposition 2).

To understand why an OBA scheme can be welfare enhancing, suppose the regulator issues a fixed number of permits such that the expected permit price is equal to the marginal harm from pollution. If in addition, the regulator issues a few permits based on firms' output, these few OBA permits will lead to both more output (the subsidy effect) and more emissions relative to the fixed allocation. The increase in output is clearly inefficient but becomes of second order as the fraction of OBA permits goes to zero. Conversely, the increase in emissions can be a good or bad depending on whether the demand/supply shocks are positive or negative. Since the extra emissions that result from increasing the OBA rate in one unit is exactly equal to output as the OBA rate goes to zero,[2] if the correlation between output and permit prices is positive the increase in emissions will be larger when the permit price is above the marginal harm than when it is below, which results in a net welfare gain.

This positive result is only reinforced as we introduce several sectors. Because firms covered by a permit market are never identical, as they belong to different productive sectors or regions, in Section 3 we look at the optimal OBA design when there are multiple sectors subject to different shocks. Sector heterogeneity introduces a covariance between permit prices and sector output —which vary from sector to sector in magnitude and sometimes direction— making OBA always worth implementing. The optimal OBA scheme has, in addition to a fraction of auctioned permits, sectors subject to bigger shocks receiving higher OBA rates (Propositions 3 and 4).

One potential implementation problem with this multi-sector OBA scheme is that it discriminates across sectors creating perverse incentives for sectors to pretend they face bigger shocks than they actually do or to simply lobby for larger OBA rates.[3] So, even

[2] Recall that we are assuming perfect compliance, so total emissions are equal to some fixed amount of permits (which can be auctioned off or grandfathered) plus the OBA rate times output.

[3] This asymmetric information problem is different from the problems of asymmetric information and pollution control we find in the literature (e.g. Spulber, 1988; Montero, 2008). One reason is that the full-information OBA scheme is not first best; another is that the sorting condition required to separate sectors

if the regulator knows sectors well, it may be unfeasible for her to sort them out without relying on self selection. Fortunately, there is a simple way for the regulator to go around this selection problem while preserving the optimal OBA outcome: to use a fraction of the auctioned permits to construct menus of OBA rates and free (i.e., lump-sum) permits (Proposition 5). Sectors facing smaller shocks are ready to take lower OBA rates because they are compensated with a larger fraction of lump-sum permits. The "single-crossing property" that allows for this separation of sectors is that more volatile sectors are willing to pay more for a marginal increase in the OBA rate.

We are obviously not the first to be interested in permit market flexibility. Since the seminal work of Weitzman (1974) much has been written on the design of permit markets subject to demand and supply shocks. Weitzman (1974) anticipated that any regulatory design, whether is based on prices or quantities, is subject to errors in the presence of market fluctuations, so the policy design challenge is to keep those errors at a minimum. Roberts and Spence (1976) propose a hybrid permit scheme in which the regulator is ready to issue extra permits if the price hits a pre-determined ceiling and buy back permits if it reaches a floor.[4] Unold and Requate (2001) propose the use of options, the strike prices of which can used to approximate the environmental damage curve.[5]

More recently, Newell and Pizer (2008) and Branger and Quirion (2014) propose to index the allocation of permits to any exogenous variable, such as GDP, that could be correlated to shocks affecting permit prices (see also Ellerman and Wing, 2003; Jotzo and Pezzey, 2007; Quirion, 2005).[6] And in the specific context of the carbon market in the EU, there are proposals to introduce a "market stability reserve" from or to which permits could be withdrawn or add as the number of unused (i.e., banked) permits in the market reaches a critical level that could push permit prices either too high or too low (Kollenberg and Taschini, 2015).[7]

is endogenous to the regulatory design.

[4]Berglann (2012) also proposes a hybrid scheme in which firms trade pollution-permit shares and pay taxes depending on their emissions and number of shares they hold.

[5]In a similar spirit, Collinge and Oates (1982) propose a menu of numbered permits with different prices, Henry (1989) suggests that the regulator can intervene directly in the permit market.

[6]Within these papers some of them consider "intensity target" in which the regulator set the future ratio between emissions and GDP (e.g. Quirion, 2005). Intensity target in this sense is a particular case of indexing the cap to GDP, and should not be confused with regulations in which the regulator sets the quantity of emissions per unit of output produced in a sector. This second class of intensity targets is closer to OBA rules.

[7]A related issue concerns inter-temporal trade of permits as a way to smooth market shocks (Kling and Rubin, 1997; Ellerman and Montero, 2007). However we are not interested by such intertemporal smoothing of yearly variations but by flexibility of the overall cap over an entire phase of trading which can be over several years or sub-period, with banking and borrowing allowed.

There is also an increasing OBA literature. To start, OBA schemes have been proposed as an alternative, albeit inferior, to border tax adjustments to deal with leakage problems (Fischer and Fox, 2007; Quirion, 2009; Monjon and Quirion, 2011; Fischer and Fox, 2012; Meunier et al., 2014).[8] They have also been proposed to deal with market power problems (Fischer, 2011; Fowlie et al., 2016). In addition to these efficiency justifications, in second-best contexts, OBAs have also been viewed as a trade-off between efficiency and compensation (Böhringer et al., 1998; Burtraw et al., 2001). Böhringer and Lange (2005b,a) compare output-based and emission-based allocations. Böhringer and Lange (2005b) determine the optimal mix between the two methods when a fraction of permits must be allocated for free for political reasons. Böhringer and Lange (2005a) stress the efficiency advantage of OBA compared to an emission-based allocation rule.

Much less attention, however, has been paid to the possibility of OBA schemes to handle market volatility.[9] One possible explanation for this lack of attention is that in the absence of leakage and/or market power there is no reason to subsidize production, which is what OBA schemes ultimately do. In this article we abstract from leakage and market power issues to focus exclusively on the performance of OBA schemes to handle market volatility.[10]

The rest of the article is organized as follows. We start in the next section with a permit market that covers a single-sector with a large number of firms to demonstrate that the optimality of OBA schemes holds quite generally. In Section 3 we extend the analysis to cover multiple sectors, each of which facing independent shocks. We conclude in Section 4.

2 A single-sector model

Consider a competitive market for an homogeneous good subject to demand and supply shocks. Inverse demand is given by $P(q;\theta)$ where q is total consumption and $\theta \in [\theta_{\min}, \theta_{\max}]$ is a demand shock. Function $P(\cdot)$ is positive, differentiable and decreasing in q and increasing in θ. The corresponding consumer gross surplus is $S(q;\theta)$ with $S_q = P(q;\theta)$.[11] The good is supplied by an industrial sector with a large number of price-taking firms. The sector's cost of producing q while polluting e is $C(q,e;\eta)$, where $\eta \in [\eta_{\min}, \eta_{\max}]$ is a supply shock.

[8] For example, Burtraw et al. (2015) suggested that OBA should be considered to mitigate leakage within the US since the implementation of the Clean Power Plan may lead to state-specific regulations.

[9] The only article that study OBA schemes under uncertainty is Meunier et al. (2014), but again the focus is on leakage not on the implication of having a flexible permit allocation.

[10] In our companion paper we extend the analysis to the possibility of leakage (Meunier et al., 2017) (Meunier et al. 2017).

[11] Partial derivatives are sometimes denoted by a subscript, e.g., $S_q \equiv \partial S/\partial q$.

Function $C(\cdot)$ is positive and increasing in q and η, decreasing in e and

$$C_{qe} < 0,\, C_{qq} > 0,\, C_{ee} > 0 \text{ and } C_{qq}C_{ee} > C_{qe}^2. \tag{1}$$

This formulation assumes that output and pollution are cost complements (see Spulber, 1988).

Damage from pollution depends on total emissions according to $D(e)$, a positive, increasing and convex function. For any given realization of θ and η, social welfare is computed as the difference between gross consumer surplus, production costs and pollution damage

$$W(q, e, \theta, \eta) = S(q; \theta) - C(q, e; \eta) - D(e) \tag{2}$$

so expected welfare is denoted by

$$\tilde{W} = \mathbb{E}_{\theta,\eta}[S(q(\cdot); \theta) - C(q(\cdot), e(\cdot); \eta) - D(e(\cdot))] \tag{3}$$

Throughout, we assume that shocks θ and η move within a range that there is positive production and pollution abatement in equilibrium for all states of demand and supply and regulatory designs (including no intervention).

2.1 OBA regulation and market equilibrium

In the absence of government intervention, the market equilibrium $P(q; \theta) = C_q(q, e; \eta)$ and $C_e(q, e; \eta) = 0$ leads to too much pollution. To correct for this, the regulator implements a permit market where the total amount of permits may not be fixed but endogenous to output. The regulator auctions off $\bar{e}$ permits and in addition allocates permits to firms based on their output.[12] For each unit of output, a firm gets α permits for free, so the total amount of pollution/permits in any given period is equal to

$$e = \bar{e} + \alpha q \tag{4}$$

In what follows, we will refer to α as the OBA rate. Most of the article is about to understand the conditions under which it is socially optimal to set $\alpha > 0$, whether there is a single sector

[12]In principle, the $\bar{e}$ permits could also be allocated for free to firms based, for example, on historic emissions. But as soon as we allow for some positive cost of public funds (Goulder et al., 1997), auctioning becomes optimal. Our implicit assumption in the article is that the cost of public funds is positive but arbitrarily small, so we do not need to explicitly model it.

like here or multiple sectors like in the next section.

At the beginning of each period firms learn θ and η, after which they decide how much to produce and pollute anticipating the additional permits they will get for their output. Since the permit market is perfectly competitive, the auction clears at the price firms expect to trade permits in the secondary market. We denote this price by r. Thus, each firm takes r and the output price $p = P(q;\theta)$ as given and solve (think of $C(\cdot)$ as the cost function of a representative firm)

$$\max_{q,e}\{pq - C(q,e;\eta) - r(e - \alpha q)\}$$

leading to the first-order equilibrium conditions

$$p = C_q(q,e;\eta) - \alpha r \tag{5}$$

and

$$r = -C_e(q,e;\eta) \tag{6}$$

Equilibrium prices p and r are in turn obtained using (4).

Since OBA is a subsidy to production, the first-order conditions are standard ones. A firm will produce to the point where the marginal cost of production is equal to the output price plus the OBA subsidy. Similarly, the firm will abate emissions to the point where the marginal cost of doing so is equal to the permit price.

Details on how the equilibrium levels of production q and pollution e respond to shocks θ and η and the regulatory variables $\bar{e}$ and α are in Appendix A.1. One aspect worth commenting here is the non-monotonic influence of the OBA rate α. From looking at the first-order conditions (5) and (6), one could decompose the effect of an increase in r on q in two opposing effects: a positive effect due to the increase in the OBA subsidy and a negative effect due to more pollution abatement.[13] When α is small, the second effect dominates, so an increase in r would lead to a drop in q. Conversely, when α is large, the first effect may dominate, and an increase in r may well lead to an increase in q as well. For these same reasons, the effect of a marginal increase in α or in $\bar{e}$ on output and pollution cannot be signed a priori when α is large. Fortunately, there is no need to delve much into these monotonicity issues. For most part we only need to focus on the welfare effects of a small α, for which the monotonicity is clear, and when we take the model to a linear world, as done in the next two sections, the monotonicity is preserved even for large α.

[13]As done in the Appendix A.1, totally differentiating (5) and (6) with respect to r yields $dq/dr = [C_{qe} + \alpha C_{ee}]/\delta_1$ and $de/dr = -[C_{qq} - P_q + \alpha C_{qe}]/\delta_1$, where $\delta_1 = C_{qq}C_{ee} - P_q C_{ee} - C_{qe}^2 > 0$.

2.2 Optimal OBA scheme

We now turn to the optimal permit design and to see whether setting $\alpha > 0$ is ever optimal. Before doing so, it helps to ask what would be the optimal number of permits $\bar{e}$ if we set $\alpha = 0$. The answer, which is well known (e.g., Weitzman, 1974), is stated in the next lemma.

Lemma 1 *When the OBA rate is set equal to zero (i.e., $\alpha = 0$), the optimal number of permits to be auctioned off, $\bar{e}$, is such that the expected permit price is equal to the marginal environmental damage.*

Proof. If $\alpha = 0$, the quantity of emissions is $\bar{e}$ in all demand states, hence, by the envelope theorem, we have that

$$\tilde{W}_{\bar{e}}(\bar{e}, \alpha = 0) = \mathbb{E}_{\theta,\eta}\left[-C_e - D'(\bar{e})\right] = \mathbb{E}_{\theta,\eta}[r] - D'(\bar{e})$$

so, at the optimum $\mathbb{E}[r] = D'(\bar{e})$. ■

By letting the permit price fluctuate around the marginal harm $D'(\bar{e})$, the authority minimizes the errors from a policy that never hits the first-best ex-post, except when the shocks θ and η are such that $r(\theta, \eta) = D'(\bar{e})$. The question now is whether setting a positive OBA rate, $\alpha > 0$, helps minimize those errors any further. Maintaining the auction allocation fixed at $\bar{e}$, the welfare effect of adding a few OBA permits to the market is

$$\tilde{W}_{\alpha}(\bar{e}, \alpha) = \mathbb{E}[-\alpha r q_{\alpha} + (r - D'(e))e_{\alpha}] \tag{7}$$

The first term captures the welfare loss from the output distortion introduced by the OBA subsidy (notice from Appendix A.1 that both q_α and e_α are positive when α is not too large, which is the relevant case for the next proposition). As captured by the second term, however, this loss can be potentially compensated by a gain from higher emissions whenever the permit price r is above $D'(\bar{e})$. But since $\mathbb{E}[r] = D'(\bar{e})$, it is not evident that the gains from allocating additional permits in periods when $r > D'(\bar{e})$ are not exactly offset by the losses from increasing emissions in periods when $r < D'(\bar{e})$. In fact, in a world of certainty, when $\bar{e}$ is such that $r = D'(\bar{e})$, setting $\alpha > 0$ only reduces welfare because you are left with just the first term.

In a world of changing supply and demand this logic may not apply, as the next proposition shows.

Proposition 1 *Consider a permit market with $\alpha = 0$ and $\bar{e}^0$ such that $D'(\bar{e}^0) = \mathbb{E}[r]$. If in that market we observe a positive correlation between permit prices and output, then it is* *optimal to introduce a positive OBA rate, $\alpha > 0$. Furthermore, the optimal OBA scheme $(\bar{e}, \alpha)$ in that case satisfies the system of equations*

$$\mathbb{E}[r - D'(e)] = \alpha \mathbb{E}[D'(e)\frac{\partial q}{\partial \bar{e}}] \tag{8}$$

$$\texttt{cov}\left(\frac{\partial W}{\partial \bar{e}}, q\right) = \alpha \mathbb{E}\left[D'(e)\frac{-C_e}{\delta_2}\right] \tag{9}$$

where $\delta_2 = -P_q + C_{qq} + 2\alpha C_{qe} + \alpha^2 C_{ee} > 0$.

Proof. Notice first that $\delta_2 > 0$ thanks to assumption (1). Now, from Lemma 1 notice that when $\alpha = 0$, it is optimal to set $\bar{e}^0$ such that $\mathbb{E}[r] = D'(\bar{e}^0)$. Next, from equation (4) obtain $e_\alpha = q + \alpha q_\alpha$, which replaced into (7) together with $\alpha = 0$ yields

$$\tilde{W}_\alpha(\bar{e}^0, \alpha = 0) = \mathbb{E}[(r - D'(\bar{e}^0))q] \tag{10}$$

Since $D'(\bar{e}^0)$ is constant and equal to the expected permit price, $\tilde{W}_\alpha(\bar{e}^0, \alpha = 0)$ is positive if

$$\mathbb{E}[(r - D'(\bar{e}^0))q] = \mathbb{E}[r - D'(\bar{e}^0)]\mathbb{E}[q] + \text{cov}(r, q) = \text{cov}(r, q) > 0 \tag{11}$$

The system (8) and (9), on the other hand, is obtained by simply rearranging the optimality conditions

$$\tilde{W}_{\bar{e}}(\bar{e}, \alpha) = \mathbb{E}[-\alpha r q_{\bar{e}} + (r - D'(e))e_{\bar{e}}] = 0 \tag{12}$$

and $\tilde{W}_\alpha(\bar{e}, \alpha) = 0$, where $\tilde{W}_\alpha(\bar{e}, \alpha)$ is given by (7). To arrive at (8), obtain first $e_{\bar{e}} = 1 + \alpha q_{\bar{e}}$ from (4) and then plug it into (12).

To arrive at (9), obtain first $e_\alpha = q + \alpha q_\alpha$ from (4) and plug it into (7) to obtain

$$\tilde{W}_\alpha(\bar{e}, \alpha) = \mathbb{E}[(r - D'(e))q - \alpha D'(e) q_\alpha] = 0 \tag{13}$$

Now, replacing $q_\alpha = q q_{\bar{e}} - C_e/\delta_2$, which is derived in Appendix A.1 (see equation (39)), in (13) yields

$$\tilde{W}_\alpha(\bar{e}, \alpha) = \mathbb{E}[\{r - D'(e) - \alpha D'(e) q_{\bar{e}}\}q + \alpha D'(e) C_e/\delta_2] = 0$$

But the term in curly brackets is $\partial W/\partial \bar{e}$, which can be seen directly from (8), so using $\mathbb{E}[\{\partial W/\partial \bar{e}\}q]$ =cov$(\partial W/\partial \bar{e}, q)$ we finally arrive at (9). ■

Production is inefficiently high with OBA given the number of emissions, however, with uncertainty this inefficiency might be worth the flexibility in the cap created by OBA. Whether it is the case can be easily checked by looking at the covariance between the permit price and the quantity produced. A positive OBA rate relaxes the overall emissions cap in all demand and supply states, which has a positive (resp. negative) welfare effect in "high" (resp. low) states of demand/supply, that is, in states in which the permit price is higher (resp. lower) than the marginal environmental damage. Therefore, the net welfare effect depends one whether the gains in high demand/supply states more than offset the losses in low demand/supply states. When the OBA rate is small, the gain (or loss) in each state is equal to the difference between the permit price and the marginal environmental damage times the number of extra permits. But this latter is exactly equal to output when $\alpha = 0$, so if output tend to be larger in periods when permit prices are high and above marginal damages, then, the net welfare effect from injecting a few extra permits in all states must be positive.

The exact choice of the regulatory variables α and $\bar{e}$, captured by equations (8) and (9), are the result of the trade-off the regulator must solve between output distortion and additional emissions. Because a marginal increase in α or $\bar{e}$ results in more emissions and output, the exact same trade-off is present in both (7) and (12). In (7), and given some $\bar{e}$, the regulator will increase α to the point in which the additional (expected) loss from the output distortion $(-\alpha r q_\alpha)$ is exactly equal to the extra gain from having relatively more emissions in periods of higher permit prices. Likewise, in (12), and given $\alpha > 0$, the regulator will increase $\bar{e}$ to the point in which the additional (expected) loss from the output distortion $(-\alpha r q_{\bar{e}})$ is exactly equal to the extra gain from having more emissions in periods of higher permit prices. Notice from (8) that $\bar{e}$ is set below the level that equalizes marginal damages $D'(e)$ to expected prices $\mathbb{E}[r]$. This is done to correct for the additional permits that are brought to the market in each period by setting $\alpha > 0$.

Proposition 1 says that observing permit prices and output to fluctuate over time is not enough to implement an OBA scheme. The latter requires output to be positively correlated with permit prices, which ultimately depends on the impact of shocks θ and η on prices and cost functions. So, the relevant question is how likely is to observe a positive correlation. We argue that it is most likely (perhaps the best example is the carbon market in Europe where the positive correlation has been reinforced by the sharp drop in permit prices during the recent international crisis). For instance, if demand is the main source of uncertainty, when demand is high both permit prices and output will be high. Similarly, if shocks affect primarily production costs (e.g., the oil price), when production costs are high both output and permit prices will be low. We cannot rule out in theory, however, cases that may exhibit a negative correlation. For example, if abatement cost are the main source of uncertainty, high abatement costs could lead to both high permit prices and low output. These examples are summarized in the following lemma.

Lemma 2 *A small positive OBA rate increases welfare (i.e., $\tilde{W}_\alpha(\alpha = 0) > 0$) when (i) $P_\theta > 0$ and $C_\eta = 0$, or (ii) $P_\theta = 0$, $C_{q\eta} > 0$ and $C_{e\eta} > 0$.*

Conversely, a small positive OBA rate may decrease welfare if the permit price does not vary with θ and the marginal abatement cost $-C_e$ increases sufficiently more than the marginal production cost C_q with respect to η (i.e., $P_\theta = 0$, $C_{q\eta} > 0$ and $-C_{e\eta} > -C_{qe}C_{q\eta}/(C_{qq} - P_q)$).

Proof. See Appendix A.2. ■

Because the implementation of an OBA scheme not only requires the regulator issuing additional permits each period but also setting different OBA rates to different groups of firms (a topic covered in Sections 3.1 and 3.2), one may argue that the gains from implementing an OBA scheme may not be sufficient to justify its implementation costs. Our numerical exercises of Section 4 show otherwise, that the gains from implementing OBA can be substantial. And having different groups of firms does not make the implementation of OBA much more difficult; it may require the regulator to allocate a fraction of the auctioned permits in a lump-sum manner to sort firms out into the scheme (this is covered in Section 3.3). Yet, some may argue that OBA permits may not be necessary if the regulator opts for an alternative (flexible) allocation scheme. The next section points otherwise.

2.3 Hybrid design

An OBA scheme is one of several ways to let the overall emissions cap to adjust to demand and supply shocks. Alternatives include the introduction of a market stability reserve (which is closely related to banking and borrowing provisions) or the use of a hybrid permit scheme with a price floor and ceiling as first proposed by Roberts and Spence (1976) and recently adopted in California (Borenstein et al., 2015). Since an optimal hybrid scheme is strictly superior to the best market stability reserve (or best banking and borrowing provisions for that matter),[14] in this section we study whether the introduction of an OBA scheme still plays a role in a well designed hybrid permit market.

[14]Unlike Roberts and Spence (1976), banking and borrowing work through firms' intertemporal optimization paying little attention to environmental damages. Introducing exchange rates to alter how firms borrow or save permits does not work either.

Consider then a hybrid design with a price ceiling and price floor, which we denote, respectively, by $\bar{r}$ and $\underline{r}$. These thresholds are equivalent to setting a penalty for not compliance equal to $\bar{r}$ and a subsidy for over-compliance equal to $\underline{r}$. The regulatory timing is as before: price thresholds $\bar{r}$ and $\underline{r}$ are set ex-ante together with the number of auctioned permits $\bar{e}$ and the OBA rate α.

To simplify the presentation we will only consider demand shocks θ, which are assumed to be drawn from the cumulative distribution function $F(\theta)$. If the demand for permits is sufficiently high (resp. low), the price of permits will be equal to $\bar{r}$ (resp. $\underline{r}$). Since the demand for permits is increasing with respect to the demand state θ (see Appendix A.1), there will be two demand states $\underline{\theta}$ and $\bar{\theta} > \underline{\theta}$ such that: (i) when $\theta \in [\theta_{\min}, \underline{\theta}]$, $r = \underline{r}$ and $e - \alpha q \leq \bar{e}$; (ii) when $\theta \in (\underline{\theta}, \bar{\theta})$, $\underline{r} < r < \bar{r}$ and $e - \alpha q = \bar{e}$; and (iii) when $\theta \in [\bar{\theta}, \theta_{\max}]$, $r = \bar{r}$ and $e - \alpha q \geq \bar{e}$.

Notice that in this hybrid scheme there will be not one but three instances in which the policy will be ex-post efficient. One is for a low demand shock $\theta \in (\theta_{\min}, \underline{\theta})$ such that $\underline{r} = D'(e(\theta) - \alpha q(\theta) < \bar{e})$; the second for an intermediate demand shock $\theta \in (\underline{\theta}, \bar{\theta})$ such that $r = D'(\bar{e})$; and the third for a high demand shock $\theta \in (\bar{\theta}, \theta_{\max})$ such that $\bar{r} = D'(e(\theta) - \alpha q(\theta) > \bar{e})$.

When $\alpha = 0$, the optimal hybrid design $(\bar{e}, \underline{r}, \bar{r})$ solves the following system of equations

$$\tilde{W}_{\bar{e}} = \int_{\underline{\theta}}^{\bar{\theta}} [r(\theta) - D'(\bar{e})] dF(\theta) = 0 \tag{14}$$

$$\tilde{W}_{\underline{r}} = \int_{\theta_{\min}}^{\underline{\theta}} [\underline{r} - D'(e(\theta))] e_{\underline{r}} dF(\theta) = 0 \tag{15}$$

$$\tilde{W}_{\bar{r}} = \int_{\bar{\theta}}^{\theta_{\max}} [\bar{r} - D'(e(\theta))] e_{\bar{r}} dF(\theta) = 0 \tag{16}$$

Equation (14) follows the same logic of the previous section. In the demand range where emissions are fixed at $\bar{e}$ and the permit price varies with θ, it is optimal to have the expected permit price be equal to the marginal damage. In the other two regions, however, where prices are fixed but emissions adjust to shocks, equations (15) and (16) show that what is optimal is to have the fixed price be equal to expected marginal damages (weighted by either $e_{\underline{r}}$ or $e_{\bar{r}}$, unless they are invariant to θ).

Introducing both a price floor and ceiling allows the equilibrium permit price to follow more closely the marginal environmental damage for different realizations of θ (cf. Roberts and Spence, 1976). Whether it is worth adding some OBA permits over this more flexible design is not obvious because extreme permit price realizations have now been truncated, which also has an effect on output outcomes. To check the optimality of OBA we proceed as before by considering the welfare effect of introducing an arbitrarily small OBA rate over this optimal hybrid scheme. Changes in expected welfare at $\alpha = 0$ are only due to changes in emissions because there is no production inefficiency when $\alpha = 0$; hence, $W_\alpha = \mathbb{E}[(r - D')e_\alpha]$.

Since $e_\alpha = q$ in the demand range where emissions are fixed at $\bar{e}$, this welfare change can be decomposed as

$$\begin{aligned}\tilde{W}_\alpha(\alpha &= 0) = \int_{\theta_{\min}}^{\underline{\theta}} (\underline{r} - D'(\theta))e_\alpha dF \\ &+ \int_{\underline{\theta}}^{\bar{\theta}} (r - D'(\bar{e}))q dF + \int_{\bar{\theta}}^{\theta_{\max}} (\bar{r} - D'(\theta))e_\alpha dF\end{aligned} \tag{17}$$

The OBA rate has a different influence in intermediary demand states than in the more extreme ones. In intermediary states, there is both a direct effect on emissions due to the increased number of free permits and an indirect effect via the rise of production due to a higher subsidy. In extreme demand states, the permit price is fixed, and free allocations alleviate the bill of the firm but not its emission choice. However, there is still the subsidy channel that induces higher emissions due to higher production. The influence of the OBA rate is positive for intermediary demand states (second term above). The magnitude of this positive effect depends on how often $\theta \in [\underline{\theta}, \bar{\theta}]$ and the covariance between output and permit prices. The first and third term are more difficult to sign because of the influence of α on emissions when the floor or ceiling bind, and how this influence compares to the influence of the permit price on emissions.

Proposition 2 *Suppose we observe a positive correlation between permit prices and output in the optimal hybrid design with $\alpha = 0$. If the influence of the OBA rate and of the price floor and ceiling on emissions do not vary with demand (i.e., $e_{\alpha\theta} = e_{\bar{r}\theta} = e_{\underline{r}\theta} = 0$), then a strictly positive OBA rate increases welfare (This is so when demand is linear, uncertainty is additive and cost is quadratic).*

Proof. When $e_{\bar{r}}$ and e_α are independent of the demand shock θ, the optimal price floor solves

$$\underline{r} = \mathbb{E}[D'(e(\theta))|\theta < \underline{\theta}]$$

so the first term in (17) is zero. Since the same applies to the third term, the welfare effect of adding a few OBA permits will be positive if the second term is positive, which requires of a positive correlation between permit prices and output. ■

Propositions 1 and 2 show that an OBA scheme is likely to be beneficial in the context of a single sector, even if there is a flexible scheme already in place. We now explains how this result extends to multiple sectors, which is the more realistic case.

3 Multiple sectors

The single-sector analysis provides us with the solution to the OBA problem when the regulator sets the same rate α to all firms, whatever similar or different they might be. In reality firms covered by a permits market are never identical for different reasons. They may use different production technologies, belong to different productive sectors or simply be located in different regions. In this section we look at the optimal OBA design when there are multiple sectors subject to different shocks (or when the regulator can assign firms to different groups). To focus on the issue of the allocation of emissions among sectors we adopt the extreme assumption that the only abatement technology available is output reduction (we do not want to leave the impression that abatement is essential to generate a correlation between permit prices and output). We start with a general set-up and then introduce some simplifying assumptions to derive additional results.

3.1 General set-up

Consider a permit market covering a large number n of sectors, each of which takes the price of permits as given. In each sector there is a continuum of identical firms. Production in sector $i = 1, ..., n$ is denoted by q_i and since the only abatement technology is output reduction, we normalize emissions to output, i.e., $e_i = q_i$. As in the previous section, inverse demand in sector i is denoted by $P_i(q_i, \theta_i)$, consumer surplus by $S_i(q_i; \theta_i) = \int_0^{q_i} P_i(x; \theta_i)dx$, and production costs by $C_i(q_i; \eta_i)$, so welfare for a given realization of $\theta = \{\theta_1, ..., \theta_n\}$ and $\eta = \{\eta_1, ..., \eta_n\}$ is equal to

$$W(\theta, \eta) = \sum_{i=1}^{n} [S_i(q_i; \theta_i) - C_i(q_i; \eta_i)] - D(e)$$

where $e = \sum_{i=1}^{n} q_i$.

An OBA scheme $\{\bar{e}, \alpha_1, ..., \alpha_n\}$ includes a fraction of $\bar{e}$ auctioned permits and OBA rates $\alpha_i \in [0, 1)$ for each sector $i = 1, ..., n$, so the total number of permits in the market will be $e = \bar{e} + \sum_{i=1}^{n} \alpha_i q_i$. The output market in each sector will clear at the price p_i that equals marginal production costs minus the OBA subsidy

$$p_i = P_i(q_i; \theta_i) = C_i'(q_i; \eta_i) + r - \alpha_i r$$

where r is permit price (common to all sectors) and $C_i'(\cdot)$ is sector i's marginal cost. Thus, the market equilibrium is described by $n+1$ equations

$$\sum_{i=1}^{n} (1-\alpha_i) q_i = \bar{e} \tag{18}$$

$$P_i(q_i, \theta_i) - C_i'(q_i; \eta_i) = \frac{1-\alpha_i}{1-\alpha_1} \left[P_1(q_1, \theta_1) - C_1'(q_1; \eta_1) \right] \tag{19}$$

for all $i = 1, ..., n$.

Lemma 3 *Any scheme $\{\bar{e}, \alpha_1, ..., \alpha_n\}$ with $\alpha_i \in [0,1)$ is equivalent (i.e., it leads to the same equilibrium outcome and payoffs) to a scheme in which the lowest OBA rate is normalized to zero*

$$\left\{ \frac{\bar{e}}{1-\alpha_j}, \frac{\alpha_1 - \alpha_j}{1-\alpha_j}, ..., \frac{\alpha_{j-1} - \alpha_j}{1-\alpha_j}, 0, \frac{\alpha_{j+1} - \alpha_j}{1-\alpha_j}, ..., \frac{\alpha_n - \alpha_j}{1-\alpha_j} \right\} \tag{20}$$

where $\alpha_j = \min_i \alpha_i$.

Proof. It is easy to see that both schemes satisfy equations (18) and (19), which implies that output levels are identical under both schemes for all demand states. In addition, the net emission price $(1-\alpha_i)r$ in sector i is identical to the price under scheme (20); more precisely, the permit price under the new scheme is $(1-\alpha_j)$ times the permit price under the original scheme. Given the equivalence in output and price levels, profits and consumer surplus must be identical across schemes. ■

Although it appears from the lemma that the regulator has some room to change OBA rates across sectors without welfare consequences, the reality is that she has none because (20) is just one of the many price normalizations she can pick,[15] any of which with the same distribution implications. Hence, one can theoretically consider schemes with negative OBA rates, since, any such scheme could be transformed into an equivalent scheme with positive OBA rates for all sectors.

When it comes to choosing the optimal OBA scheme, the same tension detected in the single-sector appears in this multiple-sector setting: letting the overall cap on emissions to adjust to shocks comes at an inefficiency cost. The inefficiency here is a misallocation of the

[15] Notice that no normalization can violate the constraint $\alpha_i < 1$. Setting $\alpha_i = 1$, for example, is equivalent to removing sector i from the permit regulation.

cap across sectors. Sectors with relatively higher OBA rates produce and pollute too much. In fact, the welfare impact of increasing the OBA rate in one sector, say k, is given by (see Appendix B.1)

$$\frac{\partial \tilde{W}}{\partial \alpha_k} = \mathbb{E}\left[(r - D'(e))q_k - D'(e)\sum_{i=1}^{n}\alpha_i \frac{\partial q_i}{\partial \alpha_k}\right] \tag{21}$$

where $e = \sum_{i=1}^{n} q_i = \bar{e} + \sum_{i=1}^{n} \alpha_i q_i$.

The first term in (21) captures the direct effect of increasing emissions in sector k by increasing α_k. This term is expected to be positive if q_k is larger when $r - D'(e) > 0$. The second term represents the effect of the change in production, not only in sector k but in all sectors. Increasing production in sector i has a net effect of $-\alpha_i D'(e)$, which is the sum of the loss $\alpha_i r$ due to the subsidy of production and the gain $\alpha_i(r - D')$ of increasing emissions. This second term can still be positive, since production in sector $i \neq k$ might well decrease with respect to α_k.

Expression (21) tells us right away that setting $\alpha_i = 0$ for all sectors cannot be optimal if there is just one sector that exhibits a correlation different from zero between output and permit prices. Notice that a negative correlation would in principle call for a negative OBA rate in that sector, but according to Lemma 3, this would be equivalent to set a null OBA rate in that sector and positive rates in all others. In any case, the presence of multiple sectors makes the optimality of an OBA scheme certain (unless all sectors are equal, which takes us back to Proposition 1).

Proposition 3 *An optimal OBA scheme $\{\bar{e}, \alpha_1, ..., \alpha_n\}$ satisfies the following system of equations*

$$\mathbb{E}[r - D'(e)] = \mathbb{E}\left[D'(e)\frac{-\partial r}{\partial \bar{e}}\sum_{i=1}^{n}\alpha_i \frac{1-\alpha_i}{C_i'' - P_i'}\right] \tag{22}$$

$$\texttt{cov}\left(\frac{\partial W}{\partial \bar{e}}, q_k\right) = \mathbb{E}\left\{D'(e)\frac{r}{C_k'' - P_k'}\frac{-\partial r}{\partial \bar{e}}\sum_{i=1}^{n}\left[\frac{1-\alpha_i}{C_i'' - P_i'}(\alpha_k - \alpha_i)\right]\right\} \tag{23}$$

for all $k = 1, ..., n$.

Proof. See Appendix B.1. ■

Equation (22) is similar to equation (8) in Proposition 1. It says that the optimal number of auctioned permits, $\bar{e}$, should be adjusted to the allocation of OBA permits, so as to keep a difference between permit prices and marginal harm equal to the marginal inefficiency cost generated by these OBA permits. If for some reason it is optimal to set all OBA rates equal to zero, then (22) reduces to the standard optimality condition that expected permit prices should be equal to marginal harm, i.e., $\mathbb{E}[r] = D'(e = \bar{e})$.

Equation (23) also follows a similar logic than equation (9) in Proposition 1. Increasing the OBA rate in sector k could be decomposed in two effects: it is like auctioning q_k more permits and shifting the demand for permits in sector k. But we know that when the total number of auctioned permits $\bar{e}$ is optimally set, the expected marginal (social) value of an extra auctioned permit in the market across all demand states is null (this applies regardless of whether we have one or multiple sectors). This implies that only the covariance matters for the first effect (i.e., the effect of auctioning additional permits) to have a positive welfare impact.

The right-hand-side of (23) can be interpreted as the marginal effect of α_k on the misallocation of the total cap when α_i's differ across sectors. The effect of α_k on the distorted allocation is a weighted difference between the OBA rate in sector α_k and the remaining sectors. Contrary to the single sector case, this effect could be negative, increasing the OBA rate in a particular sector might actually reduce distortions associated to other OBA rates.

Without uncertainty, one can see from equation (23) that it would be optimal to set all α_i equal which is equivalent to not introducing an OBA scheme. The benefit from introducing an OBA scheme necessarily requires some covariance, in at least one sector, between the benefit/cost of auctioning one extra permit in a state and the number of permits actually injected in that state.

The main difference between Propositions 1 and 3 rests on the multidimensionality of the scheme, so that one cannot, and does not need to, perform a "positive correlation test" as in Proposition 1 to determine whether a sector requires a positive OBA rate. It might be optimal to set a strictly positive OBA rate in a particular sector whatever the sign of the correlation between its output and the marginal benefits from an extra permit. We do need to know, however, the sign and size of the correlation relative to other sectors in order to compute the OBA rates.

The main message from Proposition 3 is that setting $\alpha_k = 0$ (or $\alpha_k = \alpha_i$) for all sectors would only happen if $\mathrm{cov}(\partial W/\partial \bar{e}, q_k) = 0$ for all k. It is easy to rule this out; it suffices to have sectors receiving different shocks. To fully appreciate this, and get some more palatable results, a linear specification is considered next. The linear specification will also serve as the basis for the self-selection analysis in the following section.

3.2 A linear set-up

For tractability, and without much loss of generality, in what follows we work with linear functions and shocks that only affect demand, which in addition are assumed to be independently distributed and to enter additively.[16] In particular, we let

$$D'(e) = h,\ p_i(q_i, \theta_i) = a_i + \theta_i - b_i q_i \text{ and } C_i(q_i) = \gamma_i q_i^2/2 \tag{24}$$

for all $i = 1...n$, where a_i, b_i, γ_i are all strictly positive, and $\mathbb{E}[\theta_i] = 0$, $\mathbb{E}[\theta_i^2] = \sigma_i^2 > 0$ and $\mathrm{cov}(\theta_i, \theta_j) = 0$. We also assume that shocks are such that there is always an interior solution, that is, that there is always a positive level of output in all sectors for all possible shocks and regulatory designs.

Under this linear specification, it is possible to fully describe and compute the optimal OBA scheme of Proposition 3.

Proposition 4 *Under the linear specification described above, the optimal OBA scheme considers OBA rates increasing with sector volatility (σ_i) and market size ($1/\beta_i$, where $\beta_i = b_i + \gamma_i$), that is, $\alpha_k > \alpha_l$ if $\sigma_k^2/\beta_k > \sigma_l^2/\beta_l$ for any $l \neq k = 1, ..., n$. If sector 1 is defined as $\sigma_1^2/\beta_1 = \min_i \sigma_i^2/\beta_i$, then an optimal OBA scheme is characterized by (i) $\alpha_1 = 0$, and (ii)*

$$\alpha_k = \frac{\Delta_k}{\Delta_k + \Psi} \tag{25}$$

where

$$\Delta_k = \frac{1}{h^2}\left[\frac{\sigma_k^2}{\beta_k} - \frac{\sigma_1^2}{\beta_1}\right] \tag{26}$$

and Ψ is the unique positive solution to x in equation $\sum_{i=1}^{n}\left[\beta_i(\Delta_i + x)\right]^{-1} = 1$, and (iii) a number of auctioned permits $\bar{e}$ such that

$$\mathbb{E}r = \frac{\sum_i (1-\alpha_i)/\beta_i}{\sum_i (1-\alpha_i)^2/\beta_i} h \tag{27}$$

Proof. See Appendix B.2. ■

This proposition shows quite clearly that what matters for OBA is not the absolute volatility but the relative volatility between sectors, after controlling for sector size. In fact, expression (26) indicates that the optimal design remains unchanged if the volatility in all sectors, measured by σ^2/β, change by the same amount. This relative volatility is

[16] We could have alternatively considered shocks on the supply side (e.g., $C_i = \eta_i q_i + \gamma_i q_i^2/2$). In this linear world, it is irrelevant whether shocks are on the demand or production side.

what generates a covariance between permit prices and output. Suppose there are only two sectors, 1 and 2, with $\sigma_2^2/\beta_2 > \sigma_1^2/\beta_1$. If the regulator allocates an optimal number of permits $\bar{e}$ together with $\alpha_1 = \alpha_2 = 0$, total output will be fixed, $q_1 + q_2 = \bar{e}$, but it would split between the two sectors depending on the specific shocks affecting them. This output adjustment leads to changes in permit prices r and, ultimately, to a positive covariance between permit prices and output in sector 2.

Proposition 4 also helps us to visualize more precisely the impact of changes in market conditions, for example, of adding a (volatile) sector to the regulation or of increasing the volatility of one particular sector or of a group of sectors. The first change is straightforward to evaluate. Since adding a sector would increase Ψ, this will reduce the OBA rates in all existing sectors.

In this n-sector (linear) model it is relatively easy to compute profit and welfare gains from implementing an (optimal) OBA scheme vis-a-vis the simple permit scheme of Lemma 1. The numerical exercises in Section 4 shows that these gains can be indeed substantial even for volatility levels that are not that large. These gains, however, introduce the regulator to an implementation problem as seen in the carbon market in Europe: sectors want to lobby for larger OBA rates. We turn to this implementation problem now.

3.3 Self selection

Not only in the carbon market in Europe, but allocating free permits to firms or sectors is always controversial and subject to an intense amount of lobbying activity. We expect an OBA scheme to be no different.[17] For instance, if the regulator announces the OBA scheme of Proposition 4 all sectors would lobby to get the highest available OBA rate. Therefore, even if the regulator is able to identify the characteristics of each sector well, political reality may prevent her to discriminate among sectors and implement the optimal OBA scheme. We will show that there is a simple way for the regulator to go around this selection problem while preserving the optimal OBA outcome of Proposition 4: to use a fraction of the auctioned permits $\bar{e}$ to construct a menu of OBA rates and lump-sum allocations that can sort out sectors. This allocation mechanism should be viewed as a stylized representation of the negotiation taking place between the regulator and regulated sectors. Alternatively, one can view this mechanism as the solution to an adverse selection problem in which sector's

[17] Rent seeking behavior is also present in ‘Harstad and Eskeland (2010). In their model firms signal their types by buying more permits today than what is actually efficient in order to get a more generous allocation in the future.

characteristics, most notably volatility and costs, are unknown to the regulator, so the regulator needs to communicate with sectors before designing the regulation.[18]

Our analysis builds upon the linear specification of the previous section. Since sector volatility is our central problem here, we will work with sectors that are identical except for their volatility: $a_i = a$, $b_i = b$ and $\gamma_i = \gamma$ for all $i = 1, ..., n$ and $\sigma_1 < ... < \sigma_i < ...\sigma_n$. The regulator offers a menu of permit-allocation options $\{(\alpha_j, \hat{e}_j)/j = 1, ..., n\}$, where α_j is the OBA rate in option j and $\hat{e}_j$ is the number of free lump-sum permits in that option. Whenever $\sum_j \hat{e}_j < \bar{e}$, the regulator auctions the remaining fraction $\bar{e} - \sum_j \hat{e}_j$. Since negotiations take place at the sectorial level, all firms within a sector that goes for option $j = 1, ..., n$ will receive the same allocation $(\alpha_j, \hat{e}_j)$. Sectors anticipate the effect of different OBA rates on their profit while taking the permit price r as given.

We want to determine conditions under which the optimal scheme could be indeed implemented via such a menu, that is, under which conditions the menu satisfies both (i) the balanced-budget constraint $\sum_j \hat{e}_j \leq \bar{e}$ and (ii) the self-selection constraints

$$\mathbb{E}[\pi_i(\theta_i, r, \alpha_i) + r\hat{e}_i] \geq \mathbb{E}[\pi_i(\theta_i, r, \alpha_j) + r\hat{e}_j] \tag{28}$$

for all $i = 1, ..., n$ and $j \neq i$, where

$$\pi_i(\theta_i, r, \alpha) = \frac{\gamma}{2\beta^2}\left[a + \theta_i - (1 - \alpha)r\right]^2 \tag{29}$$

is sector i's profit gross of permit transfers for a given level of demand θ_i, permit price r and OBA rate α.

Let us focus first on what it takes for the self-selection constraints to hold. This requires to establish the existence of something equivalent to a single-crossing property. Thus, take two adjacent sectors, say θ_i and θ_{i+1}, and ask which of the two is willing to pay more for a marginal increase in the OBA rate from any given level α. Differentiating sectors' payoffs with respect to α and taking the difference yields

$$\mathbb{E}\left[\frac{\partial \pi_{i+1}(\theta_{i+1}, r, \alpha)}{\partial \alpha}\right] - \mathbb{E}\left[\frac{\partial \pi_i(\theta_i, r, \alpha)}{\partial \alpha}\right] = \frac{\gamma}{\beta^2}\mathbb{E}\left[r(\theta_{i+1} - \theta_i)\right] \tag{30}$$

[18]Martimort and Sand-Zantman (2015) also study a problem of adverse selection as applied to climate change policy. One main difference with that article is that here it is mandatory for polluters to comply with the regulation. Another is that we allow initial permit allocations to be tradable. We do share with them the possibility of implementing the (full-information) optimal scheme as long as the budget-balanced constraint is not binding.

We want to establish the conditions under which this difference is positive, so that sector $i+1$ is willing to pay more than sector i for the marginal increase at any possible level α. This latter is what will allow us to separate sectors. For instance, if sector $n-1$ needs to be compensated in Δ to take its equilibrium rate α_{n-1} instead of the highest rate α_n, then sector $n-2$ needs to be compensated in strictly less than Δ to achieve the same and so on as we considers sectors with lower volatilities.

Since sectors are too small to affect the permit price, we can evaluate the profit changes in (30) at the permit price that will prevail in equilibrium, that is, at the permit price when all sectors take their equilibrium options, which is

$$r = \frac{\sum_{k=1}^{n} [(1-\alpha_k)(a+\theta_k)] - \beta\bar{e}}{\sum_{k=1}^{n}(1-\alpha_k)^2} \tag{31}$$

Plugging (31) into (30), it turns out that this latter expression is positive as long as

$$(1-\alpha_{i+1})\sigma_{i+1}^2 > (1-\alpha_i)\sigma_i^2 \tag{32}$$

which requires that the OBA rates in the optimal scheme do not grow much faster than the sector volatilities.

It is evident that condition (32) departs from the single-crossing property that we usually encounter in standard models of adverse selection, where the sorting condition depends exclusively on the functional form of the agent's utility and not on the value of a regulatory decision variable. The reason here is different is due to changes in the way a sector volatility affects permit price variations. In the absence of OBA, more volatile sectors have a greater influence on permit price variations. However, as these more volatile sectors get assigned higher OBA rates, their volatility is reduced, and hence, their influence on permit price variations. Condition (32) requires such reduction not be so large in the optimal scheme, so that more volatile sectors continue having a greater influence on permit price variations.

Whether the optimal OBA scheme of Proposition 4 can be indeed implemented not only depends on (32) holding but also on satisfying the budget constraint (i). Thanks to Lemma 3 we do not need to check a potentially large number of menus that could implement the optimal scheme. We can restrict ourselves to menus in which $\alpha_1 = 0$ and $\hat{e}_n = 0$ since any implementable menu could be transformed into a menu with these two features. Therefore, the menu that can potentially implement the optimal scheme with the minimum number of lump-sum permits being allocated consists in setting $\alpha_1 = 0$ and α_i as in Proposition 4 and

$$\begin{aligned}
\hat{e}_n &= 0 \\
\hat{e}_{n-1} - \hat{e}_n &= \mathbb{E}[\pi_{n-1}(\theta_{n-1}, r, \alpha_n) - \pi_{n-1}(\theta_{n-1}, r, \alpha_{n-1})]/\mathbb{E}[r] \\
&\dots \\
\hat{e}_i - \hat{e}_{i+1} &= \mathbb{E}[\pi_i(\theta_i, r, \alpha_{i+1}) - \pi_i(\theta_i, r, \alpha_i)]/\mathbb{E}[r] \qquad (33) \\
&\dots
\end{aligned}$$

so that each of self-selection constraints holds. Whether $\sum_{j=1}^{n} \hat{e}_j$ is smaller than $\bar{e}$ is not immediate, but it is very likely as the next proposition shows.

Proposition 5 *Under the linear specification above with sectors that are identical but for their volatility (i.e., $\sigma_1 < ... < \sigma_n$), the optimal OBA scheme in Proposition 4 can be implemented with a menu of OBA rates and lump-sum allocations $\{(\alpha_j, \hat{e}_j)/i = 1, ..., n\}$ as described in (33) as long as (i) $a > h(1 + 2\gamma/b)$ and (ii) $\sigma_1^2 < h^2 \sum_{i=1}^{n}(1 - \alpha_i)$.*

Proof. See Appendix B.4 ■

Condition (ii) is obtained from working through the $n - 1$ incentive compatibility constraints. According to this condition, it appears that all that is required is to have just one single sector in the permit system with a volatility low enough for all this to work. This should not be interpreted, however, as that all we need is to just bring a low volatility sector to the permit system; the sector must be large enough, like all the others, so that there is always a positive amount of production in the sector for any possible shock and permit price realizations. Arriving at condition (i) is more demanding, but it is also likely to hold in practice. Take the electricity sector for instance. Estimates of the value of lost load, which is a good approximation for a since it corresponds to what customers are willing to pay to avoid a disruption in their electricity service, is many times larger than the corresponding social cost of carbon, i.e., h in our model.

As we will see next, both conditions (i) and (ii) hold easily in our policy exercises, but it is nevertheless useful to explore more formally what would be the additional distortions the regulator will need to introduce to sort sectors out when these two conditions do not hold. Given that the optimal OBA scheme in Proposition 4 is already away from the first-best, it is not clear what are these least-cost extra distortions. Following a bunching-at-the-top solution, one possible option would be to maintain the optimal OBA scheme to all sectors $k < \bar{k}$ and offer the same deal $(\alpha_{\bar{k}}, \hat{e}_{\bar{k}})$ to all sectors $k \geq \bar{k}$, where the cutoff $\bar{k}$ is to be found by going down the volatility ladder until $\sum_{j=1}^{n} \hat{e}_j = \bar{e}$, while taking into account that $\bar{e}$ is not fixed but endogenously determined along with $\bar{k}$ and the rest of the menu.

3.4 A numerical two-sector example

We illustrate some of the previous results with a very simple numerical example based on the linear specification of section 3.2 with two sectors with the following numerical values for the parameters: $a_1 = a_2 = 1$, $b_1 = b_2 = 1$, $\gamma_1 = \gamma_2 = 1$, $h = 1/4$, $\theta_1 = 0$ and $\theta_2 \in \{-\lambda, \lambda\}$ with equal probability, so $\sigma_2 = \lambda$. The parameter λ will be referred as the level of volatility. The model is explored for λ moving from 0 to 1/2. We are particularly interested in large values of λ.

Corollary 1 *With the linear specification (24) for two identical sectors but for their volatility ($\sigma_1 = 0$ and $\sigma_2 > 0$) the optimal OBA scheme $\{\bar{e}, \alpha_1, \alpha_2\}$ reduces to*

$$\alpha_1 \in [0, 1)$$

$$\frac{\alpha_2 - \alpha_1}{1 - \alpha_1} = 1 - \left[(\Delta^2 + 1)^{1/2} - \Delta\right] > 0 \tag{34}$$

$$\bar{e} = \frac{1}{2}(a - h)(2 - \alpha_1 - \alpha_2) \tag{35}$$

where $\Delta = \sigma_2^2 / 2h^2 > 0$.

Proof. See Appendix B.5. ■

Corollary 1 provides the relation between α_1 and α_2 that must hold under an optimal OBA scheme, so without any loss of generality we focus on $\alpha_1 = 0$. Figure 1 depicts this optimal policy as a function of λ. It can be observed that the optimal OBA rate is increasing rapidly as the volatility increases. The level of permits to be auctioned off (which includes lump-sum permits needed to handle self selection) is also depicted, as well as the number of lump-sum permits that should be allocated to sector 1 to make the optimal OBA rate robust to self selection. These values of permits are given in percentage of the cap without OBA, but considering volatility. For example, for $\lambda = 0.5$ approximately 60% of the permits should be auctioned off, and to prevent sector 1 to lobby for sector 2's OBA rate, the regulator should give sector 1 a total of 20% of the permits in a lump-sum transfer, reducing the total number of permits to be auctioned off to 40%. This slacks applies to any level of λ, as Figure 1 shows.

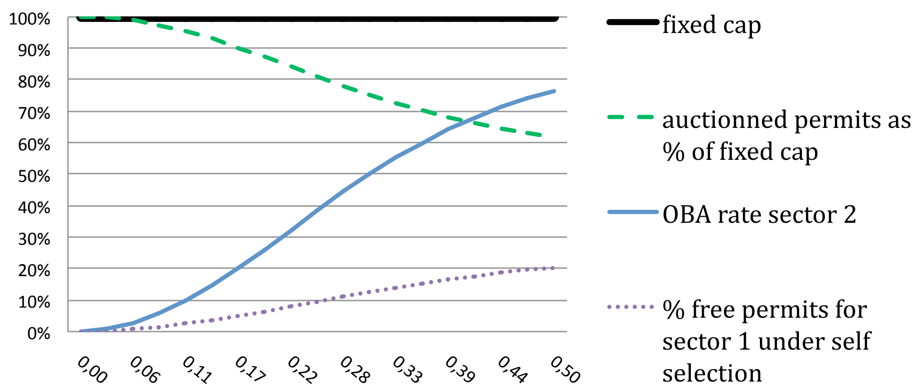

Figure 1: Optimal policy as a function of uncertainty (λ)

4 Conclusions

We have studied pollution permit markets in which a fraction of the permits are allocated to firms based on their output. We find that output-based allocations (OBAs), which are receiving increasing attention in the design of carbon markets around the world (e.g., Europe, California, New Zealand) can be optimal under demand and supply volatility despite the output distortions they may create. Take for instance the case of demand volatility and a fixed permits cap. For any given realization of demand, the cap is likely to be sub-optimal, being either too low or too high. An OBA scheme introduces some flexibility since the number of permits allocated depends on the level of demand. An OBA scheme enhances welfare by conditioning the emissions cap to economic activity, though imperfectly. This holds whether there is single productive sector covered by the permit market or, even more so, multiple sectors subject to different shocks. Even if a price floor and ceiling are introduced, an OBA scheme still enhances welfare.

Our model provides interesting insights to discuss a number of pending issues for the design of emission trading systems in general and carbon markets in particular. It would be interesting to investigate further these results in a full blown dynamic model in which the regulator commits over a given period of time and shocks occur on a frequent basis. Leakage, heterogeneous abatement technologies, and alternative policy features such as banking and market stability reserves should also be considered.

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Appendix

A Single-sector case

A.1 Equilibrium

To characterize the equilibrium for any demand-supply state θ-η we first need to establish how the demand for auctioned permits $\bar{e}$ changes with r. Take any $r > 0$, the equilibrium is fully characterized by the first-order conditions (5) and (6). Let $\tilde{q}(\alpha, r, \theta, \eta)$ and $\tilde{e}(\alpha, r, \theta, \eta)$ be the unique production and pollution levels, respectively, that solve the two first-order conditions.

Result A1. The demand for auctioned permits $\tilde{e} - \alpha\tilde{q}$ is decreasing in r.

Proof. Take the derivative of the first-order conditions (5) and (6) with respect to r:

$$\begin{bmatrix} P_q - C_{qq} & -C_{qe} \\ -C_{qe} & -C_{ee} \end{bmatrix} \begin{bmatrix} \tilde{q}_r \\ \tilde{e}_r \end{bmatrix} = \begin{bmatrix} -\alpha \\ 1 \end{bmatrix} \tag{36}$$

so that the effects of a change in the permit price on production and emissions are:

$$\begin{bmatrix} \tilde{q}_r \\ \tilde{e}_r \end{bmatrix} = \frac{1}{\delta_1} \begin{bmatrix} C_{qe} + \alpha C_{ee} \\ (P_q - C_{qq}) - \alpha C_{qe} \end{bmatrix} \tag{37}$$

where $\delta_1 = (C_{qq} - P_q)C_{ee} - C_{qe}^2$ is strictly positive by assumption (1). The derivative of the net demand for (auctioned) permits is then:

$$\begin{aligned} \tilde{e}_r - \alpha\tilde{q}_r &= \frac{-1}{\delta_1}\left[-P_q + C_{qq} + 2\alpha C_{qe} + \alpha^2 C_{ee}\right] \\ &\leq \frac{-1}{\delta_1}\left[-P_q + C_{qq} - 2\alpha\sqrt{C_{qq}C_{ee}} + \alpha^2 C_{ee}\right] \\ &\leq \frac{-1}{\delta_1}\left[-P_q + (C_{qq}^{1/2} - \alpha C_{ee}^{1/2})^2\right] < 0 \end{aligned}$$

where the second inequality is thanks to (1).■

The equilibrium permit price depends on α, $\bar{e}$ and shocks θ and η according to the function $r(\alpha, \bar{e}, \theta, \eta)$, which is the unique solution of $\tilde{e}(\alpha, r, \theta, \eta) - \alpha\tilde{q}(\alpha, r, \theta, \eta) = \bar{e}$.

The equilibrium levels of output and emissions, $q(\alpha, \bar{e}, \theta, \eta)$ and $e(\alpha, \bar{e}, \theta, \eta)$ respectively, are the (unique) solution of the system of equations:

$$P(q,\theta) - C_q(q,e,\eta) - \alpha C_e(q,e,\eta) = 0$$
$$\alpha q + \bar{e} = e$$

For the comparative static of this equilibrium, first introduce

$$\delta_2 = -P_q + C_{qq} + 2\alpha C_{qe} + \alpha^2 C_{ee} > 0.$$

See proof of Result A1 for the sign of δ_2. Let us now consider in turn the influence of $\bar{e}$, α and the shocks θ and η on both q and e.

1. The influence of $\bar{e}$ is given by:

$$\begin{bmatrix} q_{\bar{e}} \\ e_{\bar{e}} \end{bmatrix} = \frac{1}{\delta_2} \begin{bmatrix} -C_{qe} - \alpha C_{ee}) \\ q(C_{qq} + \alpha C_{qe} - P_q) \end{bmatrix} \tag{38}$$

   For small values of α both signs are positive. For large values of α there is a counteracting effect via the subsidy and free allocations. With a larger number of auctioned permits the permit price is lower (see Result A1 above) and so is the OBA subsidy. This latter effect can more than offset the increase in production that results from the lower production cost associated to cheaper permits.

2. The influence of α is given by:

$$\begin{bmatrix} q_{\alpha} \\ e_{\alpha} \end{bmatrix} = \frac{1}{\delta_2} \begin{bmatrix} -C_e - q(C_{qe} + \alpha C_{ee}) \\ -\alpha C_e + q(C_{qq} + \alpha C_{qe} - P_q) \end{bmatrix} = -\frac{C_e}{\delta_2} \begin{bmatrix} 1 \\ \alpha \end{bmatrix} + q \begin{bmatrix} q_{\bar{e}} \\ e_{\bar{e}} \end{bmatrix} \tag{39}$$

   In both lines the first term comes from the subsidy component of the scheme, and the second term is the effect via the increase in the number of permits, which is equal to the effect of the quantity of auctioned permits times output.

3. The influence of θ is given by:

$$\begin{bmatrix} q_{\theta} \\ e_{\theta} \end{bmatrix} = \frac{P_\theta}{\delta_2} \begin{bmatrix} 1 \\ \alpha \end{bmatrix} \tag{40}$$

   Both quantities are increasing in θ. However, the monotonicity of the permit price is not that clear as the next expression shows:

$$r_\theta = -C_{qe}q_\theta - C_{ee}e_\theta = \frac{P_\theta}{\delta_2}[-C_{qe} - \alpha C_{ee}] \tag{41}$$

The first term in the brackets is positive but the second one is negative and comes from the OBA subsidy. If demand increases and α is large, an increase in the number of OBA permits can more than compensate the pressure on marginal abatement cost coming from a higher output.

4. Finally, the influence of η is given by:

$$\begin{bmatrix} q_\eta \\ e_\eta \end{bmatrix} = \frac{-C_{q\eta} - \alpha C_{e\eta}}{\delta_2} \begin{bmatrix} 1 \\ \alpha \end{bmatrix} \tag{42}$$

Both quantities change in the same direction with respect to η, because of their relationship with the fixed quantity of auctioned permits: emissions can increase only if production increases and more permits are emitted. The sign of the monotonicity depends on the sign of the influence of η on the marginal production cost and the subsidy. For $\alpha = 0$ only the former matters. On the other hand, the influence of η on the permit price is the sum of a term related to the direct effect of η on abatement costs and a term related to the adjustment of production and emissions. At $\alpha = 0$ we have

$$r_\eta|_{\alpha=0} = -C_{e\eta} - C_{qe}q_\eta - C_{ee}e_\eta = -C_{e\eta} + C_{qe}\frac{C_{q\eta}}{\delta_2} \tag{43}$$

A.2 Proof of Lemma 2

We first prove that the covariance of two increasing functions of a random variable is positive.

Result A2. If ϕ and ψ are two real valued strictly increasing functions of θ then $\text{cov}(\phi(\theta), \psi(\theta)) > 0$.

Proof. Consider a second random variable ϵ independent from θ with the same distribution; then

$$2\text{cov}(\phi(\theta), \psi(\theta)) = \text{cov}(\phi(\theta), \psi(\theta)) + \text{cov}(\phi(\epsilon), \psi(\epsilon)) = \text{cov}(\phi(\theta) - \phi(\epsilon), \psi(\theta) - \psi(\epsilon))$$

and $\text{cov}(\phi(\theta) - \phi(\epsilon), \psi(\theta) - \psi(\epsilon)) = \mathbb{E}[(\phi(\theta) - \phi(\epsilon)) \times (\psi(\theta) - \psi(\epsilon))]$ and $\forall(\theta, \epsilon) \in [\theta_{\min}, \theta_{\max}]^2$ the product $(\phi(\theta) - \phi(\epsilon)) \times (\psi(\theta) - \psi(\epsilon))$ is positive and strictly so if $\theta \neq \epsilon$.■

We now use the comparative static on θ and η performed in Appendix A.1 to establish the following:

- Case 1: $P_\theta > 0$ and $C_\eta = 0$. From equations (40) and (41), r and q are both increasing in θ and invariant to changes in η. This latter implies that $\mathrm{cov}(r, q) > 0$, so from Proposition 1 we have that a small increase in α above 0 augments welfare.

- Case 2: $P_\theta = 0$, $C_{q\eta} > 0$ and $C_{e\eta} > 0$. From (42) and (43), r and q are both decreasing in η, so they covary, which, from Proposition 1, indicates that a small increase in α above 0 augments welfare.

- Case 3: $P_\theta = 0$, $C_{q\eta} > 0$ and $-C_{e\eta} > -C_{qe}C_{q\eta}/(-P_q + C_{qq})$. From (42) and (43), q is decreasing in η but and r is increasing in it, so $\mathrm{cov}(r, q) < 0$. Consequently, a small increases in α decreases welfare.

B Multi-sector case

B.1 Proof of Proposition 3

Preliminaries

To ease the presentation we will omit the shocks θ and η, unless otherwise necessary to avoid confusion. We will first determine how equilibrium output and permit prices vary with respect to $\bar{e}$ and OBA rates. For that, let us introduce the sectorial residual demand $Q_i(t)$, which is the unique solution of $P_i(x_i) - C_i'(x_i) = t$. For each sector $i = 1, ..., n$, the function $Q_i(.)$ is positive, decreasing and differentiable with $Q_i' = 1/(P_i' - C_i'')$. At the market equilibrium, sector i's output is $q_i = Q_i((1 - \alpha_i)r)$.

The equilibrium permit price is a function $r(\bar{e}, \alpha_1, .., \alpha_n)$ that solves the equation

$$\sum\nolimits_i (1 - \alpha_i)Q_i((1 - \alpha_i)r) = \bar{e}. \tag{44}$$

Taking the derivative of (44) with respect to $\bar{e}$ gives

$$r_{\bar{e}} = \left[\sum\nolimits_i (1 - \alpha_i)^2 Q_i'\right]^{-1} \tag{45}$$

and with respect to α_k for $k = 1, ..., n$

$$\sum\nolimits_i (1 - \alpha_i)^2 Q_i' r_{\alpha_k} = Q_k + (1 - \alpha_k)rQ_k'$$

so

$$r_{\alpha_k} = r_{\bar{e}}[q_k + (1-\alpha_k)rQ'_k].$$

On the other hand, changes in equilibrium output wrt to $\bar{e}$ and OBA rates are given by

$$q_{i\bar{e}} = (1-\alpha_i)Q'_i r_{\bar{e}}$$

and

$$\begin{aligned} q_{i\alpha_k} \equiv \frac{\partial q_i}{\partial \alpha_k} &= (1-\alpha_i)Q'_i r_{\alpha_k} = (1-\alpha_i)Q'_i r_{\bar{e}}[q_k + (1-\alpha_k)rQ'_k] \\ &= q_{i\bar{e}}q_k + (1-\alpha_i)Q'_i(1-\alpha_k)Q'_k r r_{\bar{e}} \end{aligned} \tag{46}$$

for all $i \neq k$, and

$$q_{k\alpha_k} = -rQ'_k + (1-\alpha_k)Q'_k r_{\alpha_k} = -rQ'_k + q_{k\bar{e}}q_k + [(1-\alpha_k)Q'_k]^2 r r_{\bar{e}} \tag{47}$$

Choice of the cap

The effect of $\bar{e}$ on welfare for any given state of demand and supply is

$$\begin{aligned} \frac{\partial W}{\partial \bar{e}} &= \sum\nolimits_i (1-\alpha_i) r q_{i\bar{e}} - D' \sum\nolimits_i q_{i\bar{e}} = (r - D') \sum\nolimits_i q_{i\bar{e}} - r \sum\nolimits_i \alpha_i q_{i\bar{e}} \\ &= (r - D') - D' \sum\nolimits_i \alpha_i q_{i\bar{e}} = (r - D') - D' \sum\nolimits_i \alpha_i (1-\alpha_i) Q'_i r_{\bar{e}} \end{aligned} \tag{48}$$

then, using $Q'_i = 1/(P'_i - C''_i)$ yields equation (22).

Choice of the OBA rates α_i

Since $\sum q_i = \bar{e} + \sum \alpha_i q_i$, we have $\sum q_{i\alpha_k} = q_k + \sum \alpha_i q_{i\alpha_k}$, so that

$$\frac{\partial W}{\partial \alpha_k} = (r - D') \sum\nolimits_i q_{i\alpha_k} - r \sum\nolimits_i \alpha_i q_{i\alpha_k} = (r - D') q_k - D' \sum\nolimits_i \alpha_i q_{i\alpha_k}$$

Which correspond to equation 21. We now use the Preliminary results above to isolate the effect of releasing q_k free permits from the "subsidy effect". Using equations (46) and (47) yields

$$\begin{aligned} \frac{\partial W}{\partial \alpha_k} &= \left[(r - D') - D' \sum\nolimits_i \alpha_i q_{i\bar{e}} \right] q_k \\ &\quad - D' \left\{ (1-\alpha_k) Q'_k \sum\nolimits_i [\alpha_i (1-\alpha_i) Q'_i] r r_{\bar{e}} - \alpha_k r Q'_k \right\} \end{aligned} \tag{49}$$

Since the first bracketed term is equal to the derivative of welfare with respect to $\bar{e}$ times output q_k, making use of (45) we obtain

$$\begin{aligned}
\frac{\partial W}{\partial \alpha_k} &= \frac{\partial W}{\partial \bar{e}} q_k - r r_{\bar{e}} Q'_k D' \sum_i \left[\alpha_i (1-\alpha_i)(1-\alpha_k) Q'_i - \alpha_k (1-\alpha_i)^2 Q'_i\right] \\
&= \frac{\partial W}{\partial \bar{e}} q_k - r r_{\bar{e}} Q'_k D' \sum_i \left[(1-\alpha_i)(\alpha_i - \alpha_k) Q'_i\right] \qquad (50)
\end{aligned}$$

Taking expectations and using the fact that $\bar{e}$ is optimal chosen we obtain

$$\mathbb{E}\left[\frac{\partial W}{\partial \bar{e}} q_k\right] = \mathbb{E}\left[\frac{\partial W}{\partial \bar{e}}\right] \mathbb{E}\left[q_k\right] + \operatorname{cov}\left(\frac{\partial W}{\partial \bar{e}}, q_k\right) = \operatorname{cov}\left(\frac{\partial W}{\partial \bar{e}}, q_k\right)$$

But $Q'_i = -1/(C''_i - p'_i)$, which shows that expression (23) holds.

B.2 Linear specification and proof of Proposition 4

Permit market equilibrium

At this stage $\bar{e}$ and $(\alpha_i)_{i\in\{1,n\}}$ are fixed, and for each realization of the n-tuple $(\theta_i)_{i\in\{1,n\}}$ output quantities and the permit price are determined by market clearing of each output market and the emission permit market.

Let $\beta_i = b_i + \gamma_i$ be the slope of the sectoral demand, $Q_i(t) = (a_i + \theta_i - t)/\beta_i$. We write $\tilde{a}_i = a_i + \theta_i$, the intersect of the demand in state θ_i. At equilibrium $q_i = (\tilde{a}_i - (1-\alpha_i)r)/\beta_i$ and the permit price clears the permit market $\sum_i q_i = \bar{e} + \sum_i \alpha_i q_i$, that is,

$$\bar{e} = \sum\nolimits_i \left[(1-\alpha_i)\frac{\tilde{a}_i}{\beta_i}\right] - r \sum\nolimits_i \left[\frac{(1-\alpha_i)^2}{\beta_i}\right]$$

so the permit price is

$$r = \frac{\sum_i \left[(1-\alpha_i)\tilde{a}_i/\beta_i\right] - \bar{e}}{\sum_i \left[(1-\alpha_i)^2/\beta_i\right]} \qquad (51)$$

Choice of the cap

From (48) and (51) we obtain that the effect of $\bar{e}$ on welfare is given by

$$\frac{\partial W}{\partial \bar{e}} = (r-h) - h \sum_i \alpha_i q_{i\bar{e}} = r - h - h \frac{\sum_i [\alpha_i (1-\alpha_i)/\beta_i]}{\sum_i (1-\alpha_i)^2/\beta_i} \qquad (52)$$

so at the optimal $\bar{e}(\alpha_1, ..., \alpha_n)$ we have the expression (27) of the expected permit price and, from (51), the optimal cap is

$$\bar{e}(\alpha_1, .., \alpha_n) = \sum_i (1-\alpha_i)(a_i - h)/\beta_i \tag{53}$$

Note that $\partial W/\partial \bar{e}$ is random and in expression (52) the random component is in r.

Choice of the OBA rates α_k

The optimal α_k satisfies equation (23). Using $D' = h$, $Q'_i = -1/\beta_i$ and $r_{\bar{e}} = 1/\sum((1-\alpha_i)^2/\beta_i)$ yields the first order condition:

$$\begin{aligned}
\mathrm{cov}\left(\frac{\partial W}{\partial \bar{e}}, q_k\right) &= \frac{h\mathbb{E}[r]}{\beta_k \sum_i [(1-\alpha_i^2)/\beta_i]} \sum_i \left[\frac{1-\alpha_i}{\beta_i}(\alpha_k - \alpha_i)\right] \\
&= \frac{h^2}{\beta_k} \frac{\sum_i (1-\alpha_i)/\beta_i}{[\sum_i (1-\alpha_i)^2/\beta_i]^2} \sum_i \left[\frac{1-\alpha_i}{\beta_i}(\alpha_k - \alpha_i)\right] \text{ using eq. (27)}
\end{aligned} \tag{54}$$

We also know that

$$\begin{aligned}
\mathrm{cov}\Big(\frac{\partial W}{\partial \bar{e}}, q_k\Big) &= \mathrm{cov}(r, (\tilde{a}_k - (1-\alpha_k)r)/\beta_k) \text{ from eq. (52)} \\
&= \mathrm{cov}\left(\frac{\sum_i [(1-\alpha_i)\theta_i/\beta_i]}{\sum_i [(1-\alpha_i)^2/\beta_i]}, \frac{1}{\beta_k}\left[\theta_k - (1-\alpha_k)\frac{\sum_i [(1-\alpha_i)\theta_i/\beta_i]}{\sum_i [(1-\alpha_i)^2/\beta_i]}\right]\right) \text{ using eq. (51)} \\
&= \frac{1/\beta_k}{[\sum_i (1-\alpha_i)^2/\beta_i]^2} \mathrm{cov}\left(\sum_i \frac{1-\alpha_i}{\beta_i}\theta_i, \theta_k \sum_i \frac{(1-\alpha_i)^2}{\beta_i} - (1-\alpha_k)\sum_i \frac{1-\alpha_i}{\beta_i}\theta_i\right) \\
&= \frac{1/\beta_k}{[\sum_i (1-\alpha_i)^2/\beta_i]^2}\left[\sigma_k^2 \frac{1-\alpha_k}{\beta_k}\sum_i \frac{(1-\alpha_i)^2}{\beta_i} - (1-\alpha_k)\sum_i \frac{(1-\alpha_i)^2}{\beta_i^2}\sigma_i^2\right] \\
&= \frac{(1-\alpha_k)/\beta_k}{(\sum_i (1-\alpha_i)^2/\beta_i)^2} \sum_i \left[\frac{(1-\alpha_i)^2}{\beta_i}\left(\frac{\sigma_k^2}{\beta_k} - \frac{\sigma_i^2}{\beta_i}\right)\right]
\end{aligned} \tag{55}$$

so, combining the two expressions above, the optimal rate α_k satisfies

$$\sum_i \left[\frac{(1-\alpha_i)^2}{\beta_i}\left(\frac{\sigma_k^2}{\beta_k} - \frac{\sigma_i^2}{\beta_i}\right)\right] = h^2 \left[\sum_i \frac{1-\alpha_i}{\beta_i}\right] \sum\nolimits_i \left[\frac{1-\alpha_i}{\beta_i}\frac{\alpha_k - \alpha_i}{1-\alpha_k}\right] \tag{56}$$

Using the latter to substract expressions associated to k and l, and writing $(\alpha_k - \alpha_i)/(1-\alpha_k) = (1-\alpha_i)/(1-\alpha_k) - 1$, yields

$$\frac{\sigma_k^2}{\beta_k} - \frac{\sigma_l^2}{\beta_l} = h^2 \left[\sum_i \frac{1-\alpha_i}{\beta_i}\right]\left[\frac{1}{1-\alpha_k} - \frac{1}{1-\alpha_l}\right] \tag{57}$$

Therefore, if $\sigma_k^2/\beta_k > \sigma_l^2/\beta_l$ then $\alpha_k > \alpha_l$.

Since α_1 is the lowest rate, we can set $\alpha_1 = 0$ and using equation (57) we obtain for all other $k = 2, ..., n$

$$\alpha_k = \frac{\Delta_k}{\Delta_k + \Psi}$$

where Δ_k is given by (26) and $\Psi = \sum_i (1-\alpha_i)/\beta_i$. Finally, summing over k gives that Ψ is a solution of the equation

$$\sum_{i=1}^{n} \frac{1}{\beta_i(\Delta_i + x)} = 1.$$

There is a unique solution to this equation between 0 and $+\infty$, it is lower than $\sum_i (1/\beta_i)$, because the left-hand-side is strictly decreasing, and, since $\Delta_1 = 0$, it is equal to $+\infty$ for $x = 0$, and for $x = \sum_i 1/\beta_i$, it is lower than 1.

Then for Ψ the unique positive solution of this equation, define $\alpha_i = \Delta_i/(\Delta_i + \Psi)$ for all $i > 1$ which is between 0 and 1, and $\bar{e}$ is given by equation (53). They all satisfy the first order conditions and thus, maximize expected welfare.

B.3 Proof of Proposition 5

Let us assume that (i) $a > h(1 + 2\gamma/b)$ and (ii) $\sigma_1^2 < h^2 \sum_{i=1}^{n} (1 - \alpha_i)$ hold, and show that the optimal OBA scheme in Proposition 4 can be implemented with a menu of OBA rates and lump-sum allocations $\{(\alpha_j, \hat{e}_j)/i = 1, ..., n\}$ as described in (33).

At the optimum scheme described in Proposition 4, for $\beta_i = \beta \; \forall i$, we have $\psi = \sum_i (1 - \alpha_i)/\beta$ and $\alpha_i = \Delta_i/(\delta_i + \psi)$ and $\sigma_i^2 - \sigma_1^2 = \alpha_i \beta \psi h^2$

We proceed in three steps: we first show that the derivative of firm $i + 1$'s profit with respect to α is higher than the derivative of firm i's. Then we show that this result ensures that the lump-sum (free) allocations (33) are sufficient for the self-selection constraints to hold. And finally we show that the regulator has enough permits to offer the lump-sum allocations (33).

We first show that the influence of the OBA rate on profit is larger for the more volatile sectors

$$\mathbb{E}\left[\frac{\partial \pi_{i+1}(\theta_{i+1}, r, \alpha)}{\partial \alpha}\right] \geq \mathbb{E}\left[\frac{\partial \pi_i(\theta_i, r, \alpha)}{\partial \alpha}\right] \tag{58}$$

Using (29) and (31), the above inequality can be written as

$$(1 - \alpha_{i+1})\sigma_{i+1}^2 \geq (1 - \alpha_i)\sigma_i^2 \tag{59}$$

for $i = 1..n-1$. Then, using $\sigma_i^2 - \sigma_1^2 = h^2\beta\alpha_i\psi/(1-\alpha_i)$, eq. (62) is equivalent to

$$
\begin{aligned}
(1-\alpha_{i+1})(\sigma_{i+1}^2 - \sigma_1^2) + (1-\alpha_{i+1})\sigma_1^2 \geq (1-\alpha_i^2)(\sigma_i^2 - \sigma_1^2) + (1-\alpha_i)\sigma_1^2 \\
(\alpha_{i+1} - \alpha_i)\beta\psi h^2 \geq (\alpha_{i+1} - \alpha_i)\sigma_1^2 \\
h^2 \sum_i (1-\alpha_i) \geq \sigma_1^2
\end{aligned}
$$

which corresponds to assumption (ii) in Proposition 5.

Thanks to this property, all differences $\hat{e}_{i+1} - \hat{e}_i$ in (33) are positive, and the self-selection constraints (28) are all satisfied. Consider $i, j = 1, ..., n$, sector i prefers its option to option j if $i < j$:

$$
\begin{aligned}
&\mathbb{E}[\pi_i(\theta_i, r, \alpha_j) - \pi_i(\theta_i, r, \alpha_i)] = \sum_{k=i}^{j-1} \mathbb{E}[\pi_i(\theta_i, r, \alpha_{k+1}) - \pi_i(\theta_i, r, \alpha_k)] \\
&\leq \sum_{k=i}^{j-1} \mathbb{E}[\pi_k(\theta_i, r, \alpha_{k+1}) - \pi_k(\theta_i, r, \alpha_k)] \text{ thanks to (61)} \\
&\leq \sum_{k=i}^{j-1} \mathbb{E}[r](\hat{e}_k - \hat{e}_{k+1}) = \mathbb{E}[r](\hat{e}_i - \hat{e}_j) \text{ by definition of the menu (33).}
\end{aligned}
$$

A similar reasoning shows that it also holds for $i > j$.

We now establish that thanks to assumption (i) the budget-balance constraint is satisfied. The regulator has enough permits to implement the optimal scheme with the allocations described in (33)

$$
\bar{e} \geq \sum_{i=1}^{n} \hat{e}_i = \sum_{i=1}^{n-1} i \times \frac{1}{\mathbb{E}[r]} \mathbb{E}[\pi_i(\theta_i, r, \alpha_{i+1}) - \pi_i(\theta_i, r, \alpha_i)] \tag{60}
$$

The difference in profit is equal to

$$
\begin{aligned}
&\mathbb{E}[\pi_i(\theta_i, r, \alpha_{i+1}) - \pi_i(\theta_i, r, \alpha_i)] = \frac{\gamma}{2\beta^2}(\alpha_{i+1} - \alpha_i)\mathbb{E}\left[r\left[2(a+\theta_i) - (2-\alpha_{i+1} - \alpha_i)r\right]\right] \\
&= \frac{\gamma}{2\beta^2}(\alpha_{i+1} - \alpha_i)\left\{\mathbb{E}[r] \times [2a - (2-\alpha_{i+1} - \alpha_i)\mathbb{E}[r]] + \text{cov}(2\theta_i - (2-\alpha_{i+1} - \alpha_i)r, r)\right\}
\end{aligned}
$$

So the constraint (63) is equivalent to

$$
\bar{e} > \frac{\gamma}{\beta^2} \sum_{i=1}^{n-1} i(\alpha_{i+1} - \alpha_i) \left\{ \left[a - (1 - \frac{\alpha_{i+1} + \alpha_i}{2})\mathbb{E}[r]\right] + \text{cov}\left(\theta_i - (1 - \frac{\alpha_{i+1} + \alpha_i}{2})r, \frac{r}{\mathbb{E}[r]}\right) \right\}
$$

Let us now establish an upper bound for the two terms on the right-hand side. Since $\alpha_n < 1$ we have that

$$\frac{\gamma}{\beta^2}\sum_{i=1}^{n-1} i(\alpha_{i+1}-\alpha_i)a = \frac{\gamma}{\beta^2}\sum_{i=1}^{n-1}(\alpha_n-\alpha_i)a < \psi\frac{\gamma}{\beta}a$$

and

$$\sum_j (1-\alpha_j)^2 \times \text{var}(r) = \frac{\sum_j (1-\alpha_j)^2\sigma_j^2}{\sum_j (1-\alpha_j)^2} = \frac{\sum_j (1-\alpha_j)^2(\sigma_j^2-\sigma_1^2)}{\sum_j (1-\alpha_j)^2} + \sigma_1^2 = \frac{\sum_j (1-\alpha_j)\alpha_j}{\sum_j (1-\alpha_j)^2}\beta\psi h^2 + \sigma_1^2$$

so that the covariance term is lower than h

$$\begin{aligned}
&\text{cov}(\theta_i - (1-\frac{\alpha_{i+1}+\alpha_i}{2})r, \frac{r}{\mathbb{E}r}) = \\
&\frac{1}{\mathbb{E}r\sum_j(1-\alpha_j)^2}\left\{(1-\alpha_i)(\sigma_i^2-\sigma_1^2) + \frac{\alpha_{i+1}-\alpha_i}{2}\sigma_1^2 - \left(1-\frac{\alpha_{i+1}+\alpha_i}{2}\right)\frac{\sum_j(1-\alpha_j)\alpha_j}{\sum_j(1-\alpha_j)^2}\beta\psi h^2\right\} \\
&< \frac{1}{\beta\psi h}\left\{\alpha_i\beta\psi h^2 + \frac{\alpha_{i+1}-\alpha_i}{2}\sigma_1^2\right\} = \frac{1}{\beta\psi h}\left\{\alpha_i(\beta\psi h^2 - \frac{\sigma_1^2}{2}) + \frac{\alpha_{i+1}}{2}\sigma_1^2\right\} \\
&< \frac{1}{\beta\psi h}\beta\psi h^2 = h
\end{aligned}$$

where the last inequality is obtained using $h^2 > \sigma_1^2$ (from (ii)), $\beta\psi > 1$, and $\alpha_i < \alpha_{i+1} < 1$. Then, the right-hand side of (63) is lower than

$$\frac{\gamma}{\beta^2}\beta\psi a + \frac{\gamma}{\beta^2}\sum_{i=1}^{n-1}\left[i(\alpha_{i+1}-\alpha_i)\right]h < \frac{\gamma}{\beta^2}\beta\psi a + \frac{\gamma}{\beta^2}\beta\psi h = (a+h)\frac{\gamma}{\beta}\psi$$

and

$$a > h(1+2\gamma/b) \Leftrightarrow (a+h)\frac{\gamma}{\beta} < a-h$$

so that, if $a > h(1+2\gamma/b)$ then (63) is satisfied, because $\bar{e} > a-h$ at the optimal scheme.

B.4 Proof of Proposition 5

Let us assume that (i) $a > h(1+2\gamma/b)$ and (ii) $\sigma_1^2 < h^2\sum_{i=1}^n(1-\alpha_i)$ hold, and show that the optimal OBA scheme in Proposition 4 can be implemented with a menu of OBA rates and lump-sum allocations $\{(\alpha_j, \hat{e}_j)/i = 1, ..., n\}$ as described in (33).

At the optimum scheme described in Proposition 4, for $\beta_i = \beta\ \forall i$, we have $\psi = \sum_i(1-\alpha_i)/\beta$ and $\alpha_i = \Delta_i/(\delta_i+\psi)$ and $\sigma_i^2 - \sigma_1^2 = \alpha_i\beta\psi h^2$

We proceed in three steps: we first show that the derivative of firm $i+1$'s profit with respect to α is higher than the derivative of firm i's. Then we show that this result ensures that the lump-sum (free) allocations (33) are sufficient for the self-selection constraints to hold. And finally we show that the regulator has enough permits to offer the lump-sum allocations (33).

We first show that the influence of the OBA rate on profit is larger for the more volatile sectors

$$\mathbb{E}\left[\frac{\partial \pi_{i+1}(\theta_{i+1}, r, \alpha)}{\partial \alpha}\right] \geq \mathbb{E}\left[\frac{\partial \pi_i(\theta_i, r, \alpha)}{\partial \alpha}\right] \tag{61}$$

Using (29) and (31), the above inequality can be written as

$$(1 - \alpha_{i+1})\sigma_{i+1}^2 \geq (1 - \alpha_i)\sigma_i^2 \tag{62}$$

for $i = 1..n-1$. Then, using $\sigma_i^2 - \sigma_1^2 = h^2 \beta \alpha_i \psi/(1 - \alpha_i)$, eq. (62) is equivalent to

$$\begin{aligned}
(1 - \alpha_{i+1})(\sigma_{i+1}^2 - \sigma_1^2) + (1 - \alpha_{i+1})\sigma_1^2 &\geq (1 - \alpha_i^2)(\sigma_i^2 - \sigma_1^2) + (1 - \alpha_i)\sigma_1^2 \\
(\alpha_{i+1} - \alpha_i)\beta\psi h^2 &\geq (\alpha_{i+1} - \alpha_i)\sigma_1^2 \\
h^2 \sum_i (1 - \alpha_i) &\geq \sigma_1^2
\end{aligned}$$

which corresponds to assumption (ii) in Proposition 5.

Thanks to this property, all differences $\hat{e}_{i+1} - \hat{e}_i$ in (33) are positive, and the self-selection constraints (28) are all satisfied. Consider $i, j = 1, ..., n$, sector i prefers its option to option j if $i < j$:

$$\begin{aligned}
\mathbb{E}[\pi_i(\theta_i, r, \alpha_j) - \pi_i(\theta_i, r, \alpha_i)] &= \sum_{k=i}^{j-1} \mathbb{E}[\pi_i(\theta_i, r, \alpha_{k+1}) - \pi_i(\theta_i, r, \alpha_k)] \\
&\leq \sum_{k=i}^{j-1} \mathbb{E}[\pi_k(\theta_i, r, \alpha_{k+1}) - \pi_k(\theta_i, r, \alpha_k)] \text{ thanks to (61)} \\
&\leq \sum_{k=i}^{j-1} \mathbb{E}[r](\hat{e}_k - \hat{e}_{k+1}) = \mathbb{E}[r](\hat{e}_i - \hat{e}_j) \text{ by definition of the menu (33).}
\end{aligned}$$

A similar reasoning shows that it also holds for $i > j$.

We now establish that thanks to assumption (i) the budget-balance constraint is satisfied. The regulator has enough permits to implement the optimal scheme with the allocations described in (33)

$$\bar{e} \geq \sum_{i=1}^{n} \hat{e}_i = \sum_{i=1}^{n-1} i \times \frac{1}{\mathbb{E}[r]} \mathbb{E}[\pi_i(\theta_i, r, \alpha_{i+1}) - \pi_i(\theta_i, r, \alpha_i)] \tag{63}$$

The difference in profit is equal to

$$\mathbb{E}[\pi_i(\theta_i, r, \alpha_{i+1}) - \pi_i(\theta_i, r, \alpha_i)] = \frac{\gamma}{2\beta^2}(\alpha_{i+1} - \alpha_i)\mathbb{E}\left[r\left[2(a+\theta_i) - (2 - \alpha_{i+1} - \alpha_i)r\right]\right]$$
$$= \frac{\gamma}{2\beta^2}(\alpha_{i+1} - \alpha_i)\left\{\mathbb{E}[r] \times [2a - (2 - \alpha_{i+1} - \alpha_i)\mathbb{E}[r]] + \text{cov}(2\theta_i - (2 - \alpha_{i+1} - \alpha_i)r, r)\right\}$$

So the constraint (63) is equivalent to

$$\bar{e} > \frac{\gamma}{\beta^2} \sum_{i=1}^{n-1} i(\alpha_{i+1} - \alpha_i) \left\{ \left[a - (1 - \frac{\alpha_{i+1} + \alpha_i}{2})\mathbb{E}[r] \right] + \text{cov}\left(\theta_i - (1 - \frac{\alpha_{i+1} + \alpha_i}{2})r, \frac{r}{\mathbb{E}[r]} \right) \right\}$$

Let us now establish an upper bound for the two terms on the right-hand side. Since $\alpha_n < 1$ we have that

$$\frac{\gamma}{\beta^2} \sum_{i=1}^{n-1} i(\alpha_{i+1} - \alpha_i)a = \frac{\gamma}{\beta^2} \sum_{i=1}^{n-1} (\alpha_n - \alpha_i)a < \psi\frac{\gamma}{\beta}a$$

and

$$\sum_j (1-\alpha_j)^2 \times \text{var}(r) = \frac{\sum_j (1-\alpha_j)^2 \sigma_j^2}{\sum_j (1-\alpha_j)^2} = \frac{\sum_j (1-\alpha_j)^2 (\sigma_j^2 - \sigma_1^2)}{\sum_j (1-\alpha_j)^2} + \sigma_1^2 = \frac{\sum_j (1-\alpha_j)\alpha_j}{\sum_j (1-\alpha_j)^2}\beta\psi h^2 + \sigma_1^2$$

so that the covariance term is lower than h

$$\text{cov}(\theta_i - (1 - \frac{\alpha_{i+1} + \alpha_i}{2})r, \frac{r}{\mathbb{E}r}) =$$
$$\frac{1}{\mathbb{E}r \sum_j (1-\alpha_j)^2} \left\{ (1-\alpha_i)(\sigma_i^2 - \sigma_1^2) + \frac{\alpha_{i+1} - \alpha_i}{2}\sigma_1^2 - \left(1 - \frac{\alpha_{i+1} + \alpha_i}{2}\right) \frac{\sum_j (1-\alpha_j)\alpha_j}{\sum_j (1-\alpha_j)^2} \beta\psi h^2 \right\}$$
$$< \frac{1}{\beta\psi h} \left\{ \alpha_i \beta\psi h^2 + \frac{\alpha_{i+1} - \alpha_i}{2}\sigma_1^2 \right\} = \frac{1}{\beta\psi h} \left\{ \alpha_i(\beta\psi h^2 - \frac{\sigma_1^2}{2}) + \frac{\alpha_{i+1}}{2}\sigma_1^2 \right\}$$
$$< \frac{1}{\beta\psi h}\beta\psi h^2 = h$$

where the last inequality is obtained using $h^2 > \sigma_1^2$ (from (ii)), $\beta\psi > 1$, and $\alpha_i < \alpha_{i+1} < 1$. Then, the right-hand side of (63) is lower than

$$\frac{\gamma}{\beta^2}\beta\psi a + \frac{\gamma}{\beta^2} \sum_{i=1}^{n-1} \left[i(\alpha_{i+1} - \alpha_i) \right] h < \frac{\gamma}{\beta^2}\beta\psi a + \frac{\gamma}{\beta^2}\beta\psi h = (a+h)\frac{\gamma}{\beta}\psi$$

and

$$a > h(1+2\gamma/b) \Leftrightarrow (a+h)\frac{\gamma}{\beta} < a-h$$

so that, if $a > h(1+2\gamma/b)$ then (63) is satisfied, because $\bar{e} > a-h$ at the optimal scheme.

B.5 Proof of Corollary 1

Let us consider the slightly more general situation, the one in Proposition 4: $p_i = a_i + \theta_i - b_i q_i$ and $C_i = \gamma_i q_i^2/2$ denoting $\beta_i = b_i + \gamma_i$ and $\tilde{a}_i = a_i + \theta_i$. Using the results in this Proposition and setting $\alpha_1 = 0$, the optimal cap is given by

$$\bar{e} = (a_1 - h)/\beta_1 + (1-\alpha_2)(a_2 - h)/\beta_2$$

and the expression in Corollary 1 is obtained by simply replacing $a_1 = a_2 = a$ and $\beta_1 = \beta_2 = 2$. Let us write $\Delta_2 = [\sigma_2^2/\beta_2 - \sigma_1^2/\beta_1]/h^2$ as in Proposition 4. The optimal α_2 is equal to $\Delta_2/(\Delta_2 + \Psi)$ with $\Psi = 1/\beta_1 + (1-\alpha_2)/\beta_2$, so $(1-\alpha_2)(\Delta_2 + \Psi) = \Psi$

$$(1-\alpha_2)(\Delta_2 + 1/\beta_1 + (1-\alpha_2)/\beta_2) = 1/\beta_1 + (1-\alpha_2)/\beta_2$$

This implies that $(1-\alpha_2)/\beta_2$ is the positive solution of the equation

$$x^2 + x\Delta - \frac{1}{\beta_1\beta_2} = 0$$

with $\Delta = \Delta_2 + 1/\beta_1 - 1/\beta_2$ (which corresponds to the value of Δ defined in Corollary 1 for $\beta_1 = \beta_2 = 2$), which yields

$$\frac{1-\alpha_2}{\beta_2} = \frac{1}{2}\left[\left(\Delta^2 + \frac{4}{\beta_1\beta_2}\right)^{1/2} - \Delta\right].$$

The equivalent expression in Corollary 1 is obtained, again, by simply replacing $\beta_1 = \beta_2 = 2$ and $\sigma_1 = 0$, and making use of Lemma 3 on the equivalence of schemes.