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Using output-based allocations to manage volatility and leakage in pollution markets

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Abstract

Output-based allocations (OBAs) are typically used in emission trading systems (ETS) with a fixed cap to mitigate leakage in sectors at risk. Recent work has shown they may also be welfare enhancing in markets subject to supply and demand shocks by introducing some flexibility in the total cap, resulting in a carbon price closer to marginal damage. We extend previous work to simultaneously include both leakage and volatility. We study how OBA permits can be implemented under an overall cap that may change with the level of production in contrast with a design that deducts OBA permits from the overall permit allocation as is the current practice in the EU-ETS and California. We show that introducing OBA permits while keeping the overall cap fixed would only increase price fluctuations and induce severe welfare losses to non-OBA sectors.

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1 Introduction

This paper is motivated by two critical issues in the design and implementation of emission trading systems (ETS), namely, that permit prices are rather often away from the level they were expected to be at the time the regulation was set (presumably the expected marginal benefit of pollution abatement for a given reference scenario) and how to allocate permits to firms. We analyze the interaction of these two issues. In the EU-ETS, for instance, a fraction of permits is allocated through auctions while the remaining fraction is allocated for free to firms in industries that are likely to face international competition from unregulated source such as in cement, petrochemicals, steel, etc. These free permits are typically allocated according to actual output based on some benchmark pollution intensity.[1]

These output-based free allocations (OBAs) are intended to solve the so-called "carbon leakage" problem, i.e., that the reduction of home carbon emissions is partly offset by a rise in foreign emissions. An OBA scheme, by subsidizing home production, reduces unregulated foreign production. There is an extensive economic literature exploring the benefit and cost of using OBAs for dealing with carbon leakage, see, for example Fischer and Fox (2007); Quirion (2009); Monjon and Quirion (2011); Fischer and Fox (2012); Meunier et al. (2014). They appear as good second best solutions in the absence of border tax adjustments.[2]

The use of OBA schemes, however, raises another important question in the design of permit markets that are subject to demand and supply shocks, which is whether the total cap should be kept fixed or flexible. In the EU-ETS and in California the cap is fixed (presumably for political reasons) and any difference between anticipated and actual permits going to OBA sectors are offset with deductions/additions of auction permits.

It would be perfectly feasible to introduce some flexibility in the total cap, in the spirit of the hybrid design of Roberts and Spence (1976), in which additional permits are either issued or bought back by the government at certain pre-specified prices.[3] The advantages of flexibility of the total cap is notably discussed in the intensity standard literature. With an intensity standard (also known as intensity target) the cap of emissions for a given country is indexed to its gross domestic product. They have been mostly introduced in the context of international negotiations on climate change following the Kyoto protocol. Such commitments appeared more acceptable for emerging countries (Dudek and Golub, 2003). The

[1] For a detailed description and empirical analysis of the EU-ETS allocation rules see Branger et al. (2015).

[2] In the US carbon leakage is likely to arise in the electricity sector because of electricity trade with neighboring states or uncovered plants. The use of OBA to mitigate leakage has been studied notably by Bushnell and Chen (2012) and Burtraw et al. (2016) in those cases.

[3] Note that Roberts and Spence (1976) collapse to a tax if the marginal damage of pollution is constant.

relevance of intensity standards has also been studied for developed economies. Ellerman and Wing (2003) develop a model to compare a fixed cap and an intensity target for Germany. Their analysis demonstrate that, due to the 2010 recession, an intensity standard (and thus a flexible cap) would have led to a more stringent carbon regulation, reflected in a higher carbon price closer to the social cost of carbon. Under uncertainty, a indexation of the cap on GDP can improve welfare by ensuring that emissions are high when they are more valuable (high marginal abatement cost) and vice-versa.[4]

The contribution of this paper is to combine the two ingredients mentioned above: carbon leakage and uncertainty, and study the performance of OBA schemes in that case. We further investigate the benefits associated with the flexibility of the cap. We built on a recent paper of ours (Meunier et al., 2016) in which it is demonstrated that, even in absence of leakage, there are good reasons, due to the induced flexible cap, to introduce OBA for sectors subject to large demand and supply shocks. The optimal OBA rate trades-off the benefits from flexibility with inefficiency associated to production subsidy.

In this paper we firstly generalize our previous results when leakage is also present. Without uncertainty the OBA rate should be equal to the leakage rate. This is no longer true under uncertainty because of the potential benefits of indexing the cap to the production of highly uncertain sectors. The larger the sector uncertainty, the higher the OBA rate for this sector should be. As a matter of fact, a large sector uncertainty should be considered as a factor as important as leakage for introducing OBA in that sector. This is an important and timely policy consideration since regulators are currently reviewing the allocation of free allocations in the EU-ETS for the period 2020-2030.

Secondly, we use the model to explore numerically how different OBA schemes manage permit price fluctuations and what are the implications of deducting OBA permits (the majority going to trade-exposed and carbon intensive sectors) from the overall permit allocation so as to keep the total cap on emissions fixed. Our numerical results show that an OBA scheme can significantly reduce carbon price fluctuation as long as its implementation considers a flexible cap on total emissions. Insisting on a fixed cap would only increase price fluctuations and induce severe welfare losses to non-OBA sectors (mainly electricity in the case of the EU-ETS). Furthermore, the introduction of OBA permits together with a flexible global cap generate almost no distortion in these non-OBA sectors.

We think that our results are particularly relevant for the current debate in the EU-ETS.

[4]The relative merit of the indexation of the cap to GDP under uncertainty has been studied notably by Quirion (2005) and Newell and Pizer (2008) in the tradition of the comparison of instruments à la Weitzman (1974).

To provide firms with some regulatory certainty regulators need to fix the contractual rules of ETSs, including the cap, long in advance, say in 2005 for the EU-ETS covering the period 2013-2020, or in 2016 for the EU-ETS covering the period 2021-2030. Back in 2005, they were unable to anticipate the uncertainties, such as the severe and durable European recession in market conditions, the new supply fuel sources such as shale gas and their implication on the price of coal, as well the new regulations that were put in place to promote renewable energy production. The unfolding of these uncertainties made the cap committed in 2005 to look little ambitious ex-post, that is, prices clearing at much lower levels than anticipated at the time of setting the cap. Furthermore, EU regulators face numerous legal and political constraints that prevent them from updating their previous commitments.

The inability to provide a long term signal for investment decisions has thrown doubts on the efficiency of the EU-ETS and various proposals to mitigate the problem such as introducing a stability mechanism are currently examined.[5] The EU-ETS is not exceptional in its inability to deliver a reasonable sequence of prices. A similar experience had been observed for the SO2 market (Schmalensee et al., 1998; Schmalensee and Stavins, 2013). More recently, Borenstein et al. (2015) reviewed the rules in place for the California CO2 market and showed that it is quite likely that future carbon prices will jump between floor and ceiling of a predetermined price corridor, which had appeared quite large at the time it was set.[6]

Our numerical results show that the introduction of some flexibility in the cap would somehow mitigate the issue of the fluctuation of the carbon price. If optimally designed, an OBA scheme together with a flexible cap ensures that carbon prices fluctuate less and remain closer to the social cost of carbon. All these results indicate that supply and demand shocks make a strong case for the use of OBAs, as long as it is associated with some flexibility in the total cap.

The rest of the paper is organized as follows. The model is presented in Section 2. Policy simulations are in Section 3. We conclude in Section 4. Some mathematical proofs are postponed to the Appendix.

2 Model

Consider an economy with two independent sectors, labeled $i = 1, 2$, each producing an homogenous good. The two sectors are covered by a common permit market, the functioning of

[5] `http://ec.europa.eu/clima/policies/ets/index_en.htm`

[6] The benefit of introducing OBA remains even with presence of a price corridor (Meunier et al., 2016).

which will be described shortly, and is the sole link between the two sectors. The total quantity consumed in sector i is q_i, which is sum of home production q_{ih} and foreign production q_{if}; this latter not subject to any pollution-control policy. Consumer gross surplus in sector $i = 1, 2$ is given by $S_i(q_i, \theta_i)$, where θ_i is a random shock, and the inverse demand function by $P_i(q_i, \theta_i) = \partial S_i(q_i; \theta_i)/\partial q_i$. Shocks θ_1 and θ_2 distribute according to some cumulative distribution function to be defined shortly. Note that these shocks can have both common and sectorial components, so one can write them as $\theta_i = v + \eta_i$, where v could be a shock affecting all sectors in the economy (e.g., recession)

We assume that production, both at home and abroad, is carried out by a group of identical price-taking firms. The cost at home in sector $i = 1, 2$ is given by $C_{ih}(q_i)$ and abroad by $C_{if}(q_{if})$. Output, whether produced domestically or internationally, leads to CO2 emissions at a rate that is normalized to one, so environmental harm is given by $D(e)$, where $e = q_1 + q_2$ are total emissions.

Denoting by $\mathbf{q}$ the quantity-quadruple $(q_{1h}, q_{2h}, q_{1f}, q_{2f})$, the social welfare function of the domestic regulator is given by

$$W(\mathbf{q}, \theta_1, \theta_2) = \sum_{i=1,2} [S_i(q_{ih} + q_{if}, \theta_i) - C_{ih}(q_{ih}) - C_{if}(q_{if})] - D(e) \tag{1}$$

Notice that in our welfare formulation foreign costs enter as if foreign plants were owned by home producers, just like domestic plants. This assumption is made mainly for methodological reasons. It allows us to exclusively focus on the regulator's incentive to control the environmental externality of production and discard any "protectionist" incentive he or she may have to displace foreign production in favor of home production. Second, we do not consider the foreign market and the possible change in foreign consumption induced by home regulation. Such a change would indeed affect world emissions and the magnitude of leakage.

The funds collected through ETSs may be used for different purposes such as reducing pre-existing taxes, financing R&D programs for green technologies or even for direct dividends back to households. Here there is no consideration of the opportunity cost of using part of these funds as a production incentive (OBA), this question can be pursued in future work.

2.1 OBA regulation and market equilibrium

In the absence of government intervention, the market equilibrium leads to too much pollution. To correct for this, the regulator implements a permit-market regulation at home where the total amount of permits may not be fixed but endogenous to output. The regu-

lator auctions off $\bar{e}$ permits and in addition allocates permits to home firms based on their output.[7] For each unit of output, a domestic firm in sector i gets α_i permits for free, so the total amount of home pollution/permits is equal to

$$e_h = \bar{e} + \alpha_1 q_{1h} + \alpha_2 q_{2h} \tag{2}$$

In what follows, we will refer to α_i as the OBA rate of sector $i = 1, 2$.

The regulator first sets the quantity of auctioned permits and the OBA rates, then firms learn shocks θ_i, after which they decide how much to produce and pollute anticipating the additional permits they will get for their output. Since the permit market is perfectly competitive, the auction clears at the price firms expect to trade permits in the secondary market. We denote this price by r. Thus, each firm at home takes r and the output price p_i as given and solves

$$\max_{q_{ih}} p_i q_{ih} - C_{ih}(q_{ih}) - r(1 - \alpha_i) q_{ih}$$

while each firm abroad solves

$$\max_{q_{if}} p_i q_{if} - C_{if}(q_{if})$$

leading to the first-order (equilibrium) conditions

$$p_i = C'_{ih}(q_{ih}) + (1 - \alpha_i) r = C'_{if}(q_{if}) \tag{3}$$

Equilibrium prices p_i and r are obtained using the inverse demand function $p_i = P_i(q_i; \theta_i)$ and the permit market constraint (2). Equilibrium productions are then a function of the regulatory variables and the demand states $\mathbf{q}^e(\bar{e}, \alpha_1, \alpha_2, \theta_1, \theta_2)$, so the expected welfare to be maximized by the (domestic) regulator is

$$\hat{W}(\bar{e}, \alpha_1, \alpha_2) = \mathbb{E} W(\mathbf{q}^e, \theta_1, \theta_2) \tag{4}$$

2.2 Optimal design in the absence of uncertainty

It is useful to consider first the case where shocks θ_1 and θ_2 are either absent or perfectly anticipated by the regulator. If the regulator could control production both at home and abroad, the first-best allocation is the quadruple $\mathbf{q}^*(\theta_1, \theta_2) = (q^*_{1h}, q^*_{2h}, q^*_{1f}, q^*_{2f})$ that satisfies

[7] In principle, the $\bar{e}$ permits could also be allocated for free to firms based, for example, on historic emissions. But as soon as we allow for some positive cost of public funds (Goulder et al., 1997), auctioning becomes optimal. Our implicit assumption in the article is that the cost of public funds is positive but arbitrarily small, so we do not need to explicitly model it.

the usual first-order conditions

$$P_i(q_{ih} + q_{if}) - C'_{ij}(q_{ij}) = D'(e) \tag{5}$$

for $i = 1, 2$ and $j = h, f$. However, the regulator can only control domestic production, in which case the second-best is given by

$$P_i(q_{ih} + q_{if}) - C'_{ih}(q_{ih}) = D'(e)\left(1 + \frac{\partial q_{if}}{\partial q_{ih}}\right) \tag{6}$$

for $i = 1, 2$ and where $\partial q_{if}/\partial q_{ih}$ is known as the leakage rate, which represents the increase in foreign production that results from a small reduction in home production. Using the equilibrium condition $P_i(q_{ih} + q_{if}) = C'_{if}(q_{if})$, the leakage rate can also be expressed as

$$l_i = -\frac{\partial q_{if}}{\partial q_{ih}} = -\frac{P'_i}{C''_{if} - P'_i}$$

The second-best solution in (6) can be implemented with a permit regulation that considers positive OBA rates as described in the following Proposition.

Proposition 1 *In the absence of uncertainty, the optimal permit scheme consists in setting OBA rates equal to leakage rates*

$$\alpha_i = l_i \tag{7}$$

and the quantity of auctioned permits $\bar{e}$ such that the equilibrium permit price is equal to marginal environmental damages

$$r = D'(e) \tag{8}$$

where $e = \bar{e} + \alpha_1 q_{1h} + \alpha_2 q_{2h} + q_{1f} + q_{2f}$.

This result establishes a welfare rationale for the implementation of OBA. To understand this result, consider unregulated foreign production as a function of domestic production $q_{if}(q_{ih})$. Then, it is *as if* there is a positive externality associated to home production equal to $-D'(e)\partial q_{if}/\partial q_{ih} = l_i \times r$, in addition to the negative externality associated to total emissions. Therefore, the permit price corrects for the negative externality and the OBA rates work as subsidies that correct for this positive externality.

The influence of any regulatory variable on welfare can be decomposed as follows

$$d\hat{W} = \sum_i [(1-\alpha_i)r - (1-l_i)D'(e)]dq_{ih} \tag{9}$$

$$= (r - D'(e))de_h - \sum_i [\alpha_i r - l_i D'(e)]dq_{ih} \tag{10}$$

where $de_h = dq_{1h} + dq_{2h}$. The first line adds the benefits and costs associated to the changes of each sector home production. The benefit is equal to the permit price corrected by the OBA rate and the cost is the marginal environmental damage corrected by the leakage rate.

With the present model, there are two quantities, q_{1h} and q_{2h}, indirectly controlled by three regulatory variables: $\bar{e}$, α_1 and α_2. Then, there is one degree of freedom and one can possibly set one of the OBA rates equal to zero and then adjust the other OBA rate and the permit price.

Corollary 1 *In the absence of uncertainty, the second-best permit scheme can be implemented by any pair of OBA rates that satisfy*

$$\frac{1-\alpha_2}{1-\alpha_1} = \frac{1-l_2}{1-l_1}$$

and a quantity of auctioned permits $\bar{e}$ such that the expected permit price is equal to the marginal environmental damage corrected by sector 1 *leakage rate :*

$$r = \frac{1-l_1}{1-\alpha_1} D'(e)$$

Proof. It can be directly seen by plugging the above expressions into the expression (9) of the derivative of welfare. ■

2.3 Optimal design under uncertainty

In presence of uncertainty, the regulator must set the OBA rate and the quantity of auctioned permits ex-ante, before shocks θ_1 and θ_2 are realized. In such a case the two OBA rates will differ from the leakage rates because OBA rates play an additional role now. As documented by Meunier et al. (2016), they also offer the possibility to partially index the total cap to actual realization of demand, which vary period after period. However, this indexing comes at a price since the introduction of a wedge between OBA and leakage rates introduces an inefficiency. The total cap is no longer optimally allocated between the two sectors.

As it could be seen from the welfare expression (10), if the OBA rates are equal to the leakage rates (assume these do not depend on the demand states), in each demand state the discrepancy between the actual cap and the optimal one is reflected in the difference between the permit price and the marginal environmental damage. Welfare could be improved by relaxing (resp. strengthening) the cap when the former is higher (resp. lower) than the latter. An adjustment of the OBA rates can help in that direction.

Proposition 2 *With uncertainty, the optimal permit scheme involves a quantity of auctioned permits $\bar{e}$ and OBA rates α_1 and α_2 that satisfy:*

$$\mathbb{E}[r - D'(e)] = \mathbb{E}\left[D'(e)\sum_i(\alpha_i - l_i)\frac{\partial q^e_{ih}}{\partial \bar{e}}\right] \tag{11}$$

$$\mathbb{E}[(r - D'(e))q_{ih}] = \mathbb{E}\left[D'(e)\sum_{j=1,2}(\alpha_j - l_j)\frac{\partial q^e_{jh}}{\partial \alpha_i}\right] \tag{12}$$

for $i = 1, 2$.

Proof. For the first equation (11), starts from equation (10) and use the relationship :

$$\frac{\partial e_h}{\partial \bar{e}} = 1 + \sum_i \alpha_i \frac{\partial q^e_{ih}}{\partial \bar{e}}$$

so that

$$\frac{\partial \hat{W}}{d\bar{e}} = \mathbb{E}\{(r - D'(e)) + \sum_i \left[\alpha_i(r - D'(e)) - (\alpha_i r - D'(e)l_i)\right]\frac{\partial q^e_{ih}}{\partial \bar{e}}\}$$

and equation (11 follows.

Equation (12), on the other hand, comes from the optimal choice of the OBA rates and is obtained through a similar manipulation using the relationship for $k = 1, 2$:

$$\frac{\partial e_h}{\partial \alpha_k} = q_{kh} + \sum_i \left[\alpha_i \frac{\partial q^e_{ih}}{\partial \alpha_k}\right]$$

■

An increase in the OBA rate of sector i has the direct effect of releasing q_{ih} permits in the market, which creates a marginal benefit and an environmental damage (left hand side of equation (12)). It also indirectly influences production by increasing the subsidy to the sector under consideration and modifying the permit price. Such changes are captured in the right-hand side of equation (12). An increase in production in a sector is detrimental if the OBA rate is above the leakage rate, because production in such a case is already high.

To see the possible benefit of setting OBA rates away from leakage rates, we can evaluate a marginal change from the situation where both rates are equal. Setting $\alpha_i = l_i$ on the right hand side of equations (11) and (12), we see no gains from such marginal change if there is no correlation between the permit price and home output quantities. Otherwise, OBA and leakage rates can differ in order to take advantage of an non-null correlation to increase (resp. decrease) the total cap when the permit price is above (resp. below) the marginal environmental damage.

At the optimal scheme, if OBA and leakage rates differ, the quantity of auctioned permits should be adjusted. The optimal quantity is such that the expected difference between the permit price and the marginal environmental damage is equal to the inefficiency cost due to the difference between the OBA and leakage rates.

2.4 Uncertainty with a quadratic specification

To perform some simulations and better grasp the consequences of introducing uncertainty, a quadratic framework is developed. Let us consider linear environmental damages, quadratic production costs, and linear demand functions:

$$\begin{aligned} D'(e) &= h \\ C_{ij}(q_i) &= \gamma_{ij} q_i^2/2 \\ P_i(q_i, \theta_i) &= a_i + \theta_i - b_i q_i \end{aligned} \tag{13}$$

with $\mathbb{E}[\theta_i] = 0,\ \mathbb{E}[\theta_i^2] = \sigma_i^2,\ \mathbb{E}[\theta_1\theta_2] = \sigma_{12}$,[8] and for $i = 1, 2$ and $j = h, f$. The leakage rate is then independent of the demand state and equal to the ratio:

$$l_i = \frac{b_i}{b_i + \gamma_{if}}$$

Let us denote by

$$s_i \equiv \frac{1}{b_i(1 - l_i) + \gamma_{ih}}$$

[8]To ensure interior solutions we assume that $\sigma_{12} < h^2$. Assuming otherwise may lead to the creation of two sector-specific ETSs.

the inverse of the slope of net surplus in sector i with respect to home production after taking into account the adjustment of foreign production,[9] which could be interpreted as the market size at home.

Lemma 1 *For any couple of OBA rates (α_1, α_2) the quantity of auctioned permits $\bar{e}$ that maximizes welfare is*

$$\bar{e}(\alpha_1, \alpha_2) = \sum_i (1-\alpha_i)(1-l_i)s_i(a_i - h) \tag{14}$$

and the expected permit price is

$$\mathbb{E}r = \frac{\sum_i (1-\alpha_i)(1-l_i)s_i}{\sum_i (1-\alpha_i)^2 s_i} h \tag{15}$$

If OBA rates are set to zero, i.e., $\alpha_1 = \alpha_2 = 0$, the optimal expected permit price is

$$\mathbb{E}r = h\left(1 - \frac{l_1 s_1 + l_2 s_2}{s_1 + s_2}\right) \tag{16}$$

Proof. See the Appendix. ■

Without OBAs, the presence of leakage implies an optimal expected permit price lower than the marginal environmental damage (the same would be true with a tax). The wedge could be interpreted as a subsidy on production to correct for the leakage positive externality. This indirect subsidy is equal to the marginal environmental damage times an aggregated leakage rate. OBA rates allow the regulator to set sector specific subsidies that are more efficient.

As can be seen from equation (15), if OBA rates are set equal to the leakage rates, so that the leakage externality is well internalized, the optimal expected permit price should be equal to the marginal environmental damage. However, because of uncertainty, there is a gain to set OBA rates otherwise as the next proposition shows.[10]

Proposition 3 *Under the specification (13) above, an optimal permit scheme satisfies:*

[9] Formally, by an implicit theorem argument, the derivative of net consumer surplus is (dropping θ_i)

$$\frac{d^2}{dq_{ih}^2}[S_i(q_{ih} + q_{if}(q_{ih})) - C_{ih} - C_{if}] = \frac{d}{dq_{ih}}[P_i - C'_{ih}] = -[b_i(1-l_i) + \gamma_{ih}]$$

[10] Note that with this linear environmental damage and leakage rates a combination of a tax and sector-specific rebates delivers the first-best and, hence, outperforms the combination of ETS and OBA. Indeed, an optimal price corridor (price floor and ceiling) would be equivalent to tax and lead to the first-best outcome. We are interested, however, in the optimal design of an ETS and not in comparison of prices vs quantities *à la* Weitzman (1974), which, to make it relevant, would required to introduce some convexity to the environmental damage function.

1. *If* $(1-l_2)^2 s_2(\sigma_2^2 - \sigma_{12}) = (1-l_1)^2 s_1(\sigma_1^2 - \sigma_{12})$*, then uncertainty does not influence the structure of the scheme and* $(1-\alpha_2)/(1-l_2) = (1-\alpha_1)/(1-l_1)$ *holds.*

2. *If* $(1-l_2)^2 s_2(\sigma_2^2 - \sigma_{12}) > (1-l_1)^2 s_1(\sigma_1^2 - \sigma_{12})$ *the relative difference between the OBA rate and the leakage rate is larger in sector* 2 *than in sector* 1*:*

$$\frac{\alpha_2 - l_2}{1-l_2} > \frac{\alpha_1 - l_1}{1-l_1} \geq 0$$

so, by setting $\alpha_1 = l_1$*, the optimal OBA rate in sector* 2 *becomes larger than* l_2 *and equal to*

$$\alpha_2 = 1 - \frac{1-l_2}{2}\left[\left(\Delta^2 + 4\frac{(1-l_1)^2 s_1}{(1-l_2)^2 s_2}\right)^{1/2} - \Delta\right] > l_2 \tag{17}$$

where

$$\Delta = \frac{1}{1-\sigma_{12}/h^2}\left\{\left(\frac{\sigma_2^2}{h^2} - 1\right) - \frac{(1-l_1)^2 s_1}{(1-l_2)^2 s_2}\left(\frac{\sigma_1^2}{h^2} - 1\right)\right\} \tag{18}$$

Proof. See the Appendix. ■

With uncertainty, OBA rates should differ from leakage rates. The relative difference is larger for sectors that are larger and riskier. The variability in those sectors is the main source of variations of the permit price, and there is a strong correlation between their output and the permit price. Such a correlation calls for an increase of the OBA rate since it helps releasing more permits precisely when the permit price is larger. And having a positive covariance ($\sigma_{12} > 0$) calls for an even larger OBA rate in sector 2. To understand this result, consider a situation in which there is no leakage (i.e., $l_1 = l_2 = 0$), two equal-size sectors (i.e., $s_1 = s_s$), and $\theta_2 = \eta + \theta_1$, where η is a shock specific to sector 2. If $\eta = 0$, so $\theta_2 = \theta_1$, it is optimal to set $\alpha_1 = \alpha_2 = 0$ (Case 1 of Proposition 3) because there is no correlation between output and permit prices.[11] But if $\eta > 0$, there will be a positive correlation between r and q_2, which calls for an increase of α_2. But doing so introduces an asymmetry in the response to the common shock θ_1. With $\alpha_2 > 0$, a positive shock θ_1 leads to an increase of q_2 and a decrease of q_1, since sector 2 is less sensitive to a change in the permit price, precisely because of $\alpha_2 > 0$. This implies that any increase in the covariance σ_{12} (or, in this example, the variance of the common shock θ_1) calls for a further increase of the optimal OBA rate α_2.

[11] As explained in Meunier et al. (2016) in more detail, it is worth introducing OBA in a sector if, absent of OBA, its output is correlated with the permit price so that the cap is increased when the permit price is above the social cost of carbon. With a common shock and identical sizes, there is no such correlation, the output of each sector being constant and equal to half the cap.

3 Simulation and policy implications

In this section we use our model to analyze the consequences of introducing an OBA scheme in permit-trading regulation and discuss its policy implications using a numerical illustration. The simulations are kept simple. The main objective is to use our model to discuss some pending issues in current ETSs.

We take the carbon market in Europe, better known as the EU-ETS, as a background for this discussion. In this context it is important to note that the regulatory decisions are made much in advance: elaborated around 2007-2008 for phase III 2013-2020, around 2016-2017 for phase IV 2021-2030. In the EU-ETS phase III a piece-wise approximation of OBA had been introduced for sectors at risk of leakage while all remaining sectors will receive no free (OBA) allowances or a decreasing lump sum (Branger et al., 2015). Our static model thus refers to the duration of a whole phase, and to the uncertainties as anticipated at the time of design. While some adaptations are contemplated for phase IV none was considered for phase III.[12]

The first issue we address concerns the difficult question of defining the sectors at risk, i.e., the sectors that should be entitled to OBA permits. It may not be easy for a regulator to distinguish uncertainty from leakage. Initially to be eligible for OBA permits, the EU-ETS required a sector to simultaneously exhibit a carbon intensity and exposition to international trade above pre-established thresholds. In the end it required the sector to comply with either requirement. As a result, more than 80% of industrial emissions (i.e., emissions covered by the EU-ETS except electricity production) became eligible for OBA permits. This has taken the EU to revise its eligibility criteria. A tiered approach is considered in which the sector OBA rate would depend on the level of leakage in that sector. Our results suggest that sector-level uncertainty is also a relevant criteria for introducing OBAs.

The second issue we consider is whether the flexibility in emissions induced by granting OBA permits to some sectors should necessarily lead to some flexibility in the overall home cap, as formulated in our analysis. Under the current EU-ETS regulation the total home cap over the period is fixed (and declining over time at a constant yearly rate). The current level of activity has dropped significantly post the commitment set in a context of high economic activity, so all sectors have emissions much lower than was originally expected. This partly explains the drop observed in the carbon price and the current debate on how to eliminate

[12] http://www.europarl.europa.eu/sides/getDoc.do?type=IM-PRESS&reference=20161215IPR56370&language=EN&format=XML

the "excess" of allowances in the market.[13] We will show that a flexible total cap would have mitigated this unbalance greatly; furthermore, it would have reduced perverse effects in non-OBA sectors (i.e., sectors for which $\alpha = 0$) due to the drop in the carbon price.

3.1 A numerical illustration

The illustration that follows is based on the quadratic specification of section 2.4 with two sectors with the following numerical values for the parameters: $a_1 = a_2 = 1$, $b_1 = b_2 = 1$, $\gamma_{1h} = 1$,$\gamma_{1f} = +\infty$ (no foreign competitors in sector 1), $\gamma_{2h} = 1$,$\gamma_{2f} = 3$, $h = 1/4$, $\theta_1 = 0$ and $\theta_2 \in \{-\lambda, \lambda\}$ with equal probability, so $\sigma_2 = \lambda$. Only sector 2 is exposed to foreign competition and to market uncertainty. For these two reasons it is eligible to OBA while sector 1 is not.[14]

For sector 2, the parameter λ will be referred as the level of uncertainty. The model is explored for λ moving from 0 to 1/2. We are particularly interested in large values of λ.[15] We are also interested in different values of γ_{2f} to cover different leakage rates for sector 2. While in this illustration the leakage rate is independent of the level of uncertainty, i.e., $l_2 = 1/(1 + \gamma_{2f})$, the optimal OBA rate, which can be obtained using Proposition 2, is not.

We turn to the first issue. Figure 1 depicts the dependence of the optimal OBA rate on the level of uncertainty for a given value of $l_2 \in \{0, 1/4, 1/2\}$, allowing γ_{2f} to vary accordingly. It shows that the optimal OBA rate increases significantly with sector uncertainty. The respective influence of the leakage rate and the level of uncertainty on the optimal OBA rate varies. For low levels of uncertainty the leakage rate is the main factor behind the optimal OBA rate, but as uncertainty increases the leakage rate becomes less of a factor to virtually disappear for large levels of uncertainty.

Figure 2 shows welfare losses in percentage terms when implementing OBA without paying attention to uncertainty for the same three levels of leakage. Consider first a case in which there is no leakage ($l_2 = 0$), if the level of uncertainty is high neglecting uncertainty means not introducing OBA for sector 2, then the expected welfare loss is of the order of 20%.

[13] `http://ec.europa.eu/clima/policies/ets/index_en.htm`

[14] Sector 2 exhibits both leakage and uncertainty, while sector 1 has none of these features. By varying theses two features this allows us to examine the relative contribution of each one to the optimal design. Adding uncertainty to sector 1 would only reduce the corresponding optimal rate.

[15] Take the cement market to have some order of magnitude for the level of uncertainty in a given sector. In Branger et al. (2015) it is observed that approximately 50% of the EU cement market has gone through a severe recession. In countries such as Ireland, Spain and Greece the level of cement consumption in 2012 was around 70% below the corresponding level of 2007, the time at which the EU-ETS had been designed. In our simulation we consider a range for the uncertainty factor of plus or minus 50%, that is a drop of 80% in consumption relative to the peak.

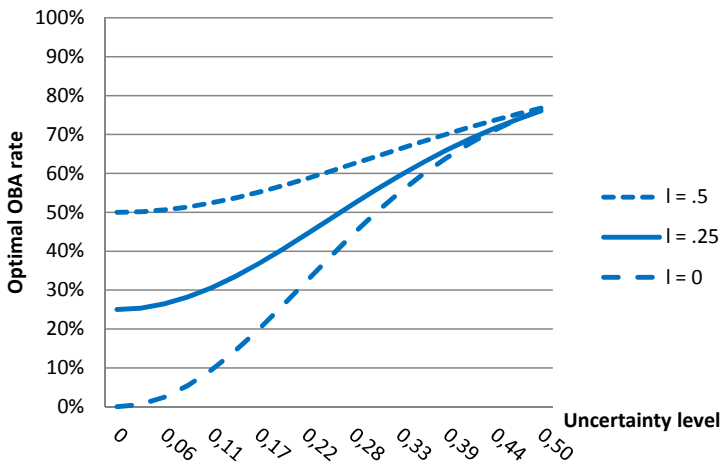


Figure 1: Optimal OBA rate as a function of uncertainty (λ) for three leakage rates

This welfare loss would decrease as the leakage rate increases, since the marginal impact of uncertainty on the OBA rate decreases, still it cannot be considered as negligible. According to our model, it is thus recommended to use a tiered taxonomy based on the sector leakage and uncertainty rates, i.e., a sector with low leakage and high uncertainty would be eligible for a similar level of OBAs as a sector with high leakage and low uncertainty.

Consider now the second issue. For this discussion we set $\gamma_{2f} = 3$ so that $l_2 = 1/4$. We compare two scenarios. The first one, Flexible Cap, corresponds to our proposal, the total cap at home is flexible and determined endogenously by our optimal policy, the OBA rate for sector 2 and the amount of auctioned permits. For the second one, Fixed Cap, the OBA rate for sector 2 is equal to the one in Flexible Cap (optimizing this rate for scenario Fixed Cap would only slightly change the results), but the total cap at home is kept constant to be the one that would be achieved with Flexible Cap in case of no uncertainty. Technically any increase (or reduction) of emissions in sector 2 is compensated with an equal reduction (or increase) of auctioned permits so that total emissions at home (but not necessarily abroad) remain constant and equal. This second scenario mimics in a static model the current practice in cap-and-trade systems in Europe and in California, where any current year's increase/reduction in the cap is compensated with an equivalent reduction/increase in a future year.

First of all Figure 3 illustrates how the range of the carbon price is exacerbated as one goes from a flexible cap to a fixed cap. In this figure the carbon price is given in relative value to the social cost of carbon that is the parameter h. This range is exacerbated as one goes from a flexible cap to a fixed cap.

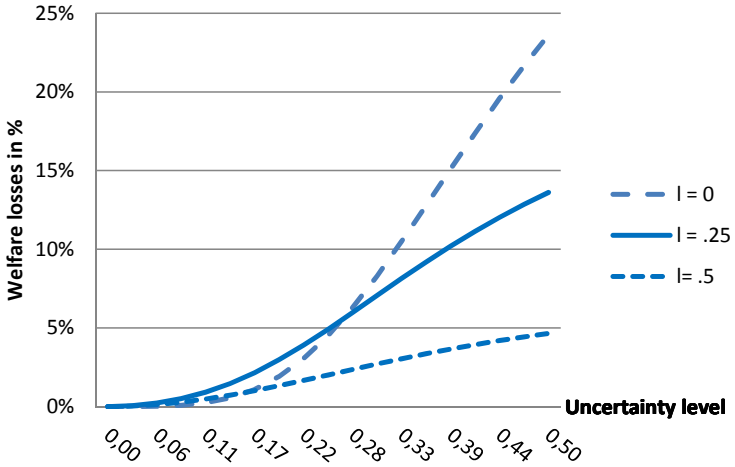
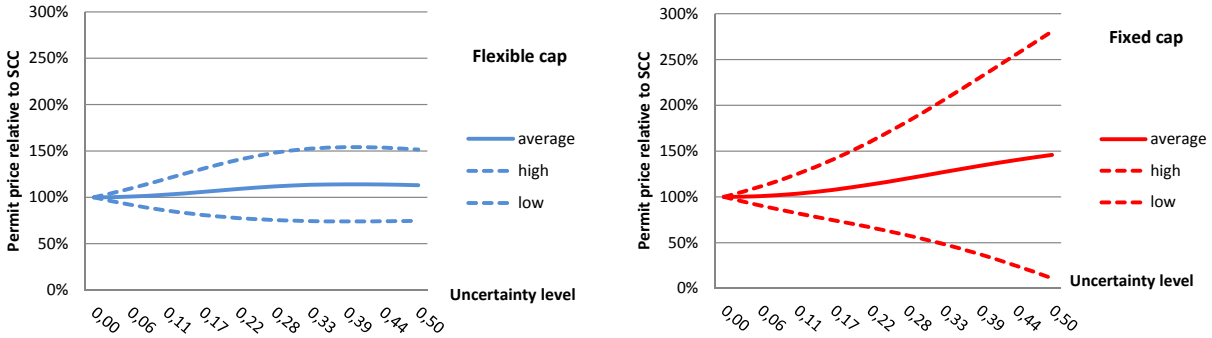


Figure 2: Welfare losses as a function of λ of OBA rate at $\alpha_2 = l_2$ vs optimal OBA rate α_2^*



(a) Flexible Cap

(b) Fixed cap

Figure 3: Premit prices as a function of uncertainty λ in the high and low demand states, with a flexible and a fixed cap.

scenario with $\lambda = 1/2$		FB	Flexible	Fixed	Flexible %	Fixed %
Welfare	world	0.174	0.158	0.141	-9	-18.6
	sector 1	0.023	0.024	0.015	3.2	-37
	sector 2	0.151	0.134	0.127	-10.9	-15.8
Emissions	world	0.8	0.9	0.85	12.1	6
	sector 1	0.375	0.359	0.379	-4.4	-15.3
	sector 2 home	0.321	0.390	0.379	21.3	17.8
	sector 2 foreign	0.107	0.152	0.155	42.3	45
Profit	sector 1	0.070	0.065	0.065	-6.9	-8.1
	sector 2 home	0.075	0.096	0.086	29	15.1

Table 1: Comparison to first best of the two scenarios

Secondly, for comparing these two scenarios relative to emissions, welfare and profits we use as benchmark the First Best scenario (i.e. a world tax equal to the social cost of carbon). The results are summarized in Table 1. They corresponds to a level of uncertainty $\lambda = 1/2$. Ordinarily introducing OBA in one sector increases emissions in that sector since production is subsidized to the detriment of the non OBA sectors (see for instance Nicolaï and Zamorano, 2016), this would suggest to reduce the home cap. These general features are also present in our model. We observe an increase in the expected level of world and home emissions, and a transfer of emissions from sector 1 to sector 2. More precisely the introduction of OBA with a flexible cap induces a greater increase in the expected world emissions than does a fixed cap scenario,the flexible cap creates better welfare outcomes than the fixed cap both at the world level and sector wise. Indeed the welfare in sector 1 now becomes greater than in the first best scenario,[16] and the welfare in sector 2 is not as far below the first best scenario as it was with a fixed cap. The flexible cap scenario dominates in all dimensions.

4 Conclusions

In a previous paper we have studied pollution permit markets in which a fraction of the permits are allocated to firms based on their output. In this paper we show that our results can be extended to the case of leakage, which for many is the primary motivation for introducing output based allocations (OBAs).

Our model provides interesting insights to discuss a number of pending issues for the design of emission trading systems in general and carbon markets in particular (e.g., Europe,

[16]Convexity of the welfare function explains this paradoxical result: in the first best the output in sector 1 is constant while it varies in the flexible cap; this does not apply to sector 2 because of leakage.

California, and forthcoming ETS around the world). A numerical illustration, motivated around discussions on how to reform the EU-ETS, is used to show the policy relevance of our results. Firstly, we show that a sector subject to demand and supply shocks should be considered as a primary criteria for using OBAs. A sector subject to such volatility should be just as eligible for OBAs as a sector with a high leakage risk but not subject to volatility.

We also show that the benefits associated with OBAs are critically dependent on the simultaneous introduction of some flexibility in the total cap at home. In the absence of this flexibility, fluctuations of the permit price would be considerably enlarged generating severe distortions in the sectors without OBAs. This may be considered as a much simpler way to control the evolution of permit prices than the complicated market-stability-reserve (MSR) approach currently followed in the EU-ETS.

The design of an ETS is subject to political economy considerations that are outside our model. This may explain why policy makers have insisted on a fixed cap at home. It is probably easier to agree on a fixed target for 2021-2030 than to let the actual cap depend on the OBA rates for the sectors at risk and their corresponding levels of economic activities during phase III. The commitment appears stronger than with a flexible cap (though as our illustration shows the fluctuations may be moderate and they could be reconsidered for setting a new flexible cap for the next period). However, ex-post, the commitment to a fixed cap for 2013-2020 has generated a sharp unexpected decline in the carbon price making the EU-ETS. As a matter of fact, a flexible cap would have led to a more stringent carbon regulation. This paradox, well emphasized in 2003 (Ellerman and Wing, 2003), should be better understood by now.

Since our primary objective was to show the role of uncertainty in the design of OBA permit schemes, some considerations were set aside during the development of our model. We see some extensions to it that can provide more precision to the numbers, but none to qualitatively change them. One possible extension is the development of a more dynamic version in which commitment periods are of limited length, as the compliance phases in the EU-ETS, with uncertainties progressively unfolding. Some limited flexibility within each commitment period may be still introduced, for example, with a price corridor, as in the Californian ETS, or with an MSR mechanism, as in the EU-ETS, or yet, with more periodic revisions of the OBA parameters (i.e. benchmarks and carbon intensities) used to compute the free allocations. The model would also benefit of more attention to short term abatement strategies, such as adapting the input mix and carbon content for electricity production, which should somehow alleviate fluctuations in the carbon price. A more elaborate economic

market model in which home and foreign markets are explicitly introduced, as well as capacity constraints and investment decisions, may be useful to better formulate leakage and its dependence on national and international shocks. The social damage may be explicitly linked to a stock and flow representation of emissions. A quantitative assessment of the results based on the calibration of an existing ETS would also help. In spite of all these limitations, we think the policy claims derived from our model have some bearing for the design of current ETS policies.

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Appendix

A Permit market equilibrium and proof of Lemma 1

A.1 Permit market equilibrium

Let us denote $\tilde{a}_i = a_i + \theta_i$ and remind that $s_i = 1/(b_{ih}(1-l_i) + \gamma_{ih})$. Foreign production as a function of home production is:

$$q_{if} = \frac{\tilde{a}_i - b_i q_{ih}}{b_i + \gamma_{if}} = \tilde{a}_i l_i / b_i - l_i q_{ih} \tag{19}$$

Then home production satisfies the equation:

$$\tilde{a}_i(1-l_i) - b_i(1-l_i)q_{ih} = \gamma_{ih} q_{ih} + (1-\alpha_i)r \tag{20}$$

so that

$$q_{ih} = s_i \left[\tilde{a}_i(1-l_i) - (1-\alpha_i)r\right] \tag{21}$$

And the equilibrium permit price clears the emission permit market, i.e., $(1-\alpha_1)q_{1h} + (1-\alpha_2)q_{2h} = \bar{e}$, it is given by

$$r = \frac{\sum_i \left[s_i(1-\alpha_i)(1-l_i)\tilde{a}_i\right] - \bar{e}}{\sum_i \left[s_i(1-\alpha_i)^2\right]} \tag{22}$$

A.2 Proof of Lemma 1

From (11), (21) and (22) the optimal quantity of auctioned permits is such that

$$\mathbb{E}(r-h) = h\sum_i (\alpha_i - l_i)\frac{\partial q_{ih}}{\partial \bar{e}} = h\frac{\sum_i (\alpha_i - l_i)(1-\alpha_i)s_i}{\sum_i (1-\alpha_i)^2 s_i}$$

therefore, at the optimal $\bar{e}(\alpha_1, \alpha_2)$ the expected permit price is given by equation (15) and, from (22), the optimal cap satisfies (14).

B Proof of Proposition 3

Let us look at the influence of α_2. We start from equation (12). From equation (21) we have that the influence of α_2 on output is

$$\frac{\partial q_{2h}}{\partial \alpha_2} = s_2 r - (1-\alpha_2) s_2 \frac{\partial r}{\partial \alpha_2}$$

and

$$\frac{\partial q_{1h}}{\partial \alpha_2} = -(1-\alpha_1) s_1 \frac{\partial r}{\partial \alpha_2}$$

from eq. (22) that its influence on the permit price is

$$\frac{\partial r}{\partial \alpha_2} = -\frac{q_2}{\sum_i (1-\alpha_i)^2 s_i} + r \frac{(1-\alpha_2) s_2}{\sum_i (1-\alpha_i)^2 s_i}$$

The derivative of welfare with respect to α_2 is

$$\frac{d\tilde{W}}{d\alpha_2} = \mathbb{E}\left\{ \left[(r-h) - \sum_i (\alpha_i - l_i) \frac{\partial q_{ih}}{\partial \bar{e}} \right] q_{2h} \right\} - h s_2 \mathbb{E}\left\{ (\alpha_2 - l_2) r - (1-\alpha_2) r \frac{\sum_i (\alpha_i - l_i)(1-\alpha_i) s_i}{\sum_i (1-\alpha_i)^2 s_i} \right\} \tag{23}$$

Let us look at each of the terms in brackets and to ease exposition we introduce:

$$z_i \equiv \frac{1-\alpha_i}{1-l_i} \text{ and } \beta_i \equiv (1-l_i)^2 s_i$$

we have:

• 1^{st} term in brackets: using equation (11), it is shown to be equal to $\text{cov}(r, q_{2h})$ which is, using equations (22) and (20):

$$\begin{aligned} & s_2(1-l_2)\text{cov}(r, \theta_2 - z_2 r) \\ & = \frac{s_1 s_2 (1-\alpha_1)}{(\sum_i (1-\alpha_i)^2 s_i)^2} (1-l_1)(1-l_2) \left\{ z_1 z_2 \left[\beta_2 \sigma_2^2 - \beta_1 \sigma_1^2 \right] + \sigma_{12} \left[\beta_1 z_1^2 - \beta_2 z_2^2 \right] \right\} \end{aligned}$$

- 2^{nd} term in brackets is equal to

$$\begin{aligned}
&h(1-\alpha_1)s_1s_2\frac{\mathbb{E}[r]}{\sum_i(1-\alpha_i)^2s_i}\left[(\alpha_2-l_2)(1-\alpha_1)-(\alpha_1-l_1)(1-\alpha_2)\right]\\
=&h(1-\alpha_1)s_1s_2\frac{\mathbb{E}[r]}{\sum_i(1-\alpha_i)^2s_i}(1-l_1)(1-l_2)\left[z_1-z_2\right]\\
=&h^2\frac{s_1s_2(1-\alpha_1)}{(\sum_i(1-\alpha_i)^2s_i)^2}(\beta_1z_1+\beta_2z_2)(1-l_1)(1-l_2)\left[z_1-z_2\right] \text{ using (15)}
\end{aligned}$$

So the derivative of welfare with respect to α_2 is

$$h^2\frac{s_1s_2(1-\alpha_1)}{(\sum_i(1-\alpha_i)^2s_i)^2}(1-l_1)(1-l_2)\left\{z_1z_2\left(\beta_2\sigma_2^2-\beta_1\sigma_1^2\right)+\sigma_{12}\left(\beta_1z_1^2-\beta_2z_2^2\right)-h^2(\beta_1z_1+\beta_2z_2)(z_1-z_2)\right\}$$

Denoting

$$z=\frac{z_2}{z_1}=\frac{(1-\alpha_2)/(1-l_2)}{(1-\alpha_1)/(1-l_1)} \text{ and } \beta=\frac{\beta_1}{\beta_2}=\frac{(1-l_1)^2s_1}{(1-l_2)^2s_2},$$

the sign of the derivative of welfare w.r.t. α_2 is equal to the sign of

$$z\left(\sigma_2^2-\beta\sigma_1^2\right)+\sigma_{12}\left(\beta-z^2\right)-h^2(\beta+z)(1-z)=\left(1-\frac{\sigma_{12}}{h^2}\right)\left\{z^2+z\Delta-\beta\right\}$$

where $\Delta=[\sigma_2^2/h^2-\beta\sigma_1^2/h^2+(\beta-1)]/(1-\sigma_{12}/h^2)$ corresponds to the definition given by equation (18). Let us assume that $\sigma_{12}<h^2$. The unique positive root of the quadratics is

$$z_+=\frac{1}{2}\left[\left(\Delta^2+4\beta\right)^{1/2}-\Delta\right]$$

The derivative of welfare w.r.t. α_2 is strictly negative for $z\in(0,z_+)$ and strictly positive for $z\in(z_+,+\infty)$. Since an increase in α_2 is equivalent to a decrease in z, welfare is quasiconcave w.r.t. z (or α_2), and maximized at z_+.

Then,

- $z_+=1$ is equivalent to $\beta-1=\Delta$ i.e. $\sigma_2^2-\beta\sigma_1^2=\beta-1$ which proves the first point.
- $z_+<1$ is equivalent to $\beta-1<\Delta$ i.e. $\sigma_2^2-\beta\sigma_1^2>\beta-1$ which proves the second point.
- And setting $\alpha_1=l_1$, $\alpha_2=1-(1-l_2)z_+$ proves the last point.