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Why conditional subsidies for risky innovative green projects should be prefered to flat subsidies

Guy Meunier and Jean-Pierre Ponssard*

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Abstract

The energy transition requires the deployment of risky programs in research and development, the vast majority of which being partially financed by public funding. This paper analyzes the potential benefit of using conditional subsidies. The relationship between the state and innovative firms is formalized in the principal agent framework. Investing in an innovative project requires an initial observable capital outlay. We introduce asymmetric information on the probability of success, which is known to the firm but not to the state agency. Furthermore the firm may influence this probability through an unobservable effort. The outcome of the project, if successful, delivers a private benefit to the firm and an external social benefit to the state. We prove that rewarding failure is a superior strategy in presence of pure adverse selection while rewarding success is superior in presence of pure moral hazard. We also identify conditions in which both forms of subsidies should be implemented. We discuss the benefit of conditional subsidies relative to flat subsidies as well as the remaining gap relative to the first best solution. Our analytical results are interpreted in view of a 10 B € investment program launched in France in 2010 to promote R&D for the energy transition over the period 2010-2020.

JEL Classification:

Keywords: Green innovation, Public financing, Asymmetric information.

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1 Introduction

We analyze the optimal way to subsidize a risky innovative project with scarce public funds. A project can generate both private and (external) social benefits, but requires an initial funding. The regulatory intervention is justified by the external social benefits. Indeed, it is common to consider that innovation activities have positive spillovers of multiple sorts that justify subsidies at the various stages of the innovation process. Notably, pilot and demonstration plants are a key step between the lab and the industrial scaling, but they are risky and capital intensive activities. Even though the analysis developed is general, the present work is motivated by the transition to a low-carbon economy. Various policies involving different forms of subsidies have been used to promote such programs.

The policies actually implemented typically exhibit some ad hoc features. The gray literature has emphasized a number of pitfalls. For instance the allocation of subsidies in the Clean Development Mechanisms is based on a counter-factual that defines baseline emissions.[1] This opens the room for financing projects that would have been deployed anyway (Gillenwater and Seres; 2011; Greiner and Michaelowa; 2003). Another example concerns the promotion of renewable energy such as solar. Governments have been late in recognizing the decline in costs so that many projects also benefited from windfall profits (Brown; 2013). The REDD program has also been critically examined in this respect (Pirard; 2008).[2] More recently, in order to finance the energetic transition under tight governmental budget constraint, Aglietta et al. (2015) have proposed a scheme based on government-backed loans that otherwise would not satisfy the regulatory rules imposed on the financial capital market, again opening the room for windfall profits.

This paper formalizes such situations using the principal agent framework (Laffont and Martimort; 2002), the principal being the agency acting on behalf of the state and the agent being the firm which carries over the project. The firm invest in a project that may succeed or fail, the probability of success

[1]The Clean Development Mechanism is a flexible mechanism in the Kyoto protocol, that allows covered (Annex I) countries to satisfy part of their abatement objective by investing in low-carbon projects (e.g. renewable electricity) in uncovered (non-Annex I) countries.

[2]The REDD (Reducing Emissions from Deforestation and Forest Degradation), or REDD+, program allows to monitor and evaluate mitigation benefits from forest conservation in developing countries.

depends upon the type of the project and the effort of the firm. The type is known by the firm but not by the agency and the effort is non contractible. The agency can propose a couple of non-negative subsidies in case of success or failure.

From a welfare point of view one needs to balance a selection bias (induce investments in projects in as much as they are socially valuable) with a risk for windfall profits (allocate funds to projects that would have been undertaken anyway) while at the same time getting the highest possible benefit for the funds allocated by the agency. We show that the effort exerted by the firm for the optimal scheme is lower than the first best effort so that less projects are selected (Proposition 1). A qualitative typology of projects is provided. Rewarding failure appears more suitable when uncertainty on the type of the firm is wide and flat and effort costly so that the risk of windfall profits prevails (Proposition 2). When the uncertainty is concentrated on low type firms (i.e. inefficient) and the cost of effort is low rewarding success will be more profitable. This typology leads to benchmark situations in which we provide a full characterization of the optimal scheme: reward failure for pure adverse selection situations (Proposition 4) and reward success for pure moral hazard ones (Proposition 5). We study in details a situation in which both adverse selection and moral hazard prevail and show how the optimal scheme progressively shifts from rewarding success to rewarding failure as a function of the distribution of types (Proposition 6). In all cases the optimal scheme performs much better than flat subsidies. The gap relative to the first best solution is identified.

Several articles deal with the issue of financing green projects under asymmetric information. Fischer (2005) provides an insightful analysis of the issue of "additionality" in the design of CDM design.[3] Mason and Plantinga (2013) consider the optimal design of contracts for carbon offsets with asymmetric information. Within the agricultural sectors, the design of agri-environmental schemes raises similar issues (e.g. Wu and Babcock; 1996; Engel et al.; 2008), and a related concerns is "stacking" of green payments: a farmer maybe rewarded twice, for biodiversity and greenhouse gases reduction for the same action (Woodward; 2011; Lankoski et al.; 2015). To our knowledge, the issue of windfall profits from innovative risky green projects has not been studied.[4]

[3]Zhang and Wang (2011) empirical analysis does cast some doubt about the additionality of Chinese CDM.

[4]There is a large literature on the coordination between environmental and innovation

From a more theoretical perspective, our analyzes lies at the intersection of several strands of the literature: optimal second best taxation with externalities, and mechanism design with both adverse selection and moral hazard.[5] In this respect our model features a risk-neutral principal and a risk neutral agent, and constrained incentive schemes. The principal (the agency) is constrained to propose a single couple of non-negative subsidies. A key element of our analysis is that some projects would be financed without the scheme, making the participation constraint dependent on the type.

Some papers have introduced some of these restrictions. Lewis and Sappington (2000a,b) considered mixed models with wealth constrained agents (see Quérou et al.; 2015, for a recent contribution). Laffont (1995) and Hiriart and Martimort (2006b,a) analyze the regulation of environmental risk under limited liability. Interestingly, the optimal design of students loan studied by Gary-Bobo and Trannoy (2015) exhibits some features similar to our results: the students are ask to reimburse their loans in case of success but not failure. However, it comes from an insurance motive due to the risk aversion of students.

Ollier and Thomas (2013) introduces ex-post participation constraint (the firm should recover its cost even if the project fails) in a mixed model relatively similar to ours.[6] They notably show that because of countervailing incentives pooling is optimal and the principal should only reward success. Which is the case in our setting when moral hazard issues dominate, that is, when one type is much more probable than the other. Otherwise, with the present model there are situations in which both subsidies are used or only a reward in case of failure. The key difference is the absence of a fixed cost in Ollier and Thomas (2013) which limits adverse selection issues: there is no need to finance low profitability projects but only motivate efforts.

The literature on optimal taxation and externalities (Sandmo; 1975; Bovenberg and de Mooij; 1994; Cremer et al.; 1998; Cremer and Gahvari; 2001) considers modified Pigouvian rules in second best setting, whether à la Ramsey or Mirrless. Depending on the instruments available and constraints

policy in dynamic models, most notably endogenous growth models (see Smulders et al.; 2014, for a survey). Subsidies for clean innovation are constant research subsidies, the details of the innovation policies are not studied, a recent exception being Gerlagh et al. (2014) who study green patents length.

[5] Mixed models are covered in chapter 7 in Laffont and Martimort (2002).

[6] In subsection 5.2. they replace the ex-post participation constraint by a limited liability constraint, making their model closer to ours.

considered (notably on the shape of the income tax) the optimal tax differs from the Pigouvian one. The fact that in our setting the optimal scheme does not consist in setting a reward equal to the marginal positive externality could be interpreted as the result of a second best situation in which public funds are costly and taxation incomplete, the profit of the successful firm cannot be taxed because of the constraint on the instruments.

Finally, it would be interesting for future research to incorporate the dynamic aspects of innovation and decompose a project in several technical steps that need be completed. The timing of these steps and the determination of a stopping time, a time at which a project is abandoned, would be worth analyzing. To such end the burgeoning literature on experimentation of new ideas under asymmetric information could be inspiring (e.g. Bergemann and Hege; 1998, 2005).

The rest of the paper is organized as follows: In Section 2 the general model is introduced and some general results derived. Section 3 analyzes in details two benchmark situations: the benefit of rewarding failure in case of pure of adverse selection and the benefit of rewarding success in case of pure moral hazard. Section 4 discusses an illustrative exemple in which both issues are present and shows how a combination of rewarding success and rewarding failure may prevail. Section 5 discusses policy implications of our results in the context of a state aid program to foster innovation for the energy transition launched in France in 2010. Some research extensions are suggested.

2 The general model with adverse selection and moral hazard

Consider the following situation. A given innovative project may or may not be initiated by a firm (the agent). If the project is initiated the firm incurs a fixed cost F, and can make additional efforts $e \in [0,1]$ at a cost $f(e,\theta)$. The project either succeeds or fails. The probability of success depends on the effort of the firm e and its type $\theta \in [0,1]$: $p(e,\theta) \in [0,1]$, and $p(0,\theta) = \theta$.

In case of success the firm gets a private revenue R and a social external benefit b is generated. In case of failure neither private nor external benefits are created. If a project is not initiated the reference payoffs are zero and no fixed cost is incurred.

The regulatory agency (henceforth the agency) knows F, R and b. Neither the type θ of the firm nor its effort are observed by the agency. Both the agency and the firm observe whether a project is initiated or not, and if it is a success or a failure.

The subsidy, if any, may only depend on the outcome of a project, i.e. whether it is a success or a failure. The subsidy is s_1 in case of success and s_2 in case of failure. They are non negative. More precisely, without loss of generality, we certainly have:

$$0 \leq s_1 \leq b$$

$$0 \leq s_2 \leq F$$

We shall refer to S^+ as this constrained class of incentive schemes (s_1, s_2), this will define our second best approach. For the sake of comparison we shall also identify the first best solution and investigate whether or not it can be achieved by an incentive scheme in which these nonnegativity constraints do not hold. The corresponding call of schemes would then be denoted as S.[7]

2.1 The model

The timing is the following:

1. Nature selects $\theta \in [0, 1]$. Types are distributed according to the cumulative distribution function $G(\theta)$, continuously differentiable with $G'(\theta) = g(\theta)$.

2. The agency proposes a couple of **non-negative** subsidy (s_1, s_2): s_1 if success and s_2 if failure.

3. For all $\theta \in [0, 1]$, a firm of type θ decides whether to accept the scheme and initiate the project at a fixed cost F and further exerts an effort $e \in (0, 1)$ at a cost $f(e, \theta)$.

4. The probability of success is $p(e, \theta)$.

    - If success: The firm gets a non negative private return R plus the subsidy s_1 The agency gets the non negative social benefit b minus the subsidy s_1

[7]The introduction of menus is only indirectly considered in Appendix C.3.

- If failure: The agency gives the subsidy s_2 to the firm.

The expected profit of a θ firm if it initiates the project and exert an effort e is

$$\pi(e,\theta,s_1,s_2) = p(e,\theta)(R+s_1) + (1-p(e,\theta))s_2 - (F + f(e,\theta)) \qquad (1)$$

so that the project is initiated if $\max_{e\in(0,1)} \pi(e,\theta,s_1,s_2) > 0$. Let the variable $\delta(\theta,s_1,s_2) \in \{0,1\}$ represents the initiating decision: it is equal to 1 if the project is initiated and 0 otherwise. The total industry profit is

$$\Pi(s_1,s_2) = \int_0^1 \delta(\theta,s_1,s_2) \max_e \pi(e,\theta,s_1,s_2) g(\theta) d\theta \qquad (2)$$

The expected surplus of the agency if the project is initiated writes:

$$v(e,\theta,s_1,s_2) = p(e,\theta)(b-s_1) - (1-p(e,\theta))s_2 \qquad (3)$$

The agency selects (s_1,s_2) ignoring the type θ of the firm and maximizes:

$$V = \int_0^1 v(e,\theta,s_1,s_2)\delta(\theta,s_1,s_2)g(\theta)d\theta \qquad (4)$$

Welfare is the sum of the agency surplus and firm profits:

$$w(e,\theta) = v + \pi = \delta[p(e,\theta)(R+b) - F - f(e,\theta)] \text{ and } W = V + \Pi \qquad (5)$$

The model might be rewritten so that the cost is a function of the probability and the type of the project: Denote $\phi(p,\theta)$ the cost, on top of F, that ensures that a project of type θ succeeds with probability p. It satisfies $\phi(p,\theta) = f(e(p,\theta),\theta)$ with $e(p,\theta)$ defined by the implicit equation $p(e(p,\theta),\theta) = p$. The assumption $p(0,\theta) = \theta$ gives $\phi(\theta,\theta) = 0$.

To get fully explicit expressions we will make use of the following linear specification:

$$p(e,\theta) = \theta + e(1-\theta) \qquad (6)$$

$$f(e,\theta) = (1-\theta)\frac{\gamma}{2}e^2 \qquad (7)$$

This formulation can be motivated by considering that a project is constituted of a continuum of technical steps, for a project of type θ a share θ of steps have already been completed (in the lab) and $(1-\theta)$ steps remain to be completed (with the pilot) to guarantee success.

With this formulation, a given level of effort has a larger impact on projects with an initially low probability of success, but it is more costly. Rewriting cost as a function of the probability of success gives:

$$\phi(p,\theta) = \frac{\gamma}{2}\frac{(p-\theta)^2}{1-\theta}, \text{ for } p > \theta, \text{ 0 otherwise}$$

which satisfies the technical assumptions in Ollier and Thomas (2013). It is increasing with respect to p and decreasing with respect to θ, the cross derivative is negative (the marginal cost to increase the probability of success is decreasing with the type), which ensures that the probability of success is decreasing with the type, for a given bonus. Indeed, with this quadratic specification the effort exerted will only depends on the bonus $s = s_1 - s_2$.

The situation without any subsidy will be refered as BAU (business as usual), the first best as FB and the second best as SB. The corresponding functions will be denoted with an exponant. For instance θ^{BAU} denotes the lowest type such that $\Pi^{BAU} > 0$. Several assumptions are further required to ensure that the problem at hand is interesting:

Assumption 1 *Some projects are profitable without subsidies:* $\Pi^{BAU} > 0$ *so that* $F < R$*.*

Assumption 2 *The BAU effort is such that* $0 \leq \theta^{BAU} \leq 1$ *hence* $R^2 \leq 2F\gamma$ *and* $F \leq R$*.*

Assumption 3 *The first best effort is less than 1:* $\gamma > R + b$*.*

2.2 First Best

The First Best is defined as the selection of projects and the efforts for each selected project that maximize welfare. The effort $e^{FB}(\theta)$ maximizes the expected welfare net of the cost for a project of type θ, if within $(0,1)$ it solves

$$p_e(e,\theta)(R+b) = f_e(e,\theta).$$

With the quadratic specification, the level of effort does not vary with θ. It is:

$$e^{FB} = (R+b)/\gamma \tag{8}$$

All projects with

$$p(e^{FB},\theta)(R+b) \geq F + f(e^{FB},\theta)$$

are initiated. Hence there is a θ^{FB} such that all projects with $\theta > \theta^{FB}$ are initiated. If θ^{FB} is positive it solves $p(e^{FB}, \theta^{FB})(R + b) = F + f(e^{FB}, \theta^{FB})$, the expected welfare gain from the marginal project is equalized with its cost.

The first best can be implemented with $s_1 = b$ and $s_2 = 0$, which corresponds to a Pigouvian subsidy.[8] However, the agency surplus is not maximized, and firms get a rent. If the agency were able to tax profits with a proportional tax then a 100% profit tax realigns the agency objective with social welfare ($V = W$ and $\Pi = 0$), and $s_1 = b$ both implements the first best and maximizes the agency surplus ($V = W^{FB}$), information asymmetry is then irrelevant. This result resonates with the fact that in a Ramsey optimal taxation framework the optimal corporate tax on pure profit is 100% (Munk; 1978) . And the present framework can be interpreted as an optimal taxation exercise with equity concerns but a limited set of instruments.

2.3 Second Best

For any subsidy couple (s_1, s_2) we have $0 < R + s_1 - s_2$ since $s_2 \leq F < R \leq R + s_1$. It follows that $\pi(e, \theta, s_1, s_2)$ is increasing with respect to θ so that there is a threshold project $\tilde{\theta}(s_1, s_2)$ such that all projects with a type above the threshold are initiated. The effort is a function of the bonus $s = s_1 - s_2$ alone such that $p_e(R+s) = f_e$. Since $\gamma > R+b$ and $b > s_1 > s_1 - s_2$ we get:

$$e(s) = (R + s)/\gamma \tag{9}$$

And $\tilde{\theta}$ is the solution of $\pi(e, \tilde{\theta}, s_1, s_2) = 0$, that is:

$$p(e, \theta)(R + s_1 - s_2) + s_2 - (F + f(e, \theta)) = 0 \tag{10}$$

The agency surplus is:

$$V = \int_{\tilde{\theta}}^{1} [p(e, \theta)(b - s_1) - (1 - p(e, \theta))s_2]g(\theta)d\theta \tag{11}$$

So that the choice of any of the subsidies s_1 and s_2 has three effects: (i) on the selection of projects via its influence on $\tilde{\theta}$, (ii) on the effort via $s_1 - s_2$, (iii) on the total expected transfer to firms. As is usual in agency problems the agency trades-off efficiency with rents. Since both the selection of projects and the effort exerted depend on efficiency, the level of efficiency

[8] The fixed cost F does not by itself justify the implementation of a subsidy because projects are infinitesimally small.

influences total welfare and not only its allocation among the agency and firms.

It is illuminating to isolate the selection of projects from the precise design of subsidies. Instead of considering the two variables s_1 and s_2, we rewrite the profit of the firm and agency surplus as functions of s and $\tilde{\theta}$. Injecting equation (10) into the expression (1) gives the profit of a firm as function of s and $\tilde{\theta}$:

$$\pi = [p(e,\theta) - p(e,\tilde{\theta})](R+s) - [f(e,\theta) - f(e,\tilde{\theta})] \tag{12}$$

And,with a slight abuse of notation, the agency surplus can be rewritten :

$$\begin{aligned} V(s,\tilde{\theta}) = \int_{\tilde{\theta}}^{1} \Big\{ & [p(e,\theta)(R+b) - (F + f(e,\theta))] \\ & - [p(e,\theta) - p(e,\tilde{\theta})](R+s) + [f(e,\theta) - f(e,\tilde{\theta})] \Big\} g(\theta) d\theta \end{aligned} \tag{13}$$

For a given $\tilde{\theta}$ a change of s as the following effect on the agency surplus:[9]

$$\begin{aligned} \frac{\partial V}{\partial s} &= \int_{\tilde{\theta}}^{1} [p_e(R+b) - f_e] e'(s) dG(\theta) - \int_{\tilde{\theta}}^{1} [p(e,\theta) - p(e,\tilde{\theta})] dG(\theta) \\ &= \int_{\tilde{\theta}}^{1} [p_e(b-s)] e'(s) dG(\theta) - \int_{\tilde{\theta}}^{1} [\theta - \tilde{\theta}] dG(\theta)(1-e) \\ &= \frac{1}{\gamma} \int_{\tilde{\theta}}^{1} \Big\{ (1-\theta)(b-s) - (\theta - \tilde{\theta})(\gamma - R - s) \Big\} g(\theta) d\theta \end{aligned} \tag{14}$$

The first line makes use of $p_e(R+s) = f_e$ for all θ which cancels the influence of s via e on the second line of eq. (13). In the second line, f_e is replaced by $p_e(R+s)$ for all θ, and the third line makes use of equation (9).

There are two effects: effort is increased (first term) and the expected subsidy transferred to firms is increased (second term). The expected subsidy is increased because a high type firm is more likely to succeed $p(e,\theta) > p(e,\tilde{\theta})$ and get the s_1 subsidy. So any change of the scheme that transfers subsidy from failure to success while keeping constant the expected subsidy of the threshold firm has a positive effect on the expected subsidy of initiated projects. This gives the following proposition which characterizes the optimal second best threshold θ^{SB}.

[9] A change of s that keeps $\tilde{\theta}$ fixed is equivalent to a change of s_1 and a corresponding change of s_2 with $ds = ds_1 - ds_2$, and from eq. (10) we get $p(e,\tilde{\theta})ds_1 + (1-p(e,\tilde{\theta}))ds_2 = 0$. A change of $\tilde{theta}$ for a given s only necessitates a change of s_2 exactly offset by a change of $ds_1 = ds_2$.

Proposition 1 *The optimal couple* (s_1^+, s_2^+) *is such that the bonus* $s_1^+ - s_2^+$ *is lower than b, the effort exerted by firms is then suboptimal and less projects are selected than in the first-best* $\theta^{SB} > \theta^{FB}$.

Furthermore if both subsidies are positive, they satisfy:

$$\int_{\tilde{\theta}}^{1} (1-\theta) dG(\theta)(b - s^+) = \int_{\tilde{\theta}}^{1} [\theta - \tilde{\theta}] dG(\theta)(\gamma - (R + s^+)), \tag{15}$$

and θ^{SB} *solves*

$$p(e,\theta)(R+b) - [F + f(e,\theta)] = [1 - G(\theta)][p_\theta(e,\theta)(R+s) - f_\theta(e,\theta)]. \tag{16}$$

The Proof is in Appendix A. In the Appendix, we also characterize the optimal menu when subsidies are not constrained and firms need the regulator's consent to initiate a project. Equation (15) exhibits the trade-off between efficiency (left-hand side) and rent extraction (right-hand-side), and is reminiscent of the equation satisfied by the optimal menu (cf Appendix A.1).

Several comments are in order that point out the significance of restricting subsidies to be non negative. Firstly, if subsidies are restricted to be non-negative, the expected subsidy received by a firm is positive whatever its type, which is not necessarily true if subsidies can be negative. If $s_1 < 0$ then high type firms will not subscribe to the scheme and their projects will be initiated with purely private funding. A second threshold should then be introduced for projects that do not subscribe to the scheme.

Secondly, the disentangling between the choice of $\tilde{\theta}$ and the bonus s is feasible as long as neither non-negativity constraints on subsidies are binding. If one of these two constraints is binding, either $s_1 = 0$ or $s_2 = 0$, then the choices of the bonus and the threshold type can no longer be made independently. It will be further illustrated in the analysis of the benchmark models.

Thirdly, the surplus of the agency $V(s, \tilde{\theta})$ is not necessarily concave with respect to s without further assumptions on the distribution of types. This non-concavity arises because a larger bonus induces more efforts which reduces the gap between the projects probability of success and thus the rent to high types. Formally, from equation (14), the agency surplus is quadratic with respect to s with a second order coefficient:

$$\int_{\tilde{\theta}}^{1} [(\theta - \tilde{\theta}) - (1-\theta)] g(\theta) d\theta = \int_{\tilde{\theta}}^{1} (\theta - \tilde{\theta}) \Big[g(\theta) - g(1 + \tilde{\theta} - \theta) \Big] d\theta \tag{17}$$

the sign of which depends on the shape of G and the threshold $\tilde{\theta}$.[10]

As a matter of fact the derivation of the optimal scheme in S^+ may be quite complex. However the following two propositions provide some hints on the qualitative structure of the solution. For instance, with a uniform distribution of types over $[0, 1]$, the coefficient of s is null, and the surplus of the agency is either everywhere increasing or decreasing with respect to s, whatever the threshold $\tilde{\theta}$. The following Proposition can then be deduced.

Proposition 2 *With a uniform distribution of types θ over $[0, 1]$, $s_1^+ = 0$ and $s_2 > 0$: failure only should be subsidized.*

The Proof is in Appendix A. This proposition may be interpreted as follows. On the one hand a positive subsidy s_1 encourages effort, which is the more valuable the lower θ and the higher b. On the other hand it opens the way for windfall profit for high type firms. If the probability distribution over θ is sufficiently flat and large these two effects annihilate each other and a positive subsidy s_2 is good enough. In this reasonning the importance of Assumption 3: $\gamma > (R + b)$ should be stressed. The cost of effort should be sufficiently large or the social benefit should be sufficiently low to ensure that it is not optimal that all projects succeed with probability 1. Otherwise, the optimal bonus should be set to ensure that effort is high enough to ensure that the probability of success equals 1 and the selection of project is only a matter of cost comparison. The following Proposition may be seen as a counterpoint to Proposition 2: if encouraging effort is not too costly, or if the social benefit is quite large, all projects should be encouraged and will succeed, the adverse selection problem evaporates. Interestingly, in that case the subsidy in case of failure is never paid, since all projects succeed, but still it is necessary to incentivize firms to exert a proper effort.

Proposition 3 *If Assumption 3 is not satisfied and $\gamma < R + b$, there is a local maximum for unconstrained scheme within S such that all projects succeed with probability 1:*

$$s = \gamma - R \Rightarrow e(s) = 1$$

[10]In general, multiple local maxima might arise, and a side consequence is that even if the optimal couple of subsidy within S^+ is composed of positive subsidies, the optimal couple of subsidies within the broader class S can be different and have a negative component.

and

$$R + b - (F + f(1,\tilde{\theta})) = \frac{1 - G(\tilde{\theta})}{g(\tilde{\theta})}\frac{\gamma}{2}. \tag{18}$$

This local optimum is within S^+ if γ is below a threshold.

See Appendix A for the proof. The significance of these general results will be further explored through two benchmark situations and a numerical example.

3 Benchmark situations

3.1 Rewarding failure: adverse selection

The problem of additionality and windfall profit appears in its simplest form in a pure adverse selection problem, in which no efforts is exerted.[11] The agency does not know the ex-ante probability of success $p = \theta$ of a given project.

The sole remaining purpose of the incentive scheme is to select projects to be initiated. We will show that the first best is achieved for schemes in S, there is no need to introduce a menu of contracts. The optimal second best scheme, with non-negativity constraints, consists in rewarding failure and it does not get the first best. We shall further discuss how it departs from a flat subsidy $s_1 = s_2$.

The threshold type $\tilde{\theta}$ at which expected profit is null $\pi(\tilde{\theta}, s_1, s_2) = 0$ is

$$\tilde{\theta}(s_1, s_2) = \frac{F - s_2}{R + s_1 - s_2} \tag{19}$$

If $R + s_1 > s_2$, as will be the case at relevant schemes, all $\theta \geq \tilde{\theta}$ will be initiated. The agency surplus can be rewritten:

$$V(s_1, s_2) = \int_{\tilde{\theta}(s_1,s_2)}^{1} [\theta(b - s_1) - (1-\theta)s_2]g(\theta)d\theta \tag{20}$$

[11] To be fully rigorous, in a standard adverse selection model effort would be exerted and contractible, the agency would propose a contract e, s_1, s_2 to firms, it would be relatively similar to the case without effort since all firms would make the same effort, but with an additional regulatory variable.

The first best threshold type is:

$$\theta^{FB} = \frac{F}{R+b}, \tag{21}$$

And the threshold type without any subsidy is:

$$\theta^{BAU} = \tilde{\theta}(0,0) = \frac{F}{R} \tag{22}$$

Again decompose the problem of the agency in two steps.

Step 1, given a targeted threshold probability θ^t the agency minimizes the expected cost of the subsidy:

$$C(\theta^t) = \min_{s_1,s_2} \int_{\theta^t}^{1} \max\{\theta s_1 + (1-\theta)s_2, 0\} dG(\theta), \; s.t. \; \tilde{\theta}(s_1,s_2) = \theta^t.$$

If subsidy can be negative, then firms might be better off investing without subscribing to the scheme, and they do so if the expected subsidy is negative. This possibility explains the maximum function in the integrand. Indeed, if the subsidies are constrained to be non negative then an investing firm subscribes to the scheme. This will give the following lemma.

Step 2, the optimal choice of θ^t maximizes $V = b\int_{\theta^t}^{1} pg(p)dp - C(p^t)$. This will give the proposition that follows.

Lemma 1 *Whatever the targeted threshold type θ^t, the scheme that minimizes the expected cost of the subsidy is:*

- *For the class S, $s_2 = F$ and $s_1 = F - R + \epsilon$ with ϵ infinitely small. Then, the profits of firms that subscribe to the subsidy is null and the surplus of the agency is equal to welfare minus the BAU profit: $W(\theta^t) - \Pi^{BAU}$.*
- *For the class S^+, $s_1 = 0$ and $s_2 = (F - \theta^t R)/(1-\theta^t)$. The profit of a firm of type $\theta \in (\theta^t, 1]$ is positive, and the agency surplus lower than welfare minus BAU profit.*

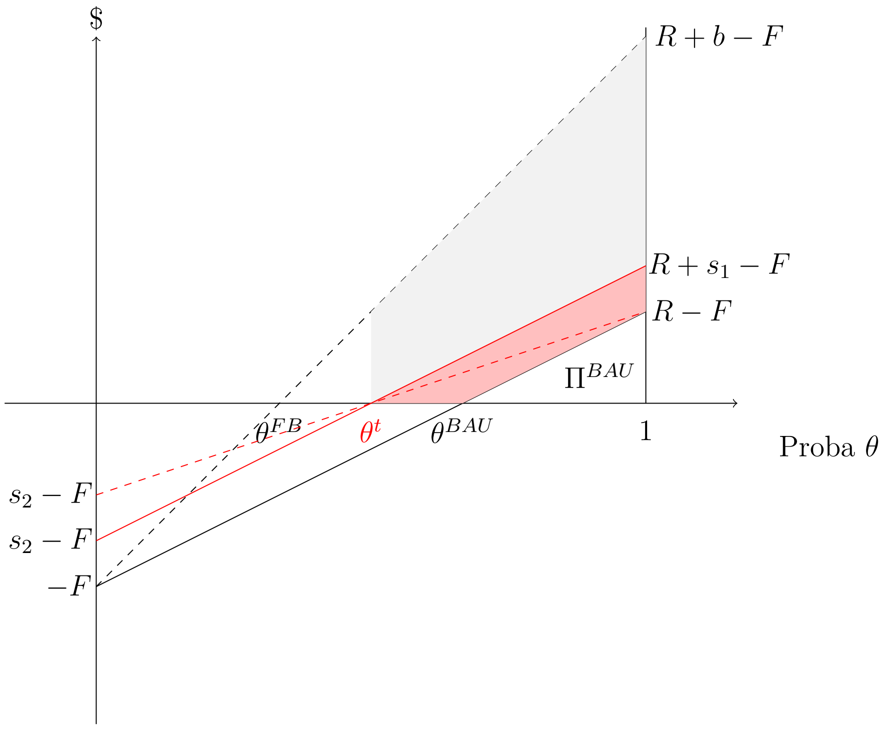


Figure 1: Expected subsidy as a function of the firm type: the red area is equal to the total expected subsidy (weighted by $g(\theta)$).

The result of Lemma 1 is illustrated Figure 1. Given a couple s_1, s_2 the red area corresponds to the total expected subsidy, the dashed line depicts a change of the subsidy line associated to an increase of s_2 and a reduction of s_1 that leaves the threshold firm unchanged. As can be seen such a change reduces the total expected subsidy by reducing the expected subsidy obtained by high type firms. High type firms succeed more frequently than the threshold type, they get more frequently the subsidy in case of success, and less frequently the subsidy in case of failure, the expected subsidy is then reduced by rewarding more failure and less success. At the extreme it is optimal to reward only failure in order to limit windfall profit.

We shall now show that without positivity constraints the optimal value of θ^t is θ^{FB} and the first best is achieved, while $\theta^{FB} \leq \theta^t \leq \theta^{BAU}$ with constraints. Let us denote θ^{SB} the optimal value of $\theta^t(s_1, s_2)$ in the second best approach. Indeed the following proposition holds:

Proposition 4 *At the optimal scheme*

- *For the class S, the optimal scheme is such that $\theta^t = \theta^{FB}$ and the first best is achieved. The profit of firms that subscribe to the scheme is null, and the agency surplus is equal to $W^{FB} - \Pi^{BAU}$.*
- *For the class S^+, the first best is not achieved, the optimal scheme rewards failure only with $s_1 = 0$ and $s_2 \geq 0$ is such that:*

  (i) $s_2 = 0$ and $\theta^{SB} = \theta^{BAU}$ if

$$b \leq \frac{R^3}{F(R-F)} \int_{F/R}^{1} (1-\theta) g(\theta) d\theta \tag{23}$$

  (ii) otherwise $s_2 > 0$ and $\theta^{FB} \leq \theta^{SB} \leq \theta^{BAU}$ with θ^{SB} defined by the following implicit equation:

$$\theta^{SB} = \theta^{FB} + \frac{1}{g(\theta^{SB})} \frac{R-F}{b+R} \int_{\theta^{SB}}^{1} \frac{1-\theta}{(1-\theta^{SB})^2} dG \tag{24}$$

The proposition may be interpreted as follows. Consider an incentive scheme and assume that $\theta^{FB} \leq \tilde{\theta} \leq \theta^{BAU}$. On the one hand projects of type θ such that $\theta^{FB} \leq \theta \leq \tilde{\theta}$ will not be implemented while they should from a first best point of view. This generates a relative loss, to be denoted as a selection bias: $W^{FB} - W(\tilde{\theta})$. On the other hand projects of type θ such that $\tilde{\theta} < \theta \leq 1$ will be implemented but with a windfall profit, which is a second loss for the regulator: $\Pi(s_1, s_2) - \Pi^{BAU}$. This gives the following result.

Corollary 4 *The optimal second best solution minimizes the sum of the selection bias and the windfall profit.*

$$V(s_1, s_2) = W^{FB} - \Pi^{BAU} - \Big[\underbrace{W^{FB} - W(\tilde{\theta})}_{\text{selection bias}} + \underbrace{\Pi(s_1, s_2) - \Pi^{BAU}}_{\text{windfall profits}} \Big]$$

Note that if Assumption 1 is not satisfied, i.e. $\Pi^{BAU} < 0$, the optimal scheme in S is non negative. The first best is achieved in S^+.

It is relatively straightforward to establish that a menu of subsidies cannot improve the situation whenever Assumption 1 holds. Whatever the initial subsidy couple proposed (s_1, s_2), there is no room for maneuver: the agency cannot propose another couple (s'_1, s'_2) that would be both more interesting to a firm of type $\theta > \tilde{\theta}(s_1, s_2)$ and less costly to the agency. The first condition being equivalent to $\theta s'_1 + (1-\theta)s'_2 > \theta s_1 + (1-\theta)s_2$ and the second to $\theta s'_1 + (1-\theta)s'_2 < \theta s_1 + (1-\theta)s_2$. Note that the above reasoning does not rest on the positivity constraints but on the risk neutrality of the principal and the agent, or the absence of moral hazard, which is analyzed in the following two sections.

Let us now consider as a direct extension a situation in which the agency observes with some noise whether the project is successful or not. The agency can only condition the subsidy on the observed signal, s_1 if it observes a success and s_2 if it observes a failure. Let α_1 be the probability of observing a signal of failure if the project is a success and α_2 the probability of observing a signal of failure if the project fails. We assume that $\alpha_2 \geq \alpha_1$, a perfect signal corresponds to $\alpha_2 = 1$ and $\alpha_1 = 0$ and an uninformative signal to $\alpha_2 = \alpha_1$. The subsidy obtained by a firm is $\alpha_1 s_2 + (1-\alpha_1)s_1$ in case of success and $\alpha_2 s_2 + (1-\alpha_2)s_1$ in case of failure. The threshold project is then:

$$\tilde{\theta}(\alpha_1 s_2 + (1-\alpha_1)s_1, \alpha_2 s_2 + (1-\alpha_2)s_1),$$

and the expected total subsidy is

$$\int_{\tilde{\theta}}^{1} \Big\{\theta\big[(1-\alpha_1)s_1 + \alpha_1 s_2\big] + (1-\theta)\big[(1-\alpha_2)s_1 + \alpha_2 s_2\big]\Big\} dG(\theta)$$

Corollary 5 *If the success and failure of a project are not perfectly observable,*

- *For the class S: the first best is achieved.*
- *For the class S^+: The optimal scheme remains of the form $s_1 = 0$ and $s_2 > 0$. The second best threshold type, the expected subsidy, the agency surplus, the welfare and the profit of firms only depend on the ratio α_1/α_2.*
  - *If $\alpha_1 = 0$ (success is perfectly observed), then, whatever α_2, at the optimal second best scheme, the threshold probability, welfare and s_2 do not depend on α_2 and correspond to perfect observability situation.*

– *Otherwise, with a uniform distribution, the threshold probability is higher, and, welfare and the agency surplus are lower than in the case with a perfect signal.*

Proof. see Appendix B.3 ■

Technically the agency always prefers to subsidize failure and not success. Whether a failure is not properly identified ($\alpha_2 < 1$) is not an issue since it can play with s_2 to increase the expected subsidy in case of failure. The agency is mainly concerned by the noise in case of success, $\alpha_1 > 0$. The ratio α_1/α_2 is the number of $ awarded to successful projects for any $ awarded for failed projects, this ratio determines the inefficiency of the subsidy scheme with noise.

Two comments are in order. Firstly, an imperfect signal may originate from a manipulation of the agent. The mere possibility of such a manipulation deteriorates the efficiency of the incentive scheme. Secondly, in case of an uninformative signal, the optimal scheme is equivalent to a flat subsidy $s_1 = s_2$ since the subsidy will be given independently of the signal received.

3.2 Rewarding success: moral hazard

In a pure moral hazard setting the type of the firm θ is known by the agency, but it can neither observe the effort nor its cost. We shall show that without the non negativity constraints the first best is achieved. With the non negativity constraints rewarding success only is the second best solution, however the first best is not achieved.

The effort that maximizes welfare e^{FB} is given by (8), the effort exerted by the firm by (9) and $e^{BAU} = e(0,0) = R/\gamma$ so that $e^{BAU} \le e^{FB}$.

We can now derive θ^{BAU} and θ^{FB} that give the respective thresholds for a firm to deploy the project without subsidy and for a first best deployment. It is easily seen that:

$$\theta^{BAU} = \frac{1}{R}\frac{2F\gamma - R^2}{2\gamma - R} \tag{25}$$

$$\theta^{FB} = max\left\{\frac{1}{R+b}\frac{2F\gamma - (R+b)^2}{2\gamma - (R+b)}, 0\right\} \tag{26}$$

Note that $\theta^{FB} \ge 0$ if and only if $(R+b)^2 \le 2F\gamma$. As b increases θ^{FB} decreases from θ^{BAU} to 0. BAU is such that for $\theta \le \theta^{BAU}$ the firm does not initiated the project, and for $\theta^{BAU} < \theta \leq 1$, it deploys the project and makes the effort e^{BAU}.

We now describe the optimal second best scheme $(s_1, s_2) \in S^+$ as a function of the type θ of the project. The agency should decide whether to ensure the deployment of a project for $\theta \leq \theta^{BAU}$, and whether to further motivate effort. Firstly, if the agency ensures the deployment of a project it is optimal to do so by rewarding success and not failure i.e. $s_1 > 0$ and $s_2 = 0$ since it maximizes the effort of the firm. Secondly, if the agency subsidizes a project it has to decide whether to solely ensure the deployment, leaving no rent to the firm, or further subsidizing success to increase the firm's effort. The occurrence of these two possibilities depends on the value of the parameters b, γ, R and F.

For small values of the external benefit b, we have $\theta^{FB} \geq 0$ and the agency does not subsidize the firm as long as $\theta \leq \theta^{FB}$. For a higher type θ, the agency subsidizes the project and calibrates the subsidy so that the firm's profit is null. For larger values of b, the agency might let a windfall profit to the firm to achieve a high probability of success. The following proposition makes this precise and proves that there will be a windfall profit as soon as $R + b \geq 2\sqrt{2F\gamma}$.

Define:

$$s_{1A}(\theta) = \frac{b - R}{2} - \gamma \frac{\theta}{1 - \theta} \tag{27}$$

and

$$s_{1B}(\theta) = \gamma \frac{\theta}{1 - \theta} \left[\sqrt{1 + \frac{2F}{\gamma} \frac{1 - \theta}{\theta^2}} - 1 \right] - R \tag{28}$$

Proposition 5 *The optimal scheme (s_1^*, s_2^*) is such that $s_2^* = 0$. The precise expression of $s_1^*(\theta)$ depends on the two following cases:*
Case 1: If $R + b \leq 2\sqrt{2F\gamma}$ ($\theta^{FB} > 0$) then

- if $\theta \leq \theta^{FB}$ the optimal subsidy is null, the project is not implemented;

- if $\theta^{FB} \leq \theta < \theta^{BAU}$ the optimal subsidy is $s_{1B}(\theta)$, the project is implemented and the firm gets no windfall profit;

- if $\theta \geq \theta^{BAU}$ the optimal subsidy is null, it is business as usual, the project is implemented and the firm gets no windfall profit.

Case 2: if $R + b \geq 2\sqrt{2F\gamma}$ ($\theta^{FB} = 0$) there is a threshold θ_A such that:

- if $\theta \leq \theta_A$ the optimal subsidy is $s_{1A}(\theta)$, the project is implemented and the firm gets a windfall profit;

- if $\theta_A \leq \theta < \theta^{BAU}$ the optimal subsidy is $s_{1B}(\theta)$, the project is implemented and the firm gets no windfall profit;

- and if $\theta \geq \max\{\theta_A, \theta^{BAU}\}$ the optimal subsidy is null, it is business as usual, the project is implemented and the firm gets no windfall profit;

Proof in Appendix B.4.

The precise expressions of θ_A cannot be determined explicitly, it is the type at which the expression s_{1A} is either equal to s_{1B} or null. Above this type θ_A, it is not worth conceding a rent to the firm in order to increase effort. Depending on the value of b this threshold is either larger or lower than θ^{BAU}.

With an unconstrained scheme, the agency implements the first best, welfare is then equal to $w(\theta, e^{FB})$, and the firm gets its BAU profit. The proof of this lemma is straightforward.

Lemma 2 *The first best is obtained with a scheme in S such that:*

- if $\theta \leq \theta^{FB}$, no subsidy is proposed and the project is not initiated,

- if $\theta \geq \theta^{FB}$, the optimal scheme is such that $s_1 - s_2 = b$ and s_2 such that $\pi = \pi^{BAU}$.

The agency surplus is then $v = p(e, \theta)(b - (s_1 - s_2)) - s_2 = -s_2$, the subsidy s_2 is negative and corresponds to a tax on profit.

4 Could it be optimal to subsidize both success and failure? An illustrative example

As already mentionned the derivation of the optimal second best scheme in the general case may be quite complex. In this section we build on the general results to provide some ideas on the qualitative structure of the solution.

Start with Proposition 2. It states that if the probability distribution of types θ is large and uniform then rewarding failure only is the solution. Now if the weight of the distribution concentrates on low types this can no longer be correct: it will be optimal to elicit efforts from these low types, i.e. to reward success. Note that this is equivalent to assuming that the social benefit b is high or that the cost of effort γ is low. We shall show through an illustrative example that the optimal scheme continuous shifts from rewarding success to rewarding failure as the weight of distribution moves from low types to high types.

More specifically, we consider only two types: θ_L and θ_H with $\theta_L < \theta_H$. The probability of type θ_H is denoted λ. We analyze the influence of the distribution of types. We shall show that there is a range for λ for which both subsidies are strictly positive while for lower λ rewarding success prevails and for higher λ rewarding failure prevails.[12]

The following assumption is introduced to get the results.

Assumption 6 *We take $(R+b) < 2\sqrt{2F\gamma}$ and θ_L and θ_H such that $\theta^{FB} < \theta_L < \theta^{BAU}$ and $\theta^{BAU} < \theta_H$.*

To get intuition on the structure of the optimal second best scheme start with a situation in which the cost of effort (γ) is very high. Rewarding failure only is optimal. As γ decreases, for low values of λ it may become worthwhile to induce a low type firm to make an effort through rewarding success, the incremental rent for the high type firm being more than compensated. How do these two situations of rewarding success and rewarding failure combine together? As λ increases the balance between the benefit accruing from a higher effort from a low type firm should exactly balance the increase in the rent of the high type firm. The following lemma precisely defines the relationship between s_1 and s_2 for these intermediary situations.

Lemma 3 *If both subsidies are strictly positive the optimal scheme (s_1^*, s_2^*) in S^+ satisfy:*

$$s_1^* - s_2^* = b - \frac{\gamma\lambda(\theta_H - \theta_L)}{(1-\theta_L) - 2\lambda(\theta_H - \theta_L)}(1 - e^{FB}) \tag{29}$$

and s_2^* is such that the profit of the low type firm is null, it solves:

$$s_2^* = F - \theta_L(R + (s_1^* - s_2^*)) - (1-\theta_L)\frac{(R + (s_1^* - s_2^*))^2}{2\gamma} \tag{30}$$

The proof is in Appendix C.

We now characterize the optimal second best scheme for all values of λ.

[12] In Appendix we show that in such a situation there are two potential benefits to using a menu in S, i.e. inducing different effort levels depending on the type of the firm and taxing profits. Still the optimal menu leaves a gap as compared with the first best: asymmetry of information generates some inefficiency independently of constraints on the incentive schemes.

Proposition 6 *The optimal scheme (s_1^*, s_2^*) depends on three thresholds λ_1, λ_2 and λ_3 as follows:*

– *for* $0 < \lambda \leq \lambda_1 :\quad s_1^* > 0$ *and* $s_2^* = 0; s_1^* = s_{1B}(\theta_L)$ *given by equation (28)*

– *for* $\lambda_1 < \lambda < \lambda_2 :\quad s_1^* > 0$ *and* $s_2^* > 0$ *given by Lemma 3 ;*

– *for* $\lambda_2 < \lambda < \lambda_3 :\quad s_1^* = 0$ *and* $s_2^* > 0$ *such that* $\pi(\theta_L, e, 0, s_2^*) = 0 :$

$$s_2^* = R - \gamma + \gamma \left[1 - \frac{2}{\gamma}\frac{R - F}{1 - \theta_L}\right]^{1/2}$$

– *for* $\lambda_3 < \lambda \leq 1 :\quad s_1^* = 0$ *and* $s_2^* = 0.$

The profit of a low type firm is always null, a high type firm gets a windfall profit as long as $\lambda < \lambda_3$.

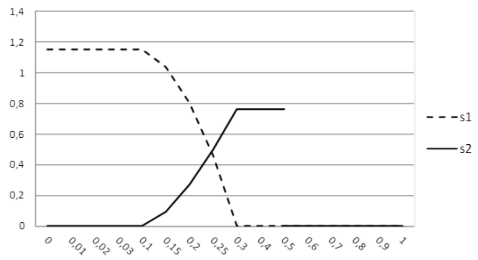
To complete our discussion we provide the detailed solution for specific values of the parameters and consider the benefit of introducing a menu. We take $F = 1$, $R = 1.5$, $b = 2$ and $\gamma = 12$. We have $\theta^{FB} = .16$ and $\theta^{BAU} = .64$. The low type is such that $\theta^{FB} < \theta_L = .3 < \theta^{BAU}$ and the high type such that $\theta_H = .75 > \theta^{BAU}$. Assumption 6 is satisfied: A low type firm would not implement the project but it would be socially valuable to do it. A high type firm would implement the project without subsidy. The parameter λ denotes the probability for the firm to be of the high type.

We can derive numerically the thresholds for λ to approximately be $\lambda_1 = .1$, $\lambda_2 = .3$ and $\lambda_3 = .6$. Figure 2(a) depicts the second best optimal solution: only reward success if $0 \leq \lambda \leq \lambda_1$, reward both success and failure if $\lambda_1 \leq \lambda \leq \lambda_2$, only reward failure if $\lambda_2 \leq \lambda \leq \lambda_3$, and provide no subsidies if $\lambda_3 \leq \lambda \leq 1$. The extreme cases correspond to intuition. Reward success if λ is small, a situation in which effort should be encouraged and windfall profit discounted by a low probability of occurence. Reward failure if λ is large for the reverse reasoning, up to a point at which type θ_L does not matter anymore and BAU should be prefered, letting no windfall profit to type θ_H. It is when $\lambda_1 \leq \lambda \leq \lambda_2$ that we expect the most from a menu. The optimal menu is derived from Proposition 7 in Appendix C.3. Figure 2(b) depicts the optimal menu. Observes that it uses negative values for subsidies so as to get back the profit of the firm.[13]

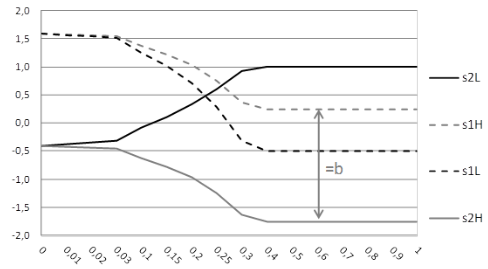
Figure 2(c) allows for comparing the second best effort and the conditional menu efforts. The second best effort decreases as λ gets into the zone $\lambda_1 \leq \lambda \leq \lambda_2$. The effort for type θ_L is sacrificed for not giving a windfall profit to

[13]We have not derived the optimal second best menu but we think that this would not qualitatively alter our discussion.

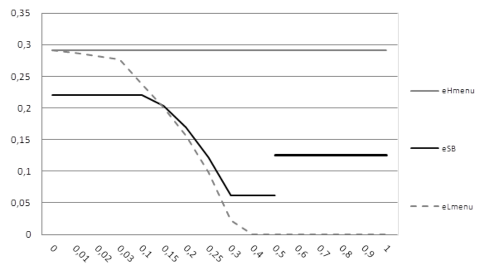
type θ_H. With a menu this is also the case but the first best effort for a high type firm is elicited, for a low benefit. Consequently we do not expect that the menu increases a lot the expected benefit for the agency. This argument does not carries over to large values of λ since negative subsidies allow the agency to recover the profit of the firm but this would no longer be true with a constrained menu. As a side comment observe that it is optimal not to induce the first best effort for the low type firm (a standard result of contract theory) this explains why the first best cannot be achieved with a menu independently of negativity constraints.



(a) Optimal second best scheme as a function of the probability λ of high type



(b) Optimal menu $\{(s_{1H}, s_{2H}), (s_{1L}, s_{2L})\}$ as a function of the probability λ of high type



(c) Efforts as a function of the probability λ of high type

Figure 2: Optimal second best scheme, menu and efforts with two types L and H as a function of the probability λ of a high type ($R = 1.5$, $F = 1$, $\gamma = 12$ and $b = 2$).

Figure 4 indeed shows that the benefit of using a menu is not significant for $\lambda_1 \leq \lambda \leq \lambda_2$. As expected the benefit of using a menu (without non-

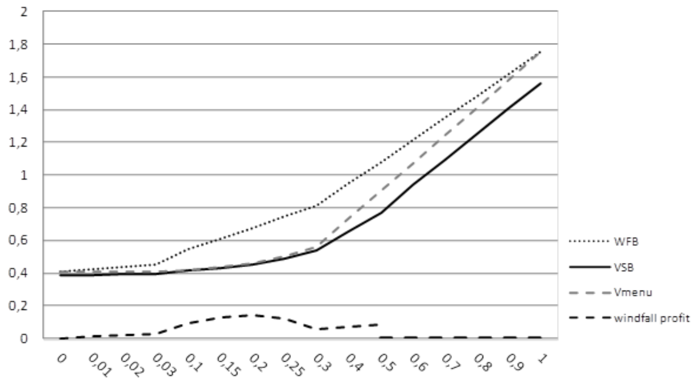


Figure 3: Payoffs as a function of the probability of a high type with First Best, Second Best and a menu (unconstrained).

negativity constraints) becomes significant for high λ through recovering the profit of the firm. In this Figure the expected windfall profit for the firm with a second best optimal scheme is also displayed (multiplied by 10 to be seen in the graph). There are two peaks reflecting the conflicting forces associated with rewarding success and rewarding failure.

5 Policy implications and research extensions

This paper is concerned with public financing of R&D programs for the energy transition that have the following characteristics: the program has an uncertain outcome and an initial sunk cost, full success or total failure, the social benefit is associated with success which also generates private gains, public financing takes the form of non negative subsidies, the state agency which monitors the subsidy allocation process has much less information about the economics of the project than the firm. We proved that rewarding failure is a superior strategy in presence of pure adverse selection while rewarding success is superior in presence of pure moral hazard. We also identified conditions in which both forms of subsidies should be implemented. We showed that conditional subsidies always perform better than flat subsidies.

The motivation comes from an in depth analysis of a state aid program

Structural parameters	Reward Failure	Flat subsidy	Reward success
Social benefit	High	High	Very high
Risk of windfall profit	High	Low	Low
Distribution of type	Flat and spread	Bimodal	Unimodal
Observability of success	High	Low	High
Impact of effort e on the probability of success $p(e, \theta)$	Small	Low	High

launched in France in 2010, the Investments for the Future programme. It covers a period of 10 years (2010-2020), for a total budget of 57 B €, and several types of activities,[14] among which innovative activities for the energetic transitions. This last part, of a total budget of 10 B €, is monitored by ADEME, a state agency.[15] Yearly, ADEME opens calls for innovative projects on some predefined areas. Each project is examined on its own merit, a selection is made. Then ADEME proposes a contract to each eligible project and the firm accepts or rejects the contract. Over 2010-2015 ADEME has financed more than 250 projects in areas such as renewable energy, zero emission vehicles, green chemistry, etc. Similar programs exist in other countries, notably the SunShot initiative in the US, launched in 2011 with the aim of driving down the cost of solar energy.[16]

Initially ADEME only used flat subsidies then introduced repayable advances for two reasons. First, a regulatory constraint from the EU that limits the level of pure subsidy at a given share of the project budget while the constraint is more flexible for repayable advances (which are less likely to bias competition). Second evidence of windfall profits appeared quite clearly in some projects. Indeed as shown in our analysis repayable advances reduce windfall profits and ensure additionality (Proposition 4). In some instances, the empirical difficulty to observe success led the agency to define intermediary technical steps and have repayable advances paid back in part along the way to avoid being manipulated and having to fall back on flat subsidies (Corollary 5).

We had the opportunity to review a large number of projects and discuss their contractual arrangements with ADEME. The economic analysis of this

[14] https://www.gouvernement.fr/secretariat-general-pour-l-investissement-sgpi

[15] http://www.ademe.fr/en/investments-for-the-future

[16] https://www.energy.gov/eere/solar/sunshot-initiative

paper allows for a better understanding of the issues at stake and offers suggestions for policy recommendations. These suggestions are summarized in Table 5 as a typology of situations. Along the lines a number of dimensions related to the project under consideration need be assessed. Depending on the significance of these dimensions three extreme contractual modes can be used as benchmarks: reward failure, flat subsidy, reward success. The proposed typology directly follows from our model. The first two lines are the key ingredients of the projects. Then, we recommend that if all type firms make low revenue from the project (R is low so that the first best is achieved with an almost flat subsidy, section 2.2) or if the observability of the outcome is low (inducing a risk of managerial gaming, corollary 5), flat subsidies should be preferred. This will also be the case whenever these two dimensions are less significant but the distribution of types is bimodal and the impact of effort is low (Proposition 6). Reward failure should be preferred when the risk of windfall profit is high, all types are equally likely and the impact of effort is low (Proposition 2). Reward success should be preferred when the impact of effort is high (Proposition 5) and the probability distribution is concentrated on low types (Proposition 6). A mixture of rewards (i.e. a scheme close to a flat subsidy) should be preferred whenever the distribution of types is bimodal (Proposition 6).

We think that this typology already provides relevant and economically grounded guidance for designing contractual arrangements. Some extensions would be worthwhile to pursue. The analysis carried on in Section 4 is preliminary: a more complete study should be made, and robustness to other functional forms be tested. Secondly we only investigated a situation in which the project leads to two extreme outcomes, failure or success. We may consider projects involving a technical phase (a proof of concept is or is not achieved) and a market phase (with a continuous outcome such as sales). The contractual arrangements should take advantage of the interim information that is revealed along the project. Thirdly, the asymmetry of information may involve another party. It appeared that in some projects ADEME plays the role of a middle man between the firm and the banking system. Indeed, at first, the asymmetry of information is much more acute between the firm and the banking system (which induces a capital market failure) than between the firm and ADEME (the state agency has a much higher technical expertise than banks). The formalization should explicitly analyze how the contractual arrangement between the state agency and the firm should evolve as the asymmetry between the bank and the firm reduces over time.

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Appendix

A General model

A.1 Optima menu without constraints

We provide a description of what would be the structure of a menu if the subsidies are not constrained to be positive, and firms cannot initiate the project without the regulator consent.

It is easier to work with the bonus $s(\theta)$ and consider $s_2(\theta)$ as a fixed transfer. The agency proposes a structured menu $(s(\theta), s_2(\theta))_{\theta\in(0,1)}$, a firm of type θ selecting the item $(s(\eta), s_2(\eta))$ has a profit $\pi(\theta, \eta) = p(e, \theta)(R +$

$s(\eta)) + s_2(\eta) - (F + f(e,\theta))$, and the first order condition necessary for self selection is $p(e,\theta)s'(\theta) + s_2'(\theta) = 0$.

Using the standard methodology in contract design, denoting $\pi^m = \pi(\theta,\theta)$, its total derivative is $d\pi^m/d\theta = p_\theta(R+s) - f_\theta(e,\theta)$ which only depends on the bonus $s(\theta)$ and not $s_2(\theta)$ because e does. Thanks to this relationship and an integration by part the surplus of the agency can be written:

$$V = \int_{\tilde{\theta}}^{1} \left\{ \Big[p(e,\theta)(R+b) - (F - f(e,\theta))\Big] g(\theta) - \frac{d\pi^m}{d\theta}[1 - G(\theta)] \right\} d\theta$$

The optimal bonus $s(\theta)$ should be such that

$$p_e(b-s)\frac{de}{ds} = \frac{1-G}{g}\frac{d}{ds}\Big[p_\theta(e,\theta)(R+s) - f_\theta(e,\theta)\Big]$$

and with our quadratic specification $s(\theta)$ solves:

$$(1-\theta)(b-s)\frac{1}{\gamma} = \frac{1-G(\theta)}{g(\theta)}(1-e(s)) = \frac{1-G(\theta)}{g(\theta)}(\gamma - R - b)\frac{1}{\gamma} \tag{31}$$

which looks like the equation (15) satisfied by a simple scheme (s, s_2). We recover the usual result that $s = b$ for high types. The selection of projects is done with the choice of $s_2(\tilde{\theta})$ the profit of the $\tilde{\theta}$ firm being nul.

A.2 Proof of Proposition 1

The agency surplus is positive for $s_1 = s_2 = 0$, and if $s_1 - s_2 \geq b$ it is non-positive, therefore, at the optimum scheme $s_1 - s_2 < b$.

Concerning the selection of projects, several cases should be distinguished according to the sign of the two subsidies at the optimum:

i If $s_1 > 0$ and $s_2 > 0$: the derivative of V with respect to s, expressed in eq. (14), is null and eq. (15) is satisfied.

  θ^{SB} cancels the derivative of V, given by eq. (13), with respect to $\tilde{\theta}$ which gives (16).

ii If $s_1 \geq 0$ and $s_2 = 0$: then $s_1 < b$ and at θ^{SB}

$$0 = p(e,\theta^{SB})(R + s_1) - [F + f(e,\theta^{SB})] \text{ from eq. (10)} \tag{32}$$

$$< p(e,\theta^{SB})(R + b) - [F + f(e,\theta^{SB})] \tag{33}$$

$$< p(e^{FB},\theta^{SB})(R + b) - [F + f(e^{FB},\theta^{SB})] \tag{34}$$

  therefore, $\theta^{SB} > \theta^{FB}$ less projects are selected than at the first best.

iii If $s_1 = 0$ and $s_2 > 0$: the above method cannot be applied, the first order condition should be considered. The threshold $\tilde{\theta}$ cannot be chosen independently from the bonus $s = -s_2$, and s_2 cancels the derivative of V given by eq. (11) so

$$p(e, \tilde{\theta})(b + s_2) - s_2 = \int_{\tilde{\theta}}^{1} [p_e.(b - s_2)e' + (1 - p)]\, dG(\theta) \Big/ \left[-\partial\tilde{\theta}/\partial s_2\right]$$

and injecting eq. (10) the left hand side is $p(e, \tilde{\theta})(R+b) - [F+f]$ which is then strictly positive, since $\partial\tilde{\theta}/\partial s_2 < 0$, and together with the fact that $e < e^{FB} =$ implies that $\theta^{SB} < \theta^{FB}$.

A.3 Proof of Proposition 2

The second order coefficient of s is given by eq. (17), and with a uniform distribution over $[0, 1]$ it is null.

Therefore, the derivative of V with respect to s is (from eq. (14)):

$$\begin{aligned} \frac{\partial V}{\partial s} &= \frac{1}{\gamma} \int_{\tilde{\theta}}^{1} \left[(1 - \theta)b - (\theta - \tilde{\theta})(\gamma - R)\right] d\theta \\ &< \frac{b}{\gamma} \int_{\tilde{\theta}}^{1} \left[(1 - \theta) - (\theta - \tilde{\theta})\right] d\theta \text{ since } \gamma - R > b \\ &= 0 \end{aligned}$$

V is strictly decreasing with respect to s so $s_1 = 0$ (otherwise s can be decreased while keeping $\tilde{\theta}$ constant).

A.4 Proof of Proposition 3

From equation (9), $e(s) = 1$ for $s \geq \gamma - R$ so that from equation (14), V is flat for $s \geq \gamma - R$, and by continuity it is increasing for s slightly below. It is then locally optimal for any targeted threshold to set $s = \gamma - R$.

With $e = 1$, the cost of a project of type θ is then $F + (1-\theta)\gamma/2$, and the agency surplus can then be written : $V = (1-G(\tilde{\theta}))[(R+b)-(F+(1-\tilde{\theta})\gamma/2)]$. The maximization of which gives equation (18).

With $s = \gamma - R$, $p(e, \theta) = 1$ and a firm profit is $\pi = (R + s) - [F + f] = \gamma + s_2 - F - (1 - \theta)\gamma/2$. The selection of projects is ensured by setting $s_2 = F - (1 + \tilde{\theta})\frac{\gamma}{2}$, and $s_1 = s + s_2 = F + (1 - \tilde{\theta})\frac{\gamma}{2} - R$.

Both are non-negative for a sufficiently small γ.

B Benchmark situations

B.1 Proof of Lemma 1

Consider a change of the subsidy couple that keeps θ^t unchanged: $\theta^t ds_1 + (1-\theta^t)ds_2 = 0$. For $\theta > \theta^t$ the effect of this change on the expected subsidy received by the firm of type θ is: $(\theta-\theta^t)(ds_1-ds_2)$ which is negative if $ds_2 > 0$. Therefore, to reduce $C(\theta^t)$ the agency should increase s_2 and reduce s_1.

B.2 Proof of Proposition 4

The threshold probability as a function of s_2 is $\tilde{\theta}(0, s_2)$, the derivative of welfare is:

$$-[\tilde{\theta}b - (1-\tilde{\theta})s_2]g(\tilde{\theta})\frac{\partial\tilde{\theta}}{\partial s_2} - \int_{\tilde{\theta}}^{1}(1-\theta)g(\theta)d\theta \tag{35}$$

the first term is the benefit from the marginal project, the second term is the increased subsidy to all more profitable projects. the derivative of the threshold probability is

$$\frac{\partial\tilde{\theta}}{\partial s_2} = \frac{1-\tilde{\theta}}{R-s_2} = \frac{(1-\tilde{\theta})^2}{R-F}$$

the derivative of welfare could then be rewritten:

$$[\tilde{\theta}(R+b) - F]g(\tilde{\theta})\frac{(1-\tilde{\theta})^2}{R-F} - \int_{\tilde{\theta}}^{1}(1-\theta)g(\theta)d\theta \tag{36}$$

At $s_2 = 0$ $\tilde{\theta} = F/R$, and the derivative of welfare is negative if

$$[F(R+b) - FR]g(F/R)\frac{1}{R}\frac{(1-F/R)^2}{R-F} \leq \int_{\tilde{\theta}}^{1}(1-\theta)g(\theta)d\theta$$

point (i) follows. Otherwise, the optimal subsidy cancels the derivative of welfare and point (ii) describes the first order condition.

B.3 Proof of corollary 5

Let us denote $\sigma_1 = \alpha_1 s_2 + (1-\alpha_1)s_2$ and $\sigma_2 = \alpha_2 s_2 + (1-\alpha_2)s_2$ the subsidies obtained in case of success and failure respectively.

• <u>For the unconstrained class S</u>: with the couple of subsidy: $s_1 = F - \alpha_2 R/(\alpha_2 - \alpha_1)$ and $s_2 = F + (1-\alpha_2)R/(\alpha_2-\alpha_1)$, the expected subsidies are $\sigma_1 = F$ and $\sigma_2 = F - R$ which implement the first best.

• <u>For the constrained class S^+</u>:

1. $s_1 = 0$ and $s_2 > 0$: The reasoning of Lemma 1 can be reproduced: an increase of σ_2 coupled with a reduction of σ_1 that leaves $\tilde{\theta}$ unchanged reduces the total expected subsidy. Consequently it is optimal to set $s_1 = 0$ and $s_2 > 0$.

2. Then, with $s_1 = 0$, $\sigma_1 = x\sigma_2$ with $x = \alpha_1/\alpha_2$ and the threshold probability is $\tilde{\theta}(x\sigma_2, \sigma_2)$, the regulator surplus is

$$V(x\sigma_2, \sigma_2) = \int_{\tilde{\theta}}^{1} \Big[\theta(b - x\sigma_2) - (1-\theta)\sigma_2\Big] dG(\theta)$$

and welfare is $W(\tilde{\theta}(x\sigma_2, \sigma_2))$.

3. If $\alpha_1 = 0$: then $x = 0$ and the surplus of the regulator, the profit of firms, and total welfare could all be written as functions of σ_2 without any other dependence on α_2. The optimum second best scheme is then similar to the scheme described by Proposition 2 with $\alpha_2 s_2$ being independent of α_2.

4. Otherwise, for $\alpha_1 > 0$: then $x > 0$,

4.1. Let us prove that θ^{SB} is increasing with respect to x, to do so we first write the first order condition:
- the total derivative of the threshold type w.r.t. σ_2 is:

$$\frac{d\tilde{\theta}}{d\sigma_2} = -\frac{1-(1-x)\tilde{\theta}}{R-(1-x)\sigma_2}$$

the first order condition satisfied at the optimal scheme is

$$\Big[\tilde{\theta} - \theta^{FB}\Big] g(\tilde{\theta}) \frac{1-(1-x)\tilde{\theta}}{R-(1-x)\sigma_2} = \int_{\tilde{\theta}}^{1} \Big[\theta x + (1-\theta)\Big] dG(\theta)$$

and with a homogeneous distribution it gives:

$$\tilde{\theta} = \theta^{FB} + \frac{R-(1-x)F}{R+b} \frac{1}{2(1-x)} \left[1 - \frac{x^2}{\Big(1-(1-x)\tilde{\theta}\Big)^2}\right]$$

- θ^{SB} increases with respect to x (brutal calculations): The right hand side of the first order condition above side is a decreasing function of $\tilde{\theta}$, and it is increasing with respect to x: Its derivative is

$$\frac{(1-\tilde{\theta})}{2(R+b)} \frac{2Fx + R[(1-\tilde{\theta})^2 - x\tilde{\theta}(1+\tilde{\theta})]}{\left(1-(1-x)\tilde{\theta}\right)^3}$$

the sign of which is the sign of $2Fx+R[(1-\tilde{\theta})^2 - x\tilde{\theta}(1+\tilde{\theta})]$ which is positive (using that $\tilde{\theta} < F/R$).

4.2. The effect of x on the regulator surplus at the optimal scheme, by an envelop argument, it is

$$\frac{\partial V}{\partial s_1}(x\sigma_2, \sigma_2)\sigma_2$$

which is negative.

Welfare is decreasing with respect to $\tilde{\theta}$ as long as $\tilde{\theta} > p^{FB}$, so it is decreasing with respect to x.

B.4 Proof of Proposition 5

• First step: $s_2^* = 0$:

The regulator maximizes its surplus (eq. 3) subject to the non-negativity constraints on profit (eq.1) and subsidy s_1 and s_2. The Lagrangien is:

$$\mathcal{L} = v(\theta, e(s_1 - s_2), s_1, s_2) + \mu_0 \pi + \mu_1 s_1 + \mu_2 s_2$$

With μ_0 the Lagrange mulltiplier associated to the participation constraint, μ_i the multiplier associated with the non-negativity constraint of s_i, $i = 1, 2$. At the optimum

$$p_e(e,\theta)[b - (s_1 - s_2)]e' - (1-\mu_0)p + \mu_1 = 0 \tag{37}$$

$$-p_e[b - (s_1 - s_2)]e' - (1-\mu_0)(1-p) + \mu_2 = 0 \tag{38}$$

And the corresponding slackness conditions. There are eight possible situations depending on whether each Lagrange multiplier is null or positive. Summing the two equations gives

$$\mu_1 + \mu_2 + \mu_0 - 1 = 0 \tag{39}$$

Let us denote s_1^* and s_2^* the optimal subsidies.

- At least one of the μ_i is positive: otherwise $\mu_1 = \mu_2 = 0$ then $\mu_0 = 1$ and $s_1^* - s_2^* = b$. Welfare is then $-(1-p)s_2^* < 0$, which cannot be optimal.

  Consequently, $\mu_1 + \mu_2 > 0$ and $\mu_0 < 1$ (from eq. (39)).

- We show by contradiction that $b > s_1^* - s_2^*$: otherwise, from equation (37) $\mu_1 = (1-\mu_0)p - p_e[b - (s_1^* - s_2^*)]e' > 0$, which implies that $s_1^* = 0$ and $s_2^* < s_1^* - b = -b < 0$ a contradiction.

- Then $s_2^* = 0$: from equation (38): $\mu_2 = (1-\mu_0)(1-p) + p_e[b - (s_1^* - s_2^*)]e' > 0$.

• Second step: Expressions of the optimal subsidy

There are four possible cases: i) $s_1^* = 0$ and $\pi > 0$, ii) $s_1^* = 0$ and $\pi = 0$, iii) $s_1^* > 0$ and $\pi > 0$, or, iv) $s_1^* > 0$ and $\pi = 0$.

Case i) corresponds to "business as usual" no subsidy is used and the project is implemented with suboptimal effort. Case ii) corresponds to a situation in which the project is not profitable and it is not worth subsidizing it.

In case iii) $s_1^* > 0$ and $\pi > 0$ then $\partial p/\partial e[b - s_1^*]e' = p$, and in case iv) $s_1^* > 0$ and $\pi = 0$ then $p_e[b - s_1^*]e' - p = -\mu_0 p \leq 0$.

The subsidy $s_{1A}(\theta)$ defined by equation (27) is the solution of $\partial p/\partial e[b - s_1^*]e' = p$. And $s_{1B}(\theta)$ is the solution of $\pi(\theta, e(s_1), s_1, 0) = 0$, replacing e by $(R + s_1)/\gamma$ in eq. (1) gives a second order equation in $(R + s_1)$ with one positive root given by equation (28).

If $s_1^* > 0$ and $\pi > 0$ then $s_1^* = s_{1A}$, and if $s_1^* > 0$ and $\pi = 0$ then $s_1^* = s_{1B}$. Furthermore, if both expressions s_{1A} and s_{1B} are positive the optimal subsidy is the larger of the two.

•Third step: Definition of the thresholds

θ^{BAU} is the solution of $s_{1B}(\theta) = 0$, it is the lowest θ at which a null subsidy ensures deployment. θ_A is the solution of $s_{1A}(\theta) = \max\{s_{1B}(\theta), 0\}$.

- From the the expressions (27) and(28) $s_{1A} > s_{1B}$ if and only if

$$\frac{(b+R)^2}{4} > \left(\frac{\gamma\theta}{1-\theta}\right)^2 + 2F\frac{\gamma}{1-\theta}$$

the right hand side being strictly increasing with respect to θ and, converging toward infinity as θ approaches 1, so that $\theta_A < 1$ and $s_{1A} > s_{1B} \Leftrightarrow \theta < \theta_A$.

- $\theta_A \geq 0$ if and only if $R + b < 2\sqrt{2F\gamma}$, otherwise, if $R + b > 2\sqrt{2F\gamma}$, then for all $\theta \in [0, 1]$ $s_{1B} > s_{1A}$, that is $\pi(\theta, e(s_{1A}), s_{1A}, 0) < 0$.
- $s_{1B}(\theta^{FB}) = b$ (if $\theta^{FB} > 0$) by definition of θ^{FB}: $p(e^{FB}, \theta^{FB})(R + b) - F - f(e^{FB}, \theta^{FB}) = 0$, so $0 = v(\theta^{FB}, e^{FB}, b, 0) + \pi(\theta^{FB}, e(b), b, 0) = \pi(\theta^{FB}, e(b), b, 0)$.

We can now look at the optimal solution

- Case 1: $R + b < 2\sqrt{2F\gamma}$:
  - Then $s_{1A} < s_{1B}$ for all θ, which implies that $p_e[b - s_1]e' < p$ for all $s_1 \geq s_{1B}$, for all θ. The optimal subsidy is then either $s_1^* = 0$ or $s_1^* = s_{1B}$.
  - The surplus of the regulator for $s_1 = s_{1B}(\theta)$ is $v = p(e(s_{1B}(\theta)), \theta)(b - s_{1B}(\theta))$, it is positive if and only if $b > s_{1B}(\theta)$, that is, $\theta > \theta^{FB}$.
  - For $\theta > \theta^{BAU}$: the project is implemented and no surplus is created by a marginal increase of effort ($p_e b - p < 0$) so $s_1^* = 0$.
- Case 2: $R + b \geq 2\sqrt{2F\gamma}$:
  - For $0 < \theta < \theta_A$, $s_{1A} > s_{1B}$ so that $\pi(\theta, e(s_{1A}), s_{1A}, 0) > 0$, and $p_e[b - s_1]e' - p > 0$ for both $s_1 = s_{1B}$ and $s_1 = 0$, so that $s_1^* = s_{1A}$.
  - For $\theta_A \leq \theta < \theta^{BAU}$: $s_{1A} < s_{1B}$ so that $\pi(\theta, e(s_{1A}), s_{1A}, 0) < 0$ and $s_{1B} > 0$ so that $s_1^* = s_{1B}$. This case might not arise if $\theta_A > \theta^{BAU}$ that is $s_{1A}(\theta^{BAU}) > 0$.
  - For $\theta \geq \max\{\theta_A, \theta^{BAU}\}$: the profit is positive for $s_1 = 0$ and $p_e(b - s_1)e' - p < 0$ at $s_1 = 0$ so that $s_1^* = 0$.

C Adverse Selection and Moral Hazard

To alleviate notation the probability $p(e, \theta_L)$ and $p(e, \theta_H)$ are denoted with subscripts: $p_L(e)$ and $p_H(e)$, and the profits π_L and π_H.

Let us denote (s_1^*, s_2^*) the optimal solution. There are four possible types of solution depending on whether each component is positive or null. Lemma (3) derive the expressions of the subsidies when they are positive, Proposition (6) consider the influence of λ on the solution.

C.1 Proof of Lemma 3

If both s_1^* and s_2^* are positive, then low type projects are implemented and their profits are null. The regulator surplus is then

$$v(s_1, s_2) = (1-\lambda)\Big[p_L(b-s_1)-(1-p_L)s_2\Big]+\lambda\Big[p_H(b-s_1)-(1-p_H)s_2\Big] \quad (40)$$

and the optimal scheme satisfies the following equation

$$\frac{\partial v}{\partial s_1}\frac{\partial \pi_L}{\partial s_2} - \frac{\partial v}{\partial s_2}\frac{\partial \pi_L}{\partial s_1} = 0$$

that is

$$\frac{\partial v}{\partial s_1}(1-p_L) - \frac{\partial v}{\partial s_2}p_L = 0$$

which gives, denoting $s^* = s_1^* - s_2^*$:

$$\begin{aligned}
\lambda[p_H(1-p_L)+(1-p_H)p_L] &= \left[(1-\lambda)\frac{\partial p}{\partial e}(e,\theta_L)+\lambda\frac{\partial p}{\partial e}(e,\theta_H)\right](b-s^*)e' \\
\lambda[p_H-p_L] &= \Big[(1-\lambda)(1-\theta_L)+\lambda(1-\theta_H)\Big](b-s^*)\frac{1}{\gamma} \\
\lambda(\theta_H-\theta_L)(\gamma-(R+s^*)) &= \Big[(1-\lambda)(1-\theta_L)+\lambda(1-\theta_H)\Big](b-s^*)
\end{aligned}$$

which then gives equation (29). The equation (30) corresponds to $\pi_L = 0$.

C.2 Proof of Proposition 6

The solution $s_1^* = s_2^* = 0$ corresponds to the situation in which L firms do not enter. The regulator surplus in that situation is:

$$V_1(\lambda) = \lambda p_H b$$

In all other situations, if one of the optimal subsidy is positive, L firms do enter (from Proposition 5, if only H firms enter then it is optimal to set $s_1 = s_2 = 0$). The regulator surplus when L firms enter is

$$V_2 = (1-\lambda)[p_L(b - s_1) - (1 - p_L)s_2] + \lambda[p_H(b - s_1) - (1 - p_H)s_2]$$

that can be equivalently defined as a function of $s = s_1 - s_2$ and s_2:

$$V_2(\lambda, s, s_2) = (1-\lambda)[p_L(b - s) - s_2] + \lambda[p_H(b - s) - s_2]$$

and the constraint $s_1 \geq 0$ is then $s + s_2 \geq 0$.

The problem of the regulator can be decomposed in two steps: first maximize V_2 and then compare the maximum obtained with V_1.

Let us consider the maximization of V_2 subject to $\pi_L \geq 0$, $s_2 \geq 0$ and $s + s_2 \geq 0$ and denote $s^{**}(\lambda)$ and $s_2^{**}(\lambda)$ the solution, and $s_1^{**} = s^{**} + s_2^{**}$. The problem can be simplified by transforming the three constraints $\pi_L \geq 0$, $s_2 \geq 0$ and $s + s_2 \geq 0$ into two constraints on s, by parameterizing everything by s.

• At the maximum $\pi_L = 0$: by contradiction, if $\pi_L > 0$ then $s_2^{**} = 0$ and s_1^{**} is larger than s_{1B} (which cancels π_L, it is defined by eq. 28) and solves

$$[(1-\lambda)\frac{\partial p_L}{\partial e} + \lambda\frac{\partial p_H}{\partial e}](b - s_1)e' = (1-\lambda)p_L + \lambda p_H$$

then $\partial p_L/\partial e(b - s_1^{**})e' > p_L$ that is $s_1^{**} < s_{1A}(\theta)$ (given by eq. 27) which is lower than $s_{1B}(\theta)$ when $(R + b) \geq 2\sqrt{2F\gamma}$ (proof of Proposition 5), a contradiction.

• We can then define $s_2(s)$:

$$s_2(s) = F - \max_e [p(e, \theta_L)(R + s) - f(e, \theta_L)]$$

it is decreasing with respect to s with $s_2'(s) = -p_L$. And $s_1(s) = s + s_2(s)$ is strictly increasing with respect to s ($s_1' = 1 - p$).

- For $s = -R$, $s_2(-R) = F$ and the associated s_1 is $F - R < 0$.
- At $s = s_{1B}$, the profit $\pi_L(e, s_{1B}, 0)$ is null so that $s_2(s_{1B}) = 0$, and $s > s_{1B} \Leftrightarrow s_2(s) < 0$. Note also that $s_{1B} < b$.
- At $s = 0$, $s_2(0)$ is positive equal to $-\pi_L(e, 0, 0)$.
- Define $\underline{s}$ the solution of $s + s_2(s) = 0$, it is between $-R$ and 0. The corresponding s_2 is such that $\pi_L(e, 0, s_2) = 0$.

The regulator's objective is then equivalent to the maximization of

$$\max_s V_2(\lambda, s, s_2(s)) \text{ s.t. } \underline{s} \leq s \leq s_{1B}$$

The derivative of the objective function with respect to s is:

$$\begin{aligned}
\mathcal{V}(\lambda, s) = &\left[(1-\lambda)\frac{\partial p_L}{\partial e} + \lambda\frac{\partial p_H}{\partial e}\right](b-s)\frac{1}{\gamma} - \lambda[p_H - p_L] \\
= &\left[(1-\lambda)(1-\theta_L) + \lambda(1-\theta_H)\right](b-s)\frac{1}{\gamma} - \lambda(\theta_H - \theta_L)(1 - \frac{R+s}{\gamma}) \\
= &\left[(1-\theta_L) - 2\lambda(\theta_H - \theta_L)\right](b-s)\frac{1}{\gamma} - \lambda(\theta_H - \theta_L)(1 - e^{FB}) \text{ using 8} \\
= &(\theta_H - \theta_L)\left[(\underline{\lambda} - \lambda)(b-s)/\gamma - \lambda(1 - e^{FB})\right]
\end{aligned}$$

in which

$$\underline{\lambda} = \frac{1-\theta_L}{2(\theta_H - \theta_L)}$$

This derivative is strictly decreasing with respect to s as long as $\lambda < \underline{\lambda}$. It is also decreasing with respect to λ for $s < s_{1B}$.

For all $s \in [\underline{s}, s_{1B}]$ we have $\mathcal{V}(0, s) = (1-\theta_L)(b-s)/\gamma > 0$ and $\mathcal{V}(\underline{\lambda}, s) < 0$.

So we already know that $s^{**}(0) = s_1^{**}(0) = s_{1B}$ and $s_2^{**}(0) = 0$, and that, $\forall \lambda > \underline{\lambda}$, $s^{**}(\lambda) = \underline{s}$: $s_1^{**}(\lambda) = 0$ and $s_2^{**}(\lambda) = s_2(\underline{s})$ the solution of

$$p_L(e)R + (1-p_L)s_2 = F + f_L(e)$$

And we can define :

- λ_1 the solution of $\mathcal{V}(\lambda, s_{1B}) = 0$
- λ_2 the solution of $\mathcal{V}(\lambda, \underline{s}) = 0$

Then the optimal solution as a function of λ is such that

- $0 \leq \lambda < \lambda_1$: $s^{**}(\lambda) = s_1^{**}(\lambda) = s_{1B}$ and $s_2^{**}(\lambda) = 0$
- $\lambda_1 \leq \lambda < \lambda_2$: $s^{**}(\lambda) \in (\underline{s}, s_{1B})$, $s_1^{**}(\lambda) > 0$ and $s_2^{**}(\lambda) > 0$
- $\lambda_2 \leq \lambda \leq 1$: $s^{**}(\lambda) = \underline{s}$, $s_1^{**}(\lambda) = 0$ and $s_2^{**}(\lambda) = s_2(\underline{s}) > 0$

Then, the regulator should compare V_2 and V_1, the difference $V_2 - V_1$ is decreasing with respect to λ and positive for $\lambda = 0$ and negative for $\lambda = 1$ (by Proposition 5). There is then a λ_3 so that $\lambda > \lambda_3$ implies $s_1^{**}(\lambda) = s_2^{**}(\lambda) = 0$.

C.3 The optimal menu for the illustrating example

We now show that even with no constraints and a menu of contracts the agency cannot implement the first best.

The optimal scheme with asymmetric information and unconstrained subsidy is a relatively standard mechanism design problem: the agency (the principal) should propose a menu of couples $\{(s_{1L}, s_{2L}), (s_{1H}, s_{2H})\}$ to the firm which self selects. The menu is designed so that a type $i = L, H$ chooses the item (s_{1i}, s_{2i}). Several cases can arise whether only a high type or both types deploy their project, and whether a low type exerts an effort.

For the sake of simplicity, and to focus on the role of asymmetric information, contrary to previous sections we assume that the agency can capture the rent from a high type.

Assumption 7 *A firm cannot deploy a project without the consent of the agency.*

Furthermore, we assume that a low type project is worth implementing, from a welfare perspective, even without effort. This assumption is satisfied in our numerical illustration. With this assumption it is always beneficial to encourage the low type firm to invest.

Assumption 8 *The low type is such that $\theta_L(R + b) - F > 0$.*

With these two additional assumptions only two situations can arise. In both situations a low type project is initiated with a sub-optimal effort, possibly null, and a high type exerts an optimal effort. In one case a high type gets a rent and the low type exerts an effort; in the other case, a high type gets no rent and a low type exerts no effort.

Proposition 7 *If the agency can propose a menu $\{(s_{1L}, s_{2L}), (s_{1H}, s_{2H})\} \in S^2$, under Assumptions 7 and 8, there is a threshold λ_{menu} that delineates two cases.*

In both cases, the high type exerts the first best effort: $s_{1H} - s_{2H} = b$, and the low type gets a null profit. Furthermore:

- *For $\lambda < \lambda_{menu}$:*
  - *The low type exerts a suboptimal effort with*

$$s_{1L} - s_{2L} = b - \frac{\lambda(\theta_H - \theta_L)}{(1-\lambda)(1-\theta_L) - \lambda(\theta_H - \theta_L)} [\gamma - R - b] \tag{41}$$

- *The high type gets a positive informational rent.*

- *For* $\lambda > \lambda_{menu}$*:*

  - *The low type exerts no effort:* $s_{1L} - s_{2L} = -R$

  - *Both the low and high type gets no profit:* $s_{2L} = F$ *and*

$$s_{2H} = F - \theta_H(R+b) - (1-\theta_H)\frac{(R+b)^2}{2\gamma} < 0$$

Proof.

The proof and the exposition are easier if we work with $s = s_1 - s_2$ and s_2. The regulator proposes a menu $\{(s_L, s_{2L}), (s_H, s_{2H})\}$, and if a firm of type i chooses the item (s_j, s_{2j}) with $i, j \in \{H, L\}$ it exerts the effort $e(s_j)$ and gets

$$\pi_{ij} = \theta_i(R+s_j) + (1-\theta_i)\frac{(R+s_j)^2}{2\gamma} - F + s_{2j}$$

and to further alleviate the exposition we denote $p_i = p(\theta_i, e(s_i))$ and $p_{HL} = p(\theta_H, e(s_L))$.

• At the optimal menu m^* both types initiate their project:

If the low type project is initiated so is the high type project.

The agency can propose a menu

$$\begin{aligned} m_0 = \{&(s_L, s_{2L}) = (-R, F), \\ &(s_H, s_{HL}) = (b, F - \theta_H(R+b) + (1-\theta_H)(R+b)^2/(2\gamma))\} \end{aligned}$$

with this menu the low type exerts no effort and gets zero profit, the agency can then extract the maximum surplus from a high type. The agency still gets a surplus from low types and cannot do better by discouraging low type and only subsidizing the deployment of high types.

• The optimal menu m^* solves:

$$\max_m \lambda[p_H(b - s_H) - s_{2H}] + (1-\lambda)[p_L(b - s_L) - s_{2L}] \tag{42}$$

subject to $\pi_{ii} \geq 0$ for $i = H, L$, and $\pi_{ii} \geq \pi_{ij}$ for $i, j \in \{H, L\}$.

Any menu with $s_L < -R$ (the effort of the low type being null) cannot generate more surplus than m_0, m^* is then either m_0 with $s_L^* = -R$ or such that $s_L^* > -R$.

Then, the only two binding constraints at the optimal menu are $\pi_{LL} \geq 0$ and $\pi_{HH} \geq \pi_{HL}$ because $R + s_L \geq 0$, so that $\pi_{HL} > \pi_{LL}$ and at the optimal scheme $s_H^* > s_L^*$ (to be checked at the end) so that $(\pi_{HH} - \pi_{HL}) - (\pi_{LH} - \pi_{LL}) = (\theta_H - \theta_L)(s_H^* - s_L^*)$.

Then, if $s_L^* > -R$ a low type firm exerts an effort, write the Lagrangien

$$\mathcal{L}(s_L, s_{2L}, s_H, s_{2H}) = \lambda[p_H(b - s_H) - s_{2H}] + (1-\lambda)[p_L(b - s_L) - s_{2L}] + \mu_L \pi_{LL} + \mu_H(\pi_{HH} - \pi_{HL}]$$

with μ_L and μ_H the Lagrange multipliers of the constraints $\pi_{LL} \geq 0$ and $\pi_{HH} \geq \pi_{HL}$ respectively. Then the optimal menu satisfies the KKT conditions:

$$\frac{\partial \mathcal{L}}{\partial s_H} = \lambda \frac{\partial p_H}{\partial e}(b - s_H)e' + (\mu_H - \lambda)p_H = 0 \tag{43}$$

$$\frac{\partial \mathcal{L}}{\partial s_{2H}} = \mu_H - \lambda = 0 \tag{44}$$

$$\frac{\partial \mathcal{L}}{\partial s_L} = (1-\lambda)\frac{\partial p_L}{\partial e}(b - s_L)e' + (\mu_L - (1-\lambda))p_L - \mu_H p_{HL} = 0 \tag{45}$$

$$\frac{\partial \mathcal{L}}{\partial s_{2L}} = -(1-\lambda) + \mu_L - \mu_H = 0 \tag{46}$$

Form eq. (43) and (44) $s_H^* = b$. From eq. (44) and (46) $\mu_L = 1$, and together with eq. (44) it gives

$$(1-\lambda)(1-\theta_L)(b - s_L)\frac{1}{\gamma} = \lambda(p_{HL} - p_L) = \lambda(\theta_H - \theta_L)(1 - \frac{R + s_L}{\gamma})$$

which, after some manipulations gives the expression (41) for s_L^* in Proposition 7. The optimal subsidy s_{2L}^* cancels a low type profit, and the subsidy s_{2H} is found with the constraint $\pi_{HH} = \pi_{HL}$.

If the above scheme can be implemented it outperformed m^0. The subsidy s_L^* is decreasing with λ, and λ_{menu} is the solution of $s_L^* = -R$ (with the expression 41). Note that the denominator in (41) is positive for $\lambda \in [0, \lambda_{menu}]$.
■