# Optimal Congestion Pricing with Diverging Long-Run and Short-Run Scheduling Preferences

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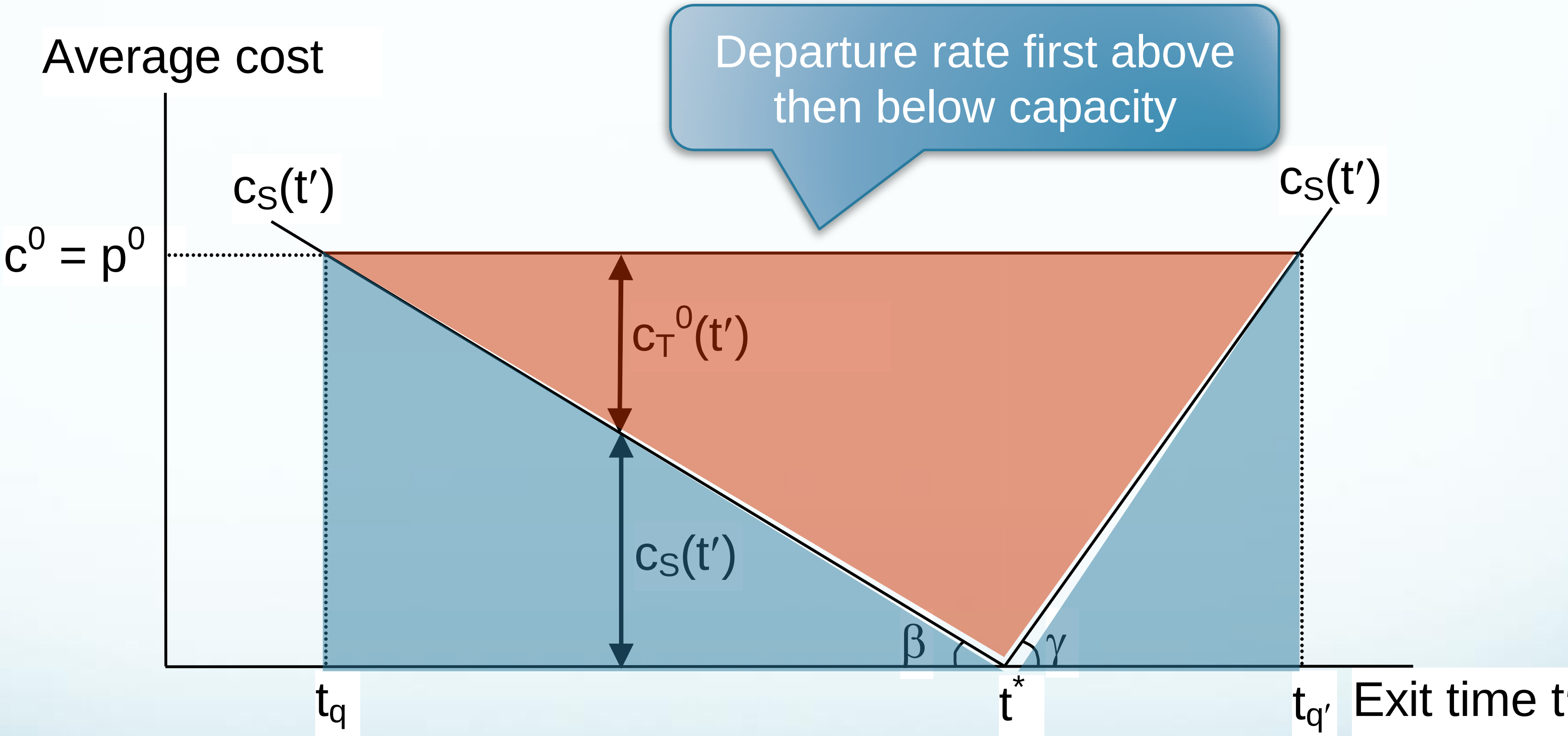
# Introduction

- Values of travel delays and schedule delays are central concepts in transport economics
- Recent evidence suggests that travellers decompose scheduling decisions into
  - long-run choices of routines
  - short-run choices of departure times
- This paper: implications for optimal congestion pricing
  - Do we need separate instruments to optimize both decisions?

# Deterministic starting point

Vickrey 1968; Small 1982



# Two dimensions of SR vs LR

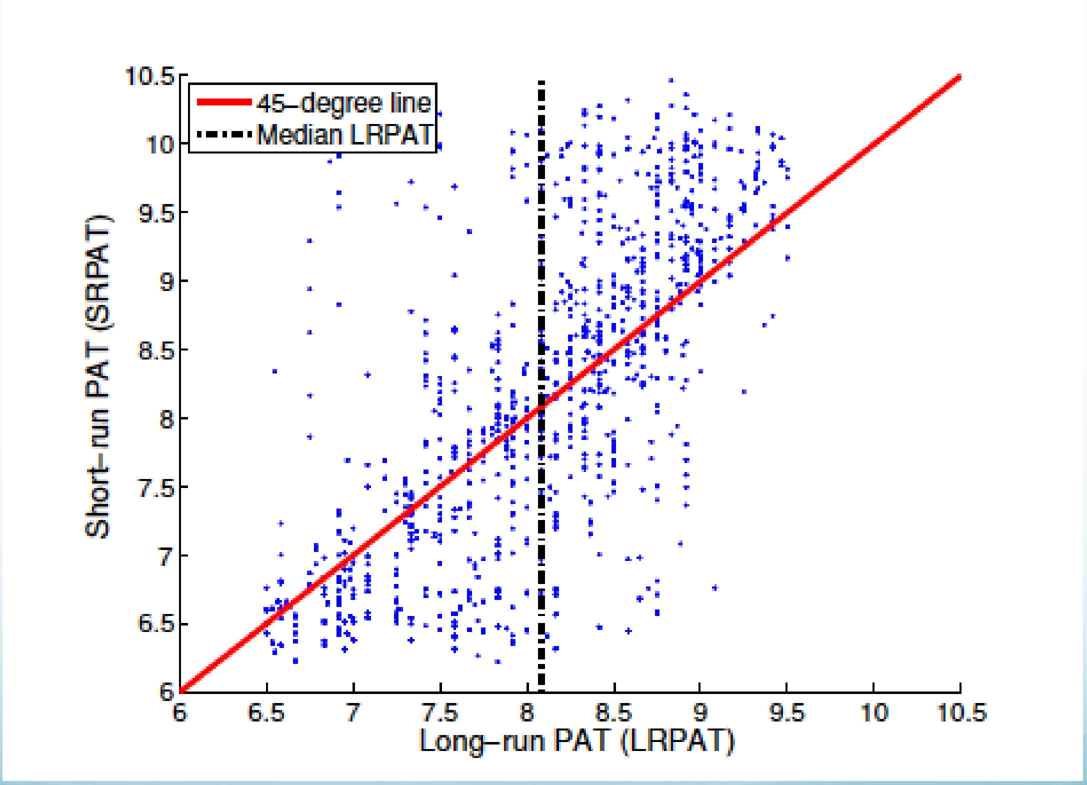
1. Different measures for preferred arrival time PAT
   - Long-run LRPAT ($t^*$): preferred arrival time if there were no congestion, ever
     - Interpretation in standard bottleneck model
   - Short-run SRPAT ($t^\#$): preferred arrival time given the expected pattern of travel times
     - Choice of 'routines' may make SRPAT deviate from LRPAT
     - With a LRPAT at 9:00, an SRPAT at 7:00, and a scheduled meeting at 7:30, an arrival time at 8:30 would bring cost of schedule delay late, not early
       - Evident: important to address in empirical modelling

# Two dimensions of SR vs LR

2. Different values of time and schedule delay, depending on ‘degree of permanentness’
   - A structural one-minute travel time gain brings more benefits *per day* than an incidental minute on a random day
   - An unanticipated schedule delay brings a greater disutility than schedule delays that are anticipated when forming routines

# Empirical confirmation

- Peer, Verhoef, Koster, Knockaert (2015)
  - Drivers plan their routines to avoid congestion



# Empirical confirmation

| Coefficient | Long-Run Value | Long-Run t-Statistic | Short-Run Value | Short-Run t-Statistic |
|---|---|---|---|---|
| $\beta_R$ | 0.22 | 4.87 | 0.13 | 5.78 |
| $\beta_T$ | -6.56 | -7.31 | -0.69 | -1.45 |
| $\beta_E$ | -2.03 | -13.28 | -2.89 | -18.38 |
| $\beta_L$ | -1.57 | -13.90 | -2.70 | -20.34 |
| $\theta$ | – | – | 0.43 | 6.25 |
| VOT (Euro/h) | 30.16 | | 5.20 | |
| VSDE (Euro/h) | 9.34 | | 21.62 | |
| VSDL (Euro/h) | 7.22 | | 20.22 | |
| Nr. Obs. | 1158 | | 5965 | |
| LogLik. | -2681 | | -10550 | |
| Pseudo $R^2$ | 0.17 | | 0.36 | |

# Implications for pricing?

- Implications for pricing
  - Is a separate regulation of choice of $t^{\#}$ desirable, above that of trip timing?
  - Peer and Verhoef 2012
    - Bottleneck model
    - Not conclusive on need for LR toll due to corner solutions

# Henderson-Chu model

- Alternative to Vickrey – ADL bottleneck
  - Demand-side and scheduling behaviour identical
    - "$\alpha$ $\beta$ $\gamma$" preferences
  - Congestion technology different
    - Vickrey: kinked performance function
    - Chu: smooth performance function
      - Delay is a function of outflow
      - E.g.: power function ("BPR")
      - Optimal toll: instantaneous application of Pigouvian toll
  - Both have closed-form solutions
    - Also for equilibrium (time-independent) cost (*c*) and price (*p*)

# Main ingredients

- $N$ identical travellers with "$\alpha$ $\beta$ $\gamma$" preferences
  - LR VoSD fraction $g$ of SR VoSD
  - LR VoT: relative premium of $a$ added to SR VoT
- SRPAT ($t^{\#}$) endogenous, LRPAT ($t^{*}$) identical and 0
- To avoid degenerate problem, we need variation between the days
  - Stochastic capacity $K$: $K_0 > K_1$
  - Probabilities: $(1-\pi)$ on state 0; $\pi$ on state 1
  - On the day itself, all travellers know the realization

# Main ingredients con’d

- BPR travel time function
  - Ignore free-flow travel time
  - Delay: $(r\ (t')/K)^{X}$
- Equilibria
  - Short-run: equilibrium distribution of arrival times $r(t')$
    - … given the distribution of SRPATs $z(t^{\#})$ and given the realization of $K$
  - Long-run: equilibrium distribution of SRPATs $z(t^{\#})$
    - … given that short-run equilibria as above will apply

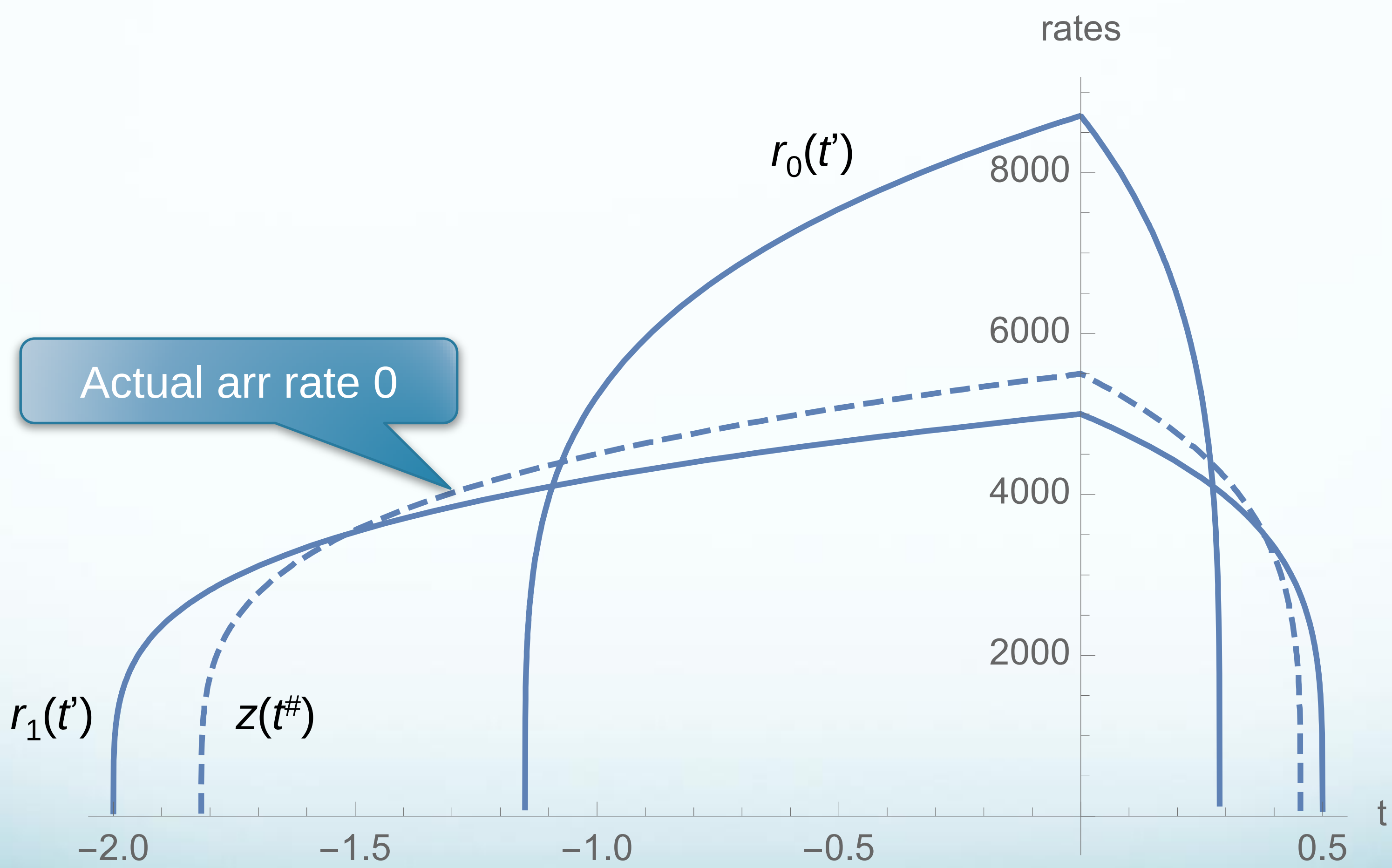
# LR equilibrium

- Three candidate types of LR equilibria
  - "Always Dispersed" (AD): values of $t^{\#}$ are chosen so dispersed that all drivers arrive at $t^{\#}$ in both states
  - "Sometimes Dispersed" (SD): density of $z(t^{\#})$ is so high that only state 0 is dispersed
    - State 1 is "condensed": early drivers arrive before their $t^{\#}$ and late drivers after their $t^{\#}$
  - "Never Dispersed" (ND): both states "condensed"
    - ND is no equilibrium: it always pays off to widen $z(t^{\#})$ to save SR SDC and accept increased LR SDC ($g$<1)

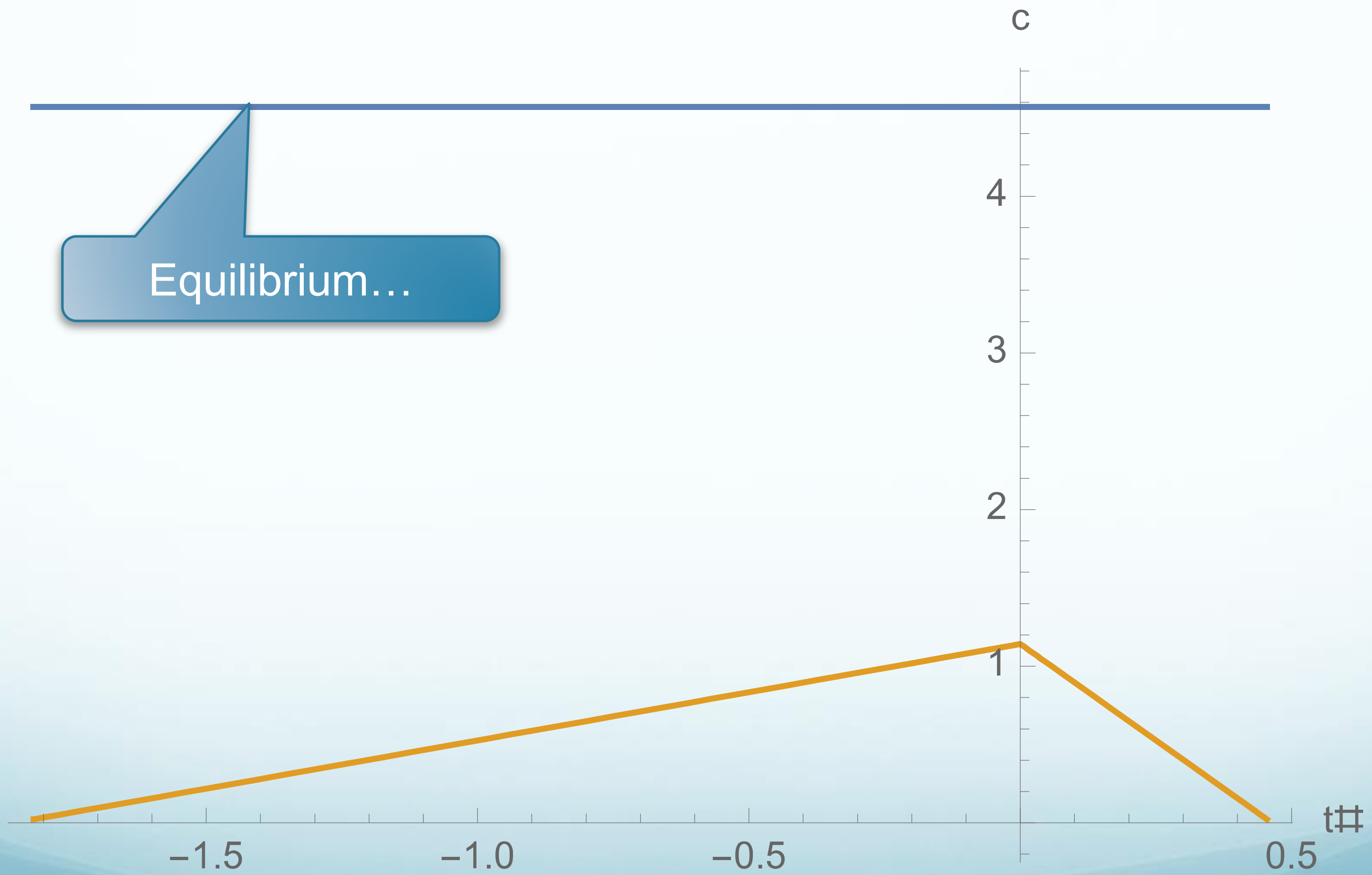
# Solution

- Important elements are the “reference arrival rate distributions” $r_0(t')$ and $r_1(t')$
  - These would apply in the basic Chu model with deterministic capacity $K_0$ or $K_1$, and with identical $t^*$
- Actual arrival pattern:
  - Condensed peak: reference arrival pattern
  - Dispersed peak: $r(t')$ equals $z(t^{\#})$
    - Everybody arrives on time (at SRPAT)

# Example: SD-NTE ($\pi$ = 0.025)



# Expected SR cost and LR cost



# Are long-run tolls needed? AD

$$c^{LR}(t^{\#}) = \alpha \cdot (1+a) \cdot \left( (1-\pi) \cdot \left( \frac{z(t^{\#})}{K_0} \right)^{\chi} + \pi \cdot \left( \frac{z(t^{\#})}{K_1} \right)^{\chi} \right) + \begin{cases} -\beta \cdot g \cdot t^{\#} \\ \gamma \cdot g \cdot t^{\#} \end{cases}$$

$$p^{LR}(t^{\#}) = c^{LR}(t^{\#}) + (1-\pi) \cdot \tau_0^{SR}(t^{\#}) + \pi \cdot \tau_1^{SR}(t^{\#}) + \tau^{LR}(t^{\#})$$

$$mc^{LR}(t^{\#}) = c^{LR}(t^{\#}) + z(t^{\#}) \cdot (1+a) \cdot \alpha \cdot \left( (1-\pi) \cdot \frac{\partial T_0(z(t^{\#}))}{\partial z(t^{\#})} + \pi \cdot \frac{\partial T_1(z(t^{\#}))}{\partial z(t^{\#})} \right)$$

$$\tau_0^{SR}(t^{\#}) = z(t^{\#}) \cdot \alpha \cdot \frac{\partial T_0(z(t^{\#}))}{\partial z(t^{\#})} \qquad \tau_1^{SR}(t^{\#}) = z(t^{\#}) \cdot \alpha \cdot \frac{\partial T_1(z(t^{\#}))}{\partial z(t^{\#})}$$

$$\tau^{LR}(t^{\#}) = z(t^{\#}) \cdot a \cdot \alpha \cdot \left( (1-\pi) \cdot \frac{\partial T_0(z(t^{\#}))}{\partial z(t^{\#})} + \pi \cdot \frac{\partial T_1(z(t^{\#}))}{\partial z(t^{\#})} \right)$$

# Intuition

- To establish short-run optimum in both states, short-run tolls must be based on short-run “$\alpha$ $\beta$ $\gamma$”
  - Through Pigouvian form, $\alpha$ in particular
- But long-run expected travel times are proportional (probability-weighted) with short-run travel times
  - Same internalization argument applies
  - Must be a long-run toll in order not to distort short-run optima
- Value of long-run toll is simply *a* times the expected value of short-run tolls

# Long-run tolls are less strongly needed in SD

$$c^{LR}(t^{\#}) = (1-\pi)\cdot\alpha\cdot(1+a)\cdot\left(\frac{z(t^{\#})}{K_0}\right)^{\chi} + \pi\cdot\left(c_1^{SR} + a\cdot\alpha\cdot\left(\frac{r_1(t'(t^{\#}))}{K_0}\right)^{\chi}\right) + \begin{cases} \beta\cdot(g-\pi)\cdot -t^{\#} \\ \gamma\cdot(g-\pi)\cdot t^{\#} \end{cases}$$

$$p^{LR}(t^{\#}) = \tau^{LR} + (1-\pi)\cdot\left(\tau_0^{SR} + \alpha\cdot(1+a)\cdot\left(\frac{z(t^{\#})}{K_0}\right)^{\chi}\right) + \pi\cdot\left(p_1^{SR} + a\cdot\alpha\cdot\left(\frac{r_1(t'(t^{\#}))}{K_0}\right)^{\chi}\right) + \begin{cases} \beta\cdot(g-\pi)\cdot -t^{\#} \\ \gamma\cdot(g-\pi)\cdot t^{\#} \end{cases}$$

$$p^{LR}(t^{\#}) - mc^{LR}(t^{\#}) = \tau^{LR} + (1-\pi)\cdot\left(\tau_0^{SR} - (1+a)\cdot\alpha\cdot\frac{\partial T_0(z(t^{\#}))}{\partial z(t^{\#})}\right) + \pi\cdot\left(p_1^{SR} - mc_1^{SR} + a\cdot\alpha\cdot T_1(t'(t^{\#}))\right)$$

$$\tau_0^{SR}(t^{\#}) = z(t^{\#})\cdot\alpha\cdot\frac{\partial T_0(z(t^{\#}))}{\partial z(t^{\#})} \qquad \tau_1^{SR}(t') = r_1(t')\cdot\alpha\cdot\frac{\partial T_1(r_1(t'))}{\partial r_1(t')}$$

$$\tau^{LR}(t^{\#}) = (1-\pi)\cdot z(t^{\#})\cdot a\cdot\alpha\cdot\frac{\partial T_0(z(t^{\#}))}{\partial z(t^{\#})} - \pi\cdot a\cdot\alpha\cdot T_1(t'(t^{\#}))$$

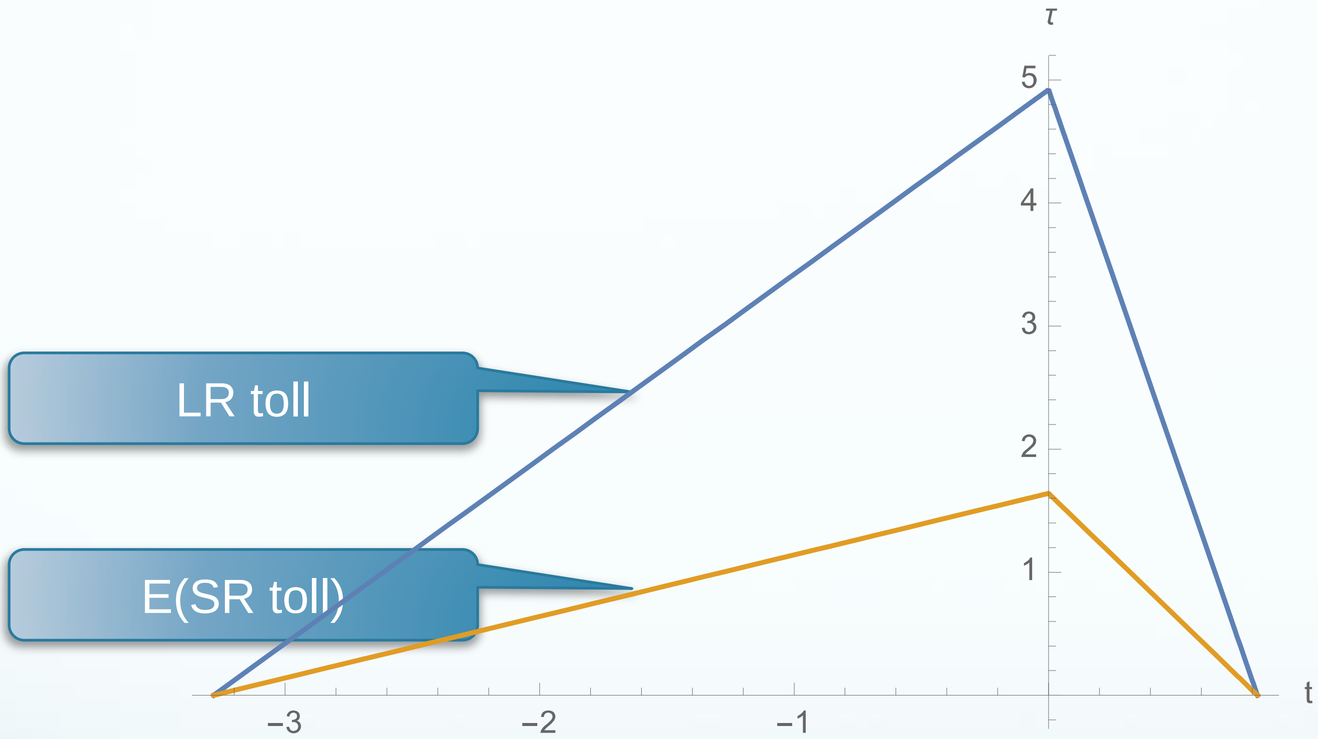
# Intuition

- For state 0, things work as in previous (AD) case
  - LR toll contains a factor $(1-\pi)\times a$ times SR toll in state 0
- But for state 1, the congestion externality is dropped
  - Marginal changes in $z(t^{\#})$ will not change traffic conditions in state 1: it is a condensed equilibrium
  - So no externality of that type enters the LR toll rule
- Instead, what is subtracted from the LR toll rule is the factor $\pi\times a\times$(travel delay in state 1)
  - It is part of the generalized price, but not of the marginal cost for $z(t^{\#})$
  - A marginal change in $z(t^{\#})$ does not change these costs

# Numerical illustration

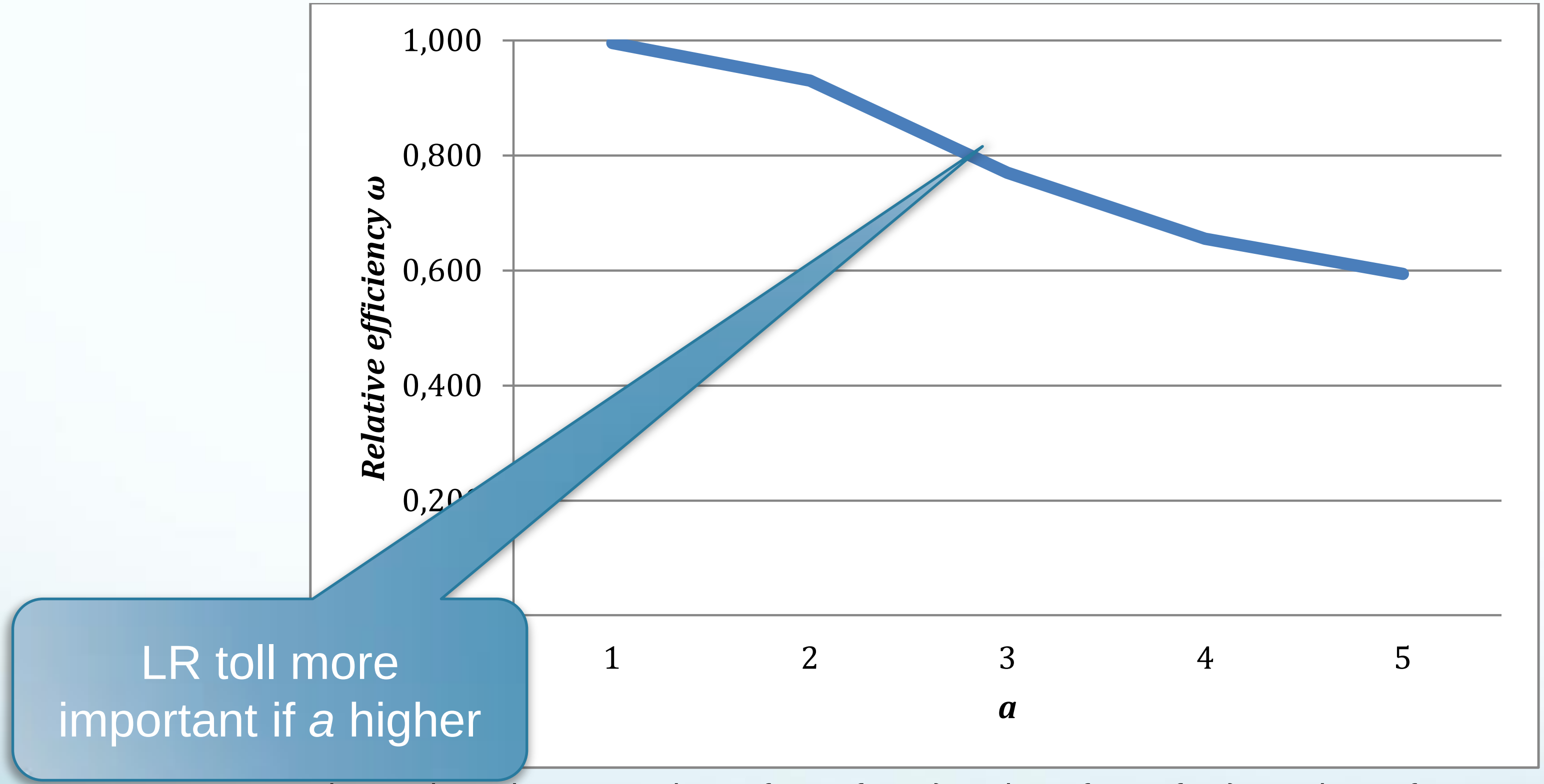
- Parameters:
  - $N = 10\ 000$
  - $K_0=10\ 000$; $K_1=5\ 000$
  - $\pi=0.25$
  - $\chi=4$
  - $\alpha=10$
  - $\beta=5$
  - $\gamma=20$
  - $\delta=4$
  - $a=3$
  - $g=0.5$

# First-best (AD)




- Still, modest cost reduction compared to QFB
  - QFB realizes 77% of FB cost reduction (SD)
  - Absence of LR toll makes SR tolls higher; E peaks near 4

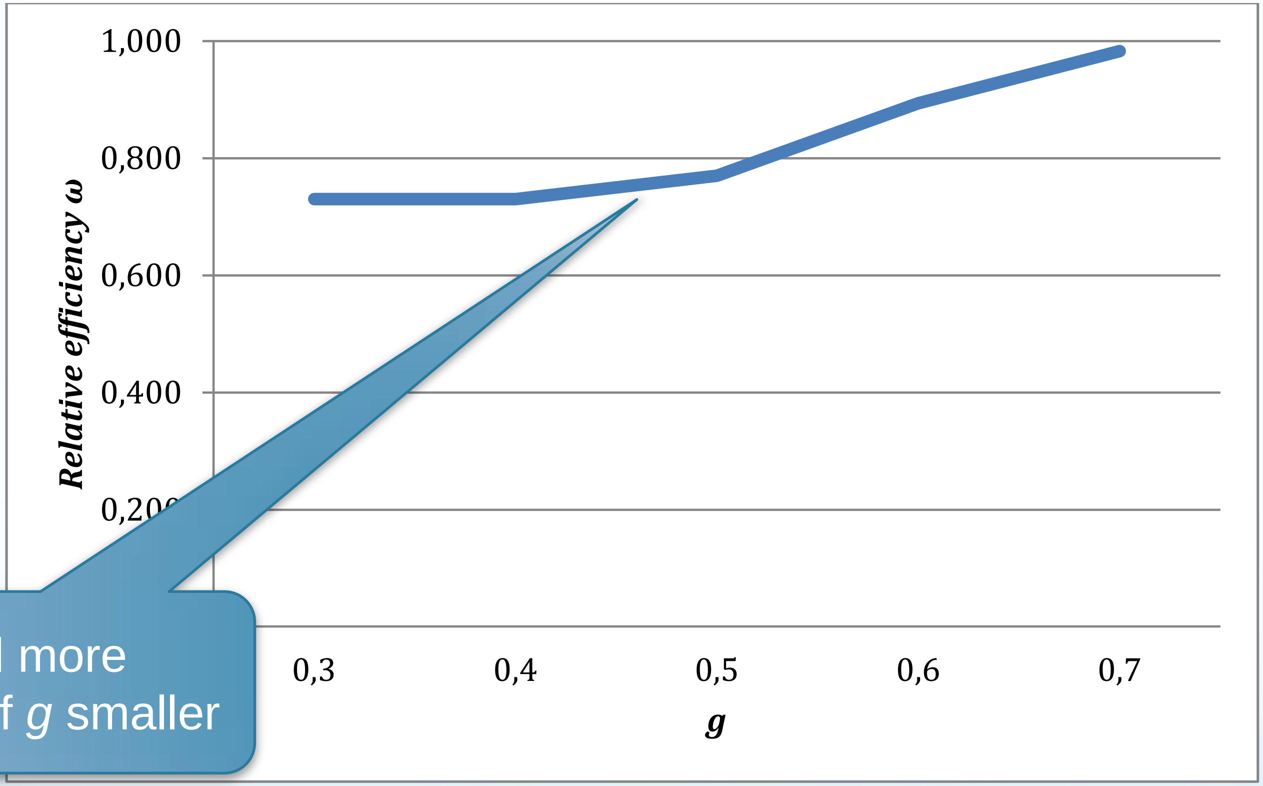
# Relative efficiency QFB: *a*




*Note: NTE is AD throughout; QFB is SD for a={1,2,3} and AD for a={4,5}; FB is SD for a=1 and AD for a={2,3,4,5}*

*Figure 3. Varying a: relative efficiency ω*

# Relative efficiency QFB: *g*

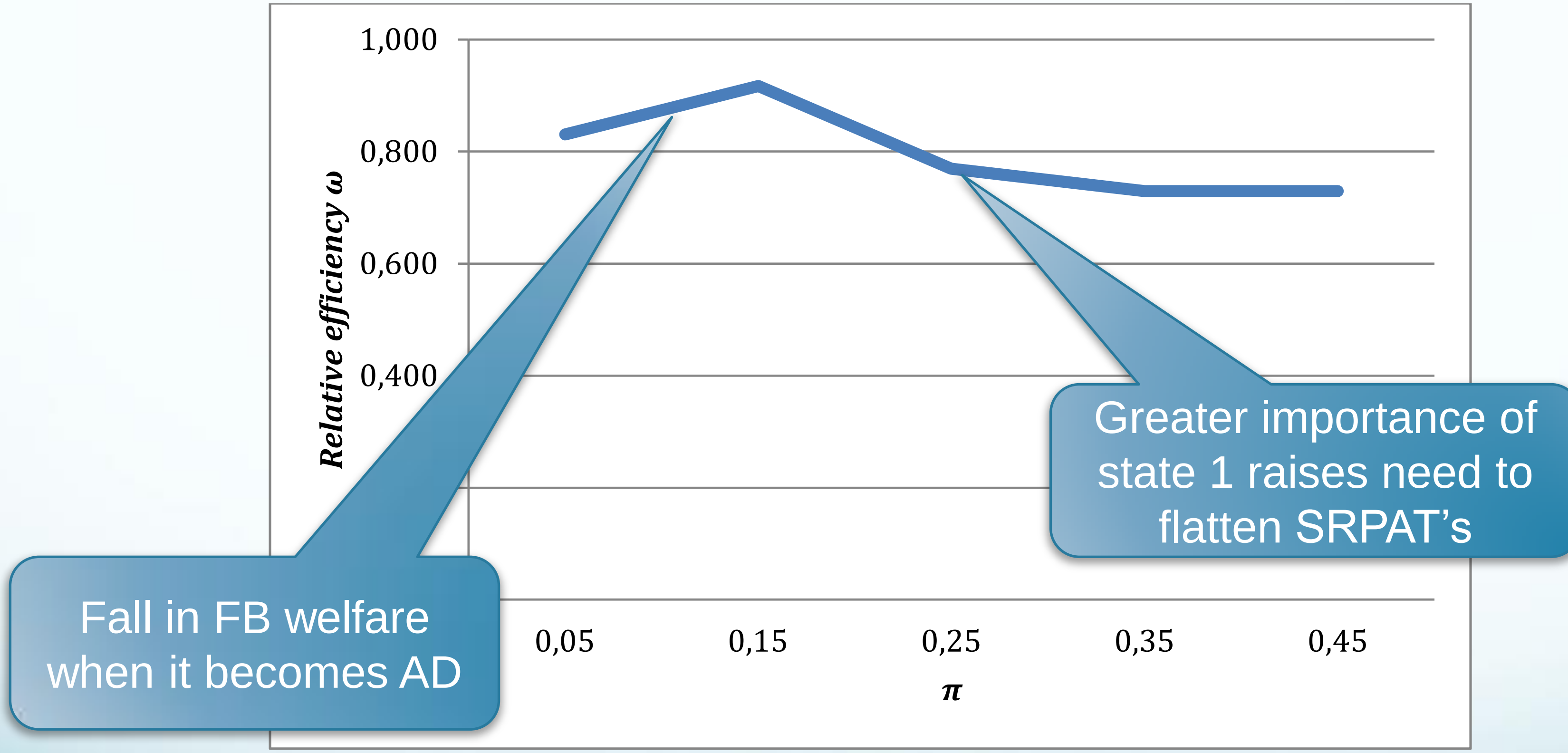



LR toll more important if *g* smaller

*Note: NTE is AD throughout; QFB is AD for g={0.3,0.4} and SD for g={0.5,0.6,0.7}; FB is AD for g={0.3,0.4, 0.5,0.6} and SD for a=0.7*

*Figure 4. Varying g: relative efficiency ω*

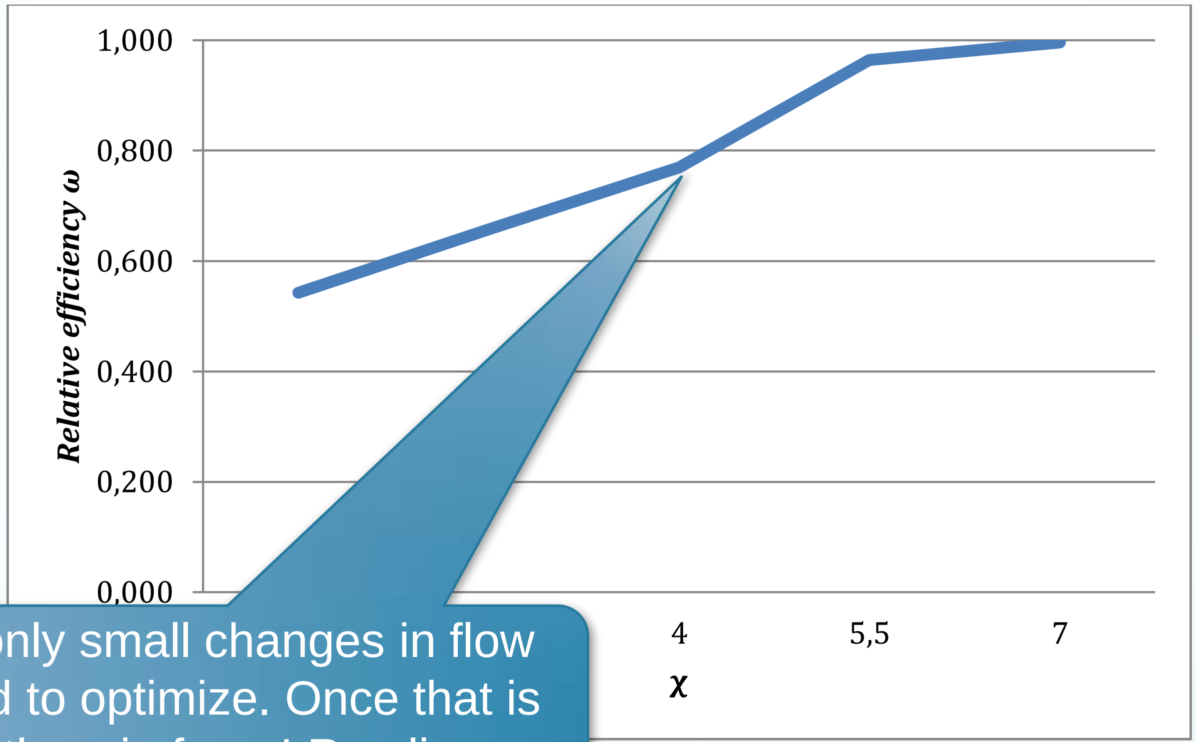
# Relative efficiency QFB: $\pi$




*Note: NTE is SD for $\pi$=0.05 and AD for $\pi$={0.15,0.25,0.3, 0.45}; QFB is SD for $\pi$={0.05,0.15,0.25} and AD for $\pi$={0.35, 0.45}; FB is SD for $\pi$=0.05 and AD for $\pi$={0.15,0.25,0.3, 0.45}*

*Figure 5. Varying $\pi$: relative efficiency $\omega$*

# Relative efficiency QFB: χ




High chi: only small changes in flow are needed to optimize. Once that is done, little gain from LR policy

*Note: NTE is AD throughout; QFB is AD for χ={1,2.5} and SD for χ={4,5.5,7}; FB is AD for χ={1,2.5,4,5.5} and SD for χ=7*

*Figure 6. Varying χ: relative efficiency ω*

# Conclusion

- Long-run toll is needed when short-run and long-run valuations of time diverge
  - Surprisingly, the need is larger for instances if the first-best is more lightly congested
  - Reason: in a condensed equilibrium, arrival pattern becomes insensitive to marginal changes in desired arrival times
- QFB has relative efficiency that may falls as low as 0.6 in the numerical example used

# Solution

- Solution proceeds technically in the same way for the three pricing regimes
  - NT = No Toll
  - QFB = Quasi First-Best: short-run Chu-tolls only
  - FB = First-Best: QFB *plus* possibly a long-run toll to optimize the choice of $t^{\#}$
- Solution differs between AD and SD
- Main steps:
  - Solve the partial differential equation for $z(t^{\#})$ that makes the long-run generalized price constant over time, given the short-run equilibria (and toll rules)
  - Solve for $t_l$ and $t_{l'}$ that guarantee $N$ drivers and equalized generalized prices at those two moments

# Solution Chu model

Generalized price always takes the form (X and d depend on AD vs SD and on pricing regime):

$$p^{LR}(t^{\#}) = X \cdot z(t^{\#})^{\chi} + Cons + \begin{cases} -d \cdot \beta \cdot t^{\#} \\ d \cdot \gamma \cdot t^{\#} \end{cases}$$

$$\dot{p}^{LR}(t^{\#}) = X \cdot \chi \cdot z(t^{\#})^{\chi-1} \cdot \dot{z}(t^{\#}) + \begin{cases} -d \cdot \beta \\ d \cdot \gamma \end{cases} = 0$$

$$z(t^{\#}) = \left( \frac{1}{X} \cdot \left( \begin{cases} d \cdot \beta \cdot \left(t^{\#} - t_l\right) \\ d \cdot \gamma \cdot \left(t_{l'} - t^{\#}\right) \end{cases} \right) \right)^{\frac{1}{\chi}}$$

$$t_l = -\left( N \cdot \frac{\gamma}{\beta+\gamma} \cdot \frac{1+\chi}{\chi} \cdot \left( \frac{X(\phi)}{d \cdot \beta} \right)^{\frac{1}{\chi}} \right)^{\frac{\chi}{1+\chi}} ; \quad t_{l'} = \left( N \cdot \frac{\beta}{\beta+\gamma} \cdot \frac{1+\chi}{\chi} \cdot \left( \frac{X(\phi)}{d \cdot \gamma} \right)^{\frac{1}{\chi}} \right)^{\frac{\chi}{1+\chi}}$$

| | No-toll equilibrium | First-best optimum |
|---|---|---|
| Short-hand $\Psi_i$ | $\Psi_i = \left(\frac{N}{K_i}\cdot\frac{1+\chi}{\chi}\cdot\frac{\delta}{\alpha}\right)^{\frac{\chi}{1+\chi}}$ | |
| Arrival rate $r(t')$ early ($t'\leq t^*$) | $r(t') = K_i\cdot\left(\frac{\beta}{\alpha}\cdot(t'-t_q)\right)^{\frac{1}{\chi}}$ | $r(t') = K_i\cdot\left(\frac{1}{1+\chi}\cdot\frac{\beta}{\alpha}\cdot(t'-t_q)\right)^{\frac{1}{\chi}}$ |
| Arrival rate $r(t')$ late ($t'>t^*$) | $r(t') = K_i\cdot\left(\frac{\gamma}{\alpha}\cdot(t_{q'}-t')\right)^{\frac{1}{\chi}}$ | $r(t') = K_i\cdot\left(\frac{1}{1+\chi}\cdot\frac{\gamma}{\alpha}\cdot(t_{q'}-t')\right)^{\frac{1}{\chi}}$ |
| Early interval: $t^*-t_q$ | $t^*-t_q = \Psi_i\cdot\frac{\alpha}{\beta}$ | $t^*-t_q = (1+\chi)^{\frac{1}{1+\chi}}\cdot\Psi_i\cdot\frac{\alpha}{\beta}$ |
| Late interval: $t_{q'}-t^*$ | $t_{q'}-t^* = \Psi_i\cdot\frac{\alpha}{\gamma}$ | $t_{q'}-t^* = (1+\chi)^{\frac{1}{1+\chi}}\cdot\Psi_i\cdot\frac{\alpha}{\gamma}$ |
| Generalized price $p$ | $p = \Psi_i\cdot\alpha$ | $p = (1+\chi)^{\frac{1}{1+\chi}}\cdot\Psi_i\cdot\alpha$ |
| Average generalized cost $\overline{c}$ | $\overline{c} = \Psi_i\cdot\alpha$ | $\overline{c} = \frac{(1+\chi)^{\frac{2+\chi}{1+\chi}}}{1+2\cdot\chi}\cdot\Psi_i\cdot\alpha$ |
| Total travel delay cost *TDC* | $TDC = \frac{1+\chi}{1+2\cdot\chi}\cdot\Psi_i\cdot\alpha\cdot N$ | $TDC = \frac{(1+\chi)^{\frac{1}{1+\chi}}}{1+2\cdot\chi}\cdot\Psi_i\cdot\alpha\cdot N$ |
| Total schedule delay cost *SDC* | $SDC = \frac{\chi}{1+2\cdot\chi}\cdot\Psi_i\cdot\alpha\cdot N$ | $SDC = \frac{\chi\cdot(1+\chi)^{\frac{1}{1+\chi}}}{1+2\cdot\chi}\cdot\Psi_i\cdot\alpha\cdot N$ |
| Total toll revenue *TR* | $TR = 0$ | $TR = \frac{\chi\cdot(1+\chi)^{\frac{1}{1+\chi}}}{1+2\cdot\chi}\cdot\Psi_i\cdot\alpha\cdot N$ |
| Total social cost *C* | $C = \Psi_i\cdot\alpha\cdot N$ | $C = \frac{(1+\chi)^{\frac{2+\chi}{1+\chi}}}{1+2\cdot\chi}\cdot\Psi_i\cdot\alpha\cdot N$ |
| Toll $\tau(t')$ | $\tau(t') = 0$ | $\begin{aligned}\tau(t') &= \alpha\cdot\chi\cdot\left(\frac{a(t')}{K_i}\right)^{\chi} \\ &= \alpha\cdot\chi\cdot(T-T_f) \\ &= \frac{\chi}{1+\chi}\cdot\begin{cases}\beta\cdot(t'-t_q) & \text{if } t'\leq t^* \\ \gamma\cdot(t_{q'}-t') & \text{if } t'>t^*\end{cases}\end{aligned}$ |

*Note: costs and prices are net of free-flow travel times $T_f$. Inclusion would require adding $\alpha\cdot T_f$ for average cost and generalized price measures, and $N\cdot\alpha\cdot T_f$ for inclusion in total costs measures.*

*Table 1. Equilibrium and first-best optimum*