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Land-sparing vs Land-sharing with incomplete policies

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Land-sparing vs Land-sharing with incomplete policies[†]

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Abstract

This article analyzes the trade-off between yield and farmed area when a valuable species is affected by agricultural practices. It revisits, from an economic perspective, the "land-sparing versus land-sharing" debate. We show that the optimal yield is either increasing or decreasing with respect to the value of the species. Land-sparing and land-sharing are not necessarily antagonistic; for sufficiently elastic demand function, both the optimal yield and the farmed area decrease with the public value of the species. A general assessment of a second-best policy is performed, and several particular policies are considered.

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1 Introduction

Agriculture is arguably the human activity with the largest impact on the environment (Green et al., 2005; Pereira et al., 2012; Tilman et al., 2017). The environmental footprint of agriculture comes both from farming practices and from the conversion of land to agriculture. Approximately one third of the Earth's ice-free land surface is used for agricultural production (Ramankutty et al., 2008).[1] There is a trade-off at the heart of lively debates between farming intensity (and associated environmental degradation) and the total land farmed. Since a seminal article by Green et al. (2005) this debate has been framed among conservation scientists as a choice between two extreme strategies: land-sparing and land-sharing. In a land-sparing strategy, farming is concentrated on the smallest possible area, with the rest being spared for nature; In a land-sharing strategy, "wildlife friendly" agriculture, with lower yield and better in-farm environmental quality, is performed over a larger area.

Green et al. (2005) analyze the trade-off between yield and farmed area from a biological point of view. They introduce the density–yield curve: the relationship between species density or abundance (number of specimens per hectare) and agricultural yield (production per hectare), and they maximize a species' total abundance subject to a food production constraint. If the density–yield curve is everywhere convex or concave, they find that the optimal strategy for species conservation is one of the extremes, either land-sparing or land-sharing. Land-sparing is optimal for a convex density–yield curve, and land-sharing is optimal for a concave one. Intuitively, with a convex density–yield curve, the species abundance drops sharply with the production of the first units, and a piece of land should be exploited intensively once it is converted to agriculture. Green et al. (2005) reignited a long-running controversy, fueled by numerous articles, in the ecology literature (see Fischer et al., 2014, for an attempt to "move forward").

The objective of the present article is to analyze the trade-off between yield and farmed area and species conservation in an economic framework encompassing consumer surplus and production costs. The analysis is theoretical, it does not provide definite answers but identifies the economic mechanisms at stake. The model can be used to address the questions of what might be the impact of the value of the environment on the optimal policy, and which (second-best) policies could be welfare enhancing.

The model used is a partial equilibrium model of the market for an agricultural good produced in the habitat of a valuable species. The total size of the habitat is split between farmed and unfarmed land. The density of the species per hectare is a function of the yield. The optimal welfare-maximizing yield and production are described. The optimal yield is either increasing or decreasing with respect to the value of the species, depending on the shape of the density yield curve. It is increasing (decreasing) if the density–yield curve is convex (concave), a situation associated with land-sparing (land-sharing) in Green et al. (2005). In both cases (convex and concave density–yield curve), the benefits from the species conservation are traded-off by a higher production cost, which is overlooked in the conservation literature.

[1] Agricultural land is the sum of arable land (12%) and pastures (22%). A pedagogical presentation of figures can be found at: `https://ourworldindata.org/yields-and-land-use-in-agriculture/#agriculture-land-use-over-the-long-run`.

The introduction of the demand for food allows us to consider its adjustment. Indeed, food consumption decreases when the value of the species is internalized, irrespective of the shape of the density–yield curve. The demand price elasticity determines whether the total farmed area actually increases when the yield decreases. If demand for food is elastic, both lower yield and larger unfarmed area are optimal when the density–yield curve is concave. This result shows that land-sparing and land-sharing are not mutually exclusive strategies. Furthermore, the reduction of food consumption might be the main channel through which the environmental cost from farming should be reduced.

This article then considers the policy consequences in a second-best setting. Indeed, the first-best optimal allocation can be implemented with a policy that rewards land-owners for the value of environmental goods both on farmed and unfarmed lands. However, such a policy might not be available. For instance, in the EU, under the Common Agricultural Policy, most agri-environmental schemes are "management-based" and reward the adoption of an agricultural practice (Hasund and Johansson, 2016). Their environmental performance, even on farms, has been criticized (e.g. Kleijn et al., 2006). However, even a "result-based" agri-environmental scheme (Wittig et al., 2006; Matzdorf and Lorenz, 2010; Kaiser et al., 2010, e.g.) that rewards environmental quality only on farms still constitutes a subsidy to farming even though not a direct subsidy to production.

The analysis of the second-best policy highlights that the question of whether a particular agricultural practice should be promoted depends on the policy instrument available. For instance, even if the agricultural yield should be increased in the first-best solution it should not necessarily be subsidized in a second best setting in which natural reserves cannot be enforced.

Whether a policy is welfare-enhancing will depend on the shape of the density–yield curve and the elasticity of the demand function.[2] For instance, even if land-sharing (reduced yield and increased area) is optimal in a first-best setting, it may be welfare enhancing to implement natural reserves if the demand is elastic. In such a case, the quantity of food consumed should decrease sufficiently to ensure that the environmental benefit on unfarmed land compensates for the loss on farmed land.

Land-sparing has been criticized for the potential difficulty of implementing it. Once land has been converted to intensive farming, it may be difficult to enforce the actual sparing of the remaining land (Godfray, 2011; Ewers et al., 2009). The present analysis of second-best policies partly addresses this concern. If land-sparing cannot be enforced, subsidizing intensive farming induces an over-expansion of farming (compared to the first-best policy), which can compensate for the benefits from an increased yield. This negative result arises if the demand for food is sufficiently price elastic. If the demand for food is inelastic then subsidizing intensive farming enhances welfare.[3]

Green et al. (2005) are not the first to argue that intensive agriculture can be good for the environment despite its local environmental cost by sparing land for nature (e.g. Waggoner, 1996; Borlaug, 2002). The land-sparing vs land-sharing debate on the potential

[2] The role of the elasticity of demand has been mentioned in several articles (e.g. Green et al., 2005; Phalan, Balmford, Green and Scharlemann, 2011; Lambin and Meyfroidt, 2011), but not considered analytically in an integrated framework.

[3] See Muhammad et al. (2011) for estimates of the demand for food in different countries. The demand for food is lower in wealthier countries, mainly because of the revenue effect (Slutsky elasticity is U-shaped).

benefits of intensive farming echoes the debate on the environmental consequences of the green revolution and the associated intensification of agriculture in some developing countries (see Paarlberg, 2013, Chapter 6, for a brief exposition). Some authors have studied whether an increase of the yield (as an exogenous shock) actually spares land for nature (e.g., Rudel et al., 2009; Ewers et al., 2009). The causes of deforestation have also been analyzed both theoretically and empirically.[4] Whether an increase in agricultural productivity leads to a reduction in farmed area depends notably upon the price elasticity of demand for food. This empirical question is different from the optimal way to internalize environmental value. The optimal policy should be concerned with both agricultural practices and farmed area and not focus exclusively on the former.

The density–yield curve is a key methodological innovation of Green et al. (2005), but few estimates exist. Phalan, Balmford, Green and Scharlemann (2011) construct density–yield curves for bird and tree species in southwest Ghana and northern India.[5] They conclude that land-sparing is the optimal strategy (see also Balmford et al., 2005; Kleijn et al., 2009; Teillard et al., 2015). In the environmental economics literature, Ruijs et al. (2017) propose a semi-parametric technique to estimate production possibility frontiers between agricultural production, biodiversity and carbon sequestration, possibly non-concave. They apply their framework to Central and Eastern Europe, and establish that production functions are non-concave, which implies benefits from specialization, as in a land-sparing strategy.

On the theoretical side, several articles in the economic literature consider the relationship between land use and the environment.

Hart et al. (2014) reframe the Green et al. (2005)'s analysis as a cost minimization problem. They minimize the cost to farmers to reach a target for wild nature.[6] They show that if the cost function is convex or concave everywhere, the optimal solution is either land sharing or land sparing, a result similar to that of Green et al. (2005). For more general cost functions, they establish that intermediary efforts could be optimal for a subset of farmers.[7] They apply their framework to analysis of bird protection in mown grasslands in Sweden. In the present article, the food demand is explicitly modeled, the optimal yield is also intermediary because of the fixed cost associated with farming and not with nature protection, and the cost of further increasing yield beyond the laissez-faire situation is considered.

Desquilbet et al. (2017) stress the role of agricultural markets but consider only two farm-

[4]See (Angelsen and Kaimowitz, 1999) for a review, and Leblois et al. (2016) for a recent empirical analysis that stresses the role of international a trade.

[5]The relationship between agricultural practices and some species densities has been investigated (Fuller et al., 2005; Chamberlain et al., 2010; Firbank et al., 2008). In particular, the comparison of organic and conventional agriculture has received considerable attention. Many studies conclude that organic farming enhances biodiversity on farms, even though some species might be adversely affected (see the meta-analysis by Bengtsson et al., 2005). Most studies also conclude that the yield of organic farming is lower than the yield of conventional farming, but the results are highly variable (de Ponti et al., 2012; Seufert et al., 2012).

[6]Hart et al. (2014) analyze the dual problem of the problem considered by Green et al. (2005). The density–yield curve of Green et al. (2005) corresponds to the cost function of Hart et al. (2014) which could be interpreted as a profit loss on an agricultural market in which the price is implicitly assumed fixed.

[7]The case of more general density–yield curves is also briefly considered in the supplementary material of Green et al. (2005), and the graphical discussion therein parallel the analysis of Hart et al. (2014) but Green et al. (2005) do not consider the possibility of split solutions.

ing systems (intensive and extensive farming). They compare the environmental and market outcomes of the two systems, and they stress the negative environmental consequences associated with the rebound effect. The equilibrium production is lower with extensive farming than with intensive farming because of the larger cost of extensive farming. Consequently, even with a convex density–yield curve environmental quality can be higher with extensive farming rather than intensive farming thanks to the reduction of the quantity produced. They further compare consumers surplus and producer profits. Compared to Desquilbet et al. (2017), we consider a continuum of yields, and it is costly to either increase or decrease the yield compared to its laissez-faire level. Our result showing that, even if the density–yield curve is convex, intensive farming should not be subsidized if demand is sufficiently elastic, has similarities to their results. However, the converse also holds, in that, even with a concave density–yield curve extensive farming should not be subsidized if the demand is elastic.

Martinet (2013) also considers these two types of farming and introduces heterogeneous land productivity. He analyzes the food and wildlife production possibility set. Because of heterogeneous soil quality, and the possibility of reallocating production from less productive to more productive land, he shows that the total abundance associated with a given level of food production might be maximized through coexistence of intensive farming on the most productive land, extensive farming on intermediately productive land, and natural reserve on the least productive land. This situation can arise when the implicit density–yield curve is concave (a case corresponding to land-sharing in the framework of Green et al., 2005).[8] In the present article, farmers are assumed identical, as in Green et al. (2005), and the introduction of heterogeneity as in Martinet (2013) is a path for future research but is not critical to realizing the aims of this paper.

Literature that is less related includes the work of Eichner and Pethig (2006) in which a general equilibrium of the economy is linked to a general equilibrium of an ecosystem (Tschirhart, 2000). In their model, land is used either for human activity or for wildlife; there is no intermediate level (see also Christiaans et al., 2007; Pethig, 2004, on pesticide use). They do not analyze the trade-off between the area used and the intensity of human activity. However, a natural extension of the present work would be to develop the biological side of the model in the spirit of these works.

The rest of the article is organized as follows. The model is introduced in Section 2. The optimal (first-best) policy is described in Section 3. Second-best policies are considered in Section 4. The main limitations of the model are discussed in Section 5. Section 6 concludes.

2 Model

2.1 Framework

The model is kept as simple as possible voluntarily to encompass the framework of Green et al. (2005), namely the density–yield curve, into an economic model with variable total production. We consider the market for one food product; the total quantity produced is

[8] Legras et al. (2018) also consider the allocation of heterogeneous land between two farming techniques, they maximize environmental quality subject to a budget- constraint, and provide a numerical analysis.

F (in tons), the consumption of which generates the gross consumer surplus $S(F)$ (in \$), a positive increasing and concave function. The corresponding price function $P(F)$ (\$/ton), equal to $S'(F)$, is positive and decreasing. The price elasticity of the demand for food is denoted ϵ:

$$\epsilon(F) = \Big|\frac{P}{P'(F)F}\Big|. \tag{1}$$

On the supply side, the yield is denoted y (in tons per hectare) and the total quantity of land farmed L (in hectares), so that $F = yL$. The cost of farming is $c(y)$ (\$ per hectare); this is the cost to produce y tons of food on a hectare of land. The total cost to produce F is then $c(y)L = c(y)F/y$. The cost $c(y)$ is positive, increasing and convex.[9] It is assumed that there is a fixed cost associated with land conversion, $c(0) > 0$, so that average costs first decrease and then increase. This fixed cost can also be interpreted as the opportunity cost of farming.

There is one valuable species, and the size of its population on a particular piece of land is a function of the yield. The total size of the habitat and potential area of farmed land is $\bar{L}$. The density on a hectare of farmland is $b(y)$ (specimen/hectare), a positive and decreasing function of the yield. We consider a species that is hurt by farming, some species might indeed benefit from it. There is a maximum yield $\bar{y}$ at which b is null, and production cost is sufficiently large at this yield that it is never optimal to adopt it. The total population on the habitat under consideration is the sum of the population on unfarmed land $b(0)(\bar{L} - L)$ and farmed land $b(y)L$.

The marginal value of the species is β (\$/specimen); it is assumed constant for the sake of simplicity. The impact of β on optimal farming decisions will be studied.[10] Total welfare is then

$$W = S(yL) - c(y)L + \beta[b(0)(\bar{L} - L) + b(y)L].$$

It will prove easier to work with F and y rather than L and y, welfare should be written as a function of yield and food production, replacing L with F/y:

$$W(y, F) = S(F) - \left[\frac{c(y)}{y} + \beta\frac{b(0) - b(y)}{y}\right] F + \beta b(0)\bar{L}. \tag{2}$$

To ensure that there is a unique interior optimum, the value of the species is assumed sufficiently small so that $c''(y) > \beta b''(y)$ for all y. If the density–yield curve is concave ($b'' < 0$) this condition is satisfied for all $\beta > 0$. If the density–yield curve is convex β should be lower than c''/b''. Otherwise, welfare is not concave everywhere and there can be several local optima. We rule out this possibility.

[9] Convexity is related to decreasing return to scale on an hectare, or a typical farm, and not to land heterogeneity as in Desquilbet et al. (2017) and Martinet (2013).

[10] Indeed, it is very likely that the marginal value is decreasing with respect to the total size of the population, an endangered species being more valuable than an abundant one. Since we study the impact of β, to introduce a concave function $B(.)$ and consider a welfare function : $W = S(yL) - c(y)L + \beta B\left(b(0)(\bar{L} - L) + b(y)L\right)$ would not have deep implications on our analysis, the equilibrium marginal value of a specimen would be $\beta B'$ instead of β. The main difference would be that an increase of the demand for food would endogenously trigger an increase of the environmental marginal value, we are not interested in such changes but focus on the internalization of an external cost.

Only density–yield curve that are either convex or concave everywhere are considered in order to focus on the influence of demand and cost functions. More complex curves (e.g., first convex then concave) are possible and would be more realistic. The situation in which there is a drop around zero, for instance, due to deforestation, could be considered as an extreme case of convexity.

Let us make a few remarks on the scale of the model and the density–yield curve. Several other limitations and possible extensions are discussed further in Section 5: we further discuss the scales of the market and area considered, the explicit introduction of inputs within a production function and the function $b(.)$, and dynamic issues.

Concerning the scale, farming decisions, whether to farm or not, and the yield choice, are made at the scale of a farm.[11] The scale of the area $\bar{L}$ is more problematic, Balmford et al. (2015) argue that the relevant scale of their model is that of a region or province, within a country. The scale of their approach is determined by the relevant biological scale. Since the present analysis focuses on the role of demand, the market should determine the relevant scale. It is then very likely that multiple heterogeneous ecological regions supply food to that market. This heterogeneity is not modelled, but it should be in future research. The tension between the two scales, of an ecological region and of a market, raises issues associated with the allocation of production among several regions, international trade and the coordination of policies. These issues are discussed further in Section 5.

The density–yield curve is the key ingredient imported from Green et al. (2005), and it has been subject to many criticisms (e.g. Fischer et al., 2014), addressed partly by the authors in their original article and following articles. Most notably, the framework does not consider habitat fragmentation and the role of the spatial distribution of farming activities, other negative externalities from intensive farming, and jointness of production between food and the environment. The lack of a spatial structure is inherent in the framework;[12] the two other points are less fundamental.

Firstly, the function $b(.)$ can represent any impact of farming on the environment; from greenhouse gases to eutrophication, a similar trade-off would arise. In a recent article, Balmford et al. (2018) stress that externalities per unit of output, and not unit of area, should be compared, that few estimates exist, and that these estimates tend to favor land-sparing ($b(.)$ is likely to be convex).

Secondly, even though we consider a function $b(.)$ that is decreasing everywhere, jointness of production could be modelled with a function that is first increasing. We shall come back to this point and its consequences on the optimal policy at the end of section 3.

Jointness of production might be particularly relevant for some species in regions with centuries-long history of farming, in which traditional agricultural practices (notably pastoral-based) and the environment have co-evolved into so-called cultural landscapes (Bignal and McCracken, 2000).[13] Together with the fact that the impact of agriculture on the environ-

[11]Indeed, the size of farms exhibits a great variability across countries, from few hectare in China to hundreds of hectare in the US and the EU (Lowder et al., 2016), and could itself be an endogenous variable of the model.

[12]See Kremen (2015) for an ecological discussion of the issues of scale and Lewis and Plantinga (2007); Lewis et al. (2009) for an economic analysis of optimal policies to address habitat fragmentation.

[13]The UNESCO has its own list of cultural landscape: `https://whc.unesco.org/en/culturallandscape/`

ment is likely to be greatest in developing countries, it suggests that the framework is better suited for an expanding agricultural region in a developing country,[14] than for a situation of farmland contraction in a cultural landscape. Navarro and Pereira (2015) offer a critical discussion of cultural landscapes and question the environmental benefits of traditional agriculture in Europe. They support an ambitious program of active restoration on a large scale on abandoned farmlands. Formally, in the present model, their point boils down to the choice of $b(0)$, the environmental situation of a non farmed hectare, in case of abandonment, which depends on a possible active restoration strategy that is not explicit in the present model.

2.2 Laissez-faire and comparative statics

The market works as a textbook competitive market; each land-owner is price taker and entry is free. If no regulation is implemented, the total profit from land-use is

$$\Pi(p, y, L) = [py - c(y)]L = pF - \frac{c(y)}{y}F. \tag{3}$$

The 'laissez-faire' yield, to be denoted y_0, corresponds to the minimum efficient scale in a textbook competitive market. It minimizes the average cost $c(y)/y$, so it equalizes the marginal and the average costs:

$$c'(y_0) = c(y_0)/y_0. \tag{4}$$

The supply curve is determined by the decisions of price-taking land owners and it is *as if* a representative landowner were maximizing the profit given by equation (3), subject to the constraint on land $L \leq \bar{L}$. If the price is below $c(y_0)/y_0$, then farming is unprofitable and no land is converted, if the price is above $c(y_0)/y_0$, all land is farmed, and if the price is equal to $c(y_0)/y_0$, then each land owner is indifferent between either farming or not farming.

At equilibrium, the price clears the market. If $\bar{L}$ is sufficiently large, such that $P(y_0\bar{L}) < c(y_0)/y_0$, then land is abundant and some land will remained unfarmed. At equilibrium, the yield is y_0, and the quantity of land farmed, to be denoted L_0, is such that the price of food $p = P(y_0L_0)$ is equal to the average cost $c(y_0)/y_0$. The profit of the representative farmer is then zero. If $P(y_0\bar{L}) > c(y_0)/y_0$, land is scarce, the whole region is farmed, y is above the laissez-faire yield y_0, and farmers obtain a positive profit.[15]

Let us consider the following quadratic specification to fix ideas:

$$c(y) = c_0 + c_1y + \frac{c_2}{2}y^2 \tag{5}$$

[14] The few published estimates of density–yield curves only concern three regions in developing countries : Southern Uganda (Phalan, Onial and Balmford, 2011), and, Southwest Ghana and Northern India (Hulme et al., 2013). However, according to Balmford et al. (2015) other indirect evidence suggest a convex density–yield curve in other contexts, notably for butterflies in the UK.

[15] The decentralization of this result operates as follow: Each owner of a piece of land decides whether to farm it or not and chooses its yield taking the price as given. For a given price p, the yield chosen maximizes $p.y - c(y)$: If the price is strictly lower than $c(y_0)/y_0$, farmers do not produce. If the price is above, the yield chosen equalizes the marginal cost with the price $p = c'(y)$, farming is profitable and the whole region is farmed. The equilibrium farmed area is $\bar{L}$ if $P(y_0\bar{L}) > c(y_0)/y_0$, and it is $L_0 < \bar{L}$ such that $P(y_0L_0) = c(y_0)/y_0$ otherwise.

In which c_0 is the fixed cost associated with farming, per hectare (we consider the size of a farm as being fixed and not endogenously chosen).[16] The parameter c_2 is the curvature of the cost function and captures the inherent decreasing return to scale of farming on a fixed amount of land. With the quadratic specification, the laissez-faire yield is

$$y_0 = \sqrt{\frac{2c_0}{c_2}}. \tag{6}$$

The laissez-faire yield does not depend on the linear coefficient of the cost function; it is increasing with respect to the fixed cost and decreasing with respect to the curvature of the cost function. The equilibrium area farmed L_0, assuming that land is abundant, is such that

$$P(y_0 L_0) = \frac{c(y_0)}{y_0} = c_1 + \sqrt{2c_0c_2} \tag{7}$$

Any increase of one of the cost parameters c_i induces a reduction of the total food production, the effect on the yield and farmed area depends on the component considered. The comparative statics results are summarized in the following lemma.

Lemma 1 *If the production cost is given by equation (5), total food production is decreasing with respect to c_i for $i = 0, 1, 2$, and, concerning the equilibrium yield and farmed area:*

- *An increase of the fixed cost c_0 induces an increase of the yield and a reduction of the total farmed area;*
- *An increase of the linear component c_1 does not influence the yield and reduces the farmed area;*
- *An increase of the curvature c_2 induces a reduction of the yield and a reduction of the farmed area if and only if demand is sufficiently elastic:*

$$\epsilon > \frac{c_1 + \sqrt{2c_0c_2}}{\sqrt{2c_0c_2}}.$$

See Appendix A. If the fixed cost of farming increases, the laissez-faire yield is larger and total food production is lower; both effects play in the same direction to reduce the farmed area and spare land. Concerning the curvature c_2, it has opposite effects on the yield and the total food production, and the elasticity of the demand function determines which one dominates. If the demand function is inelastic (ϵ close to zero), the quantity produced does not change much, and farmed area increases to compensate for the reduction of the yield.

[16] A change of units of L amounts to a rescaling of the cost function. For instance, since the relevant unit for farming decisions is the farm, the size of a farm and the number of farms can be introduced. If l is the size of a farm, there are L/l farms, and the cost for a farm to produce a quantity $\tilde{y} = ly$ is $lc(\tilde{y}/l) = lc_0 + c_1\tilde{y} + 0.5c_2\tilde{y}^2/l$.

3 The optimal solution

In this section we describe the optimal solution and the influence of the environmental value β. The analysis of Green et al. (2005) could be re-framed as a special case with an inelastic demand function.

The optimal solution consists of a pair of yield and quantity of food $(y^*(\beta), F^*(\beta))$ that maximizes welfare subject to the constraint $L \leq \bar{L}$. If the constraint is not binding, the optimal policy is characterized by the two first-order conditions:[17]

$$P(F) = \frac{c(y)}{y} + \beta \frac{b(0) - b(y)}{y} \tag{8}$$

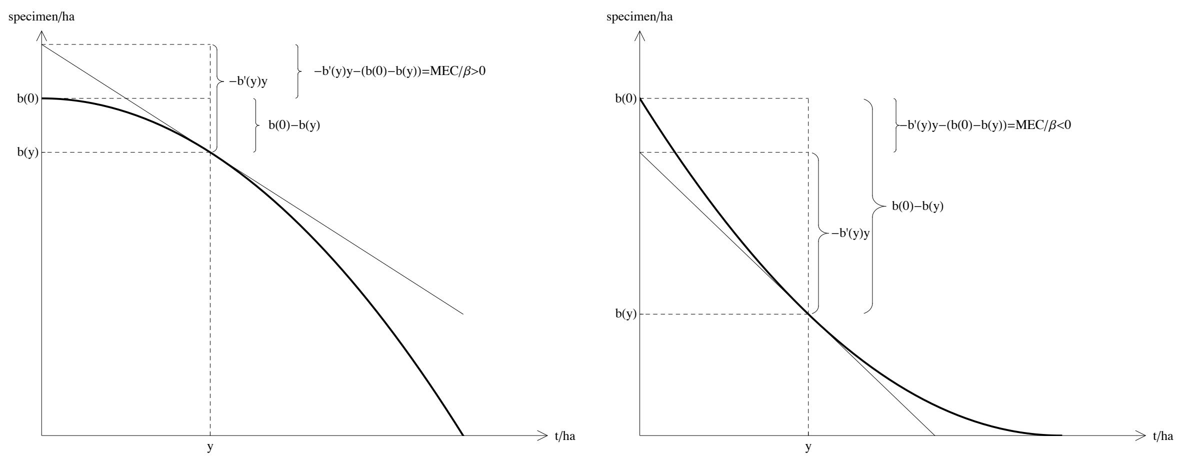
$$c(y) - c'(y)y = \beta \left[-b'(y)y - (b(0) - b(y)) \right]. \tag{9}$$

The first equation states that the price of food should be equalized with its marginal cost, which includes the value of the environment. The second equation represents the arbitrage made when choosing the optimal yield between economic and environmental costs.

The two sides of equation (9) represent similar trade-offs between average and marginal cost. The left-hand side of equation (9) is the marginal benefits from production cost reduction due to increased yield. It is decreasing and null at the laissez-faire yield y_0.

The right-hand side of equation (9) is the marginal environmental cost from an increase in the yield. It is the difference between the direct environmental cost on farms ($-\beta b'(y) > 0$) and the indirect gain obtained from the reduction of the farmed area ($\beta(b(0) - b(y))$). The shape of the density–yield curve determines whether this marginal environmental cost is either increasing and positive, or decreasing and negative. It is illustrated in Figure 1 for both a concave (1(a)) and a convex (1(b)) density–yield curve.

[17] The assumption on β: $c'' < \beta b''$ ensures that welfare is quasi-concave. Let us show that if y and F satisfy the pair of conditions (8) and (9), then the second-order conditions are satisfied. This is because i) $\partial W / \partial y$ is linear in F, so $\partial W^2 / \partial y \partial F = [\partial W / \partial y] / F$ is null if y satisfies the equation (9), and ii) the second-order derivative w.r.t. to the yield is $\partial W^2 / \partial y^2 = F/y^2(c'' - \beta b'') - 2F/y \partial W / \partial y = F/y^2(c'' - \beta b'') < 0$ if y satisfies (9).



(a) With a concave density–yield curve, the marginal environmental cost (MEC) is positive

(b) With a convex density–yield curve, the marginal environmental cost (MEC) is negative

Figure 1: Impact of a marginal increase of the yield on the environmental loss $-(b(0) - b(y))/y$. The density–yield $b(y)$ curve is bold, the slope of the straight line is $b'(y) < 0$.

Proposition 1 *The quantity of food produced decreases with respect to the value of the species β.*

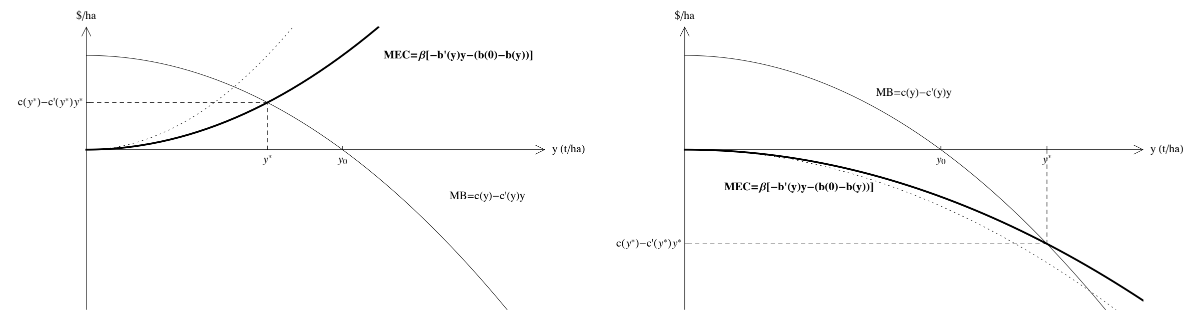
If land is abundant, i.e., $F^* < y^*\bar{L}$,

- *If the density–yield curve is concave, the optimal yield decreases with respect to the value of the species β*
- *if the density–yield curve is convex, the optimal yield increases with respect to the value of the species β*

If land is scarce, i.e., $F^* = y^*\bar{L}$, the optimal yield is decreasing with respect to β.

Thr Proof is in Appendix B. The proposition is illustrated by Figure 2. With a concave density–yield curve (Figure 2(a)), the situation looks familiar: the marginal environmental cost is increasing with respect to the yield, and the optimal yield should be lower than is the laissez-faire yield. The case of the convex density–yield curve is different, as illustrated in Figure 2(b). With a convex density–yield curve, the species is very sensitive to the first increase of the yield, so the benefits from land sparing $((b(0) - b(y))/y)$ compensate for the environmental loss on farmland. The marginal environmental cost is then decreasing with respect to the yield, and it is optimal to spare land by increasing the yield on farmland.[18]

[18]If the density–yield curve is convex, the environmental cost is concave, and the second term of (9) is decreasing with respect to the yield. Multiple local optima may exist for a sufficiently large value of β. All interior optima are on the right side of the laissez-fairescale. When the value of the species β increases, one may jump from one interior equilibrium to another. Proposition 1 is still true because the new equilibrium is situated to the right of the old one.



(a) With a concave density–yield curve, the optimal yield is lower than the laissez-faire yield

(b) With a convex density–yield curve, the optimal yield is higher than the laissez-faire yield

Figure 2: The determination of the optimal yield with a concave (resp. convex) density–yield curve. The dotted line represents the effect of an increase of the value of the species.

In Figure 2, it is assumed that land is abundant for farming, so that any change of the yield for a given food production is associated with an adjustment of the farmed area. If land is scarce and already fully exploited, a marginal increase in the yield does not trigger a reduction in the area farmed, except at the threshold. In such a case, a marginal increase in the value of the species β induces a reduction in the yield, regardless of the shape of the density–yield curve.

In their article, Green et al. (2005) compute the optimal strategy that maximizes abundance subject to constraints on both the quantity of food produced and the quantity of land available. In the absence of production cost, it can be seen in Figure 2 that the optimal strategy is in a corner; it is optimal to set the lowest (resp. highest) possible yield with a concave (resp. convex) density–yield curve. Within the present framework, their analysis can be reframed as an optimal reaction to an increase of the value of the species, when the demand for food is inelastic.

Corollary 1 *(Green et al., 2005)*
If the demand for food is inelastic ($\epsilon = 0$), so that F is maintained fixed;

- ***Land sharing**: If the density–yield curve is concave then the yield decreases and the farmed area increases with respect to β*

- ***Land sparing**: If the density–yield curve is convex, then the yield increases and the farmed area decreases with respect to β*

With an elastic demand for food, the quantity of food produced is reduced by the internalization of the environmental cost, and the total farmed area depends upon demand elasticity.

Corollary 2 *If the demand for food is elastic ($\epsilon > 0$);*

- *If the density–yield curve is convex, then the yield increases and the farmed area decreases with respect to β (land sparing)*

- *If the density–yield curve is concave, then the yield decreases and the farmed area increases if $\epsilon < \tilde{\epsilon}$ (land sharing) and decreases if $\epsilon > \tilde{\epsilon}$*

$$\begin{aligned}\tilde{\epsilon} &= \frac{\beta\text{-elasticity of the optimal yield}}{\beta\text{-elasticity of the total cost}} \\ &= \frac{\left|b'y^* + (b(0) - b(y^*))\right|}{(c'' - \beta b'')y^{*2}} \times \frac{c(y^*) + \beta(b(0) - b(y^*))}{b(0) - b(y)}\end{aligned} \tag{10}$$

The Proof is in Appendix B. Even if the optimal yield decreases with respect to the value of the species, the optimal quantity of farmland can decrease if demand is sufficiently price elastic. The threshold price elasticity is the ratio of the β elasticity of the optimal yield and the β elasticity of the total environment cost. The former is related to the curvature of the density–yield and cost functions.[19] The latter is the share of the environment in the total cost. An increase in the value of the species reduces the yield and increases the production cost. The total farmed area decreases despite the reduction in the yield if the increase in the cost is sufficiently large (a large denominator in eq. (10)) to trigger a large reduction in the quantity consumed.

There are two channels through which the environmental externalities should be internalized: the yield and the demand channels. The relative environmental role of each of these channels could be assessed by looking at the derivative of the total species abundance with respect to β when y and F are optimally set :

$$\begin{aligned}\frac{d}{d\beta}\left[\frac{b(y) - b(0)}{y}F\right] &= \frac{F}{y}\left\{\Big[b'(y)y + b(0) - b(y)\Big]\frac{y^{*\prime}}{y} + \Big[b(0) - b(y)\Big]\frac{-F^{*\prime}}{F}\right\} \\ &\propto \left|1 + \frac{b'(y)y}{b(0) - b(y)}\right|\tilde{\epsilon} + \epsilon\end{aligned}$$

the threshold elasticity $\tilde{\epsilon}$, given by eq. (10), measures the relative importance of the yield channel. At $\beta = 0$, for a quadratic density–yield curve, using eq. (10) and that $c(y_0) = c'(y_0)y_0$, the environmental benefits from a small increase of β is proportional to:

$$\left[\frac{b''y_0^2/2}{b(0) - b(y_0)}\right]^2 \frac{c'(y_0)}{c''y_0} + \epsilon$$

The relative role of each channel could be assessed by comparing the price elasticity of demand with the first term. The first term is the product of two factors related to the convexity of the density–yield and the cost functions. The first factor is the square of the relative weight of the second-order term into the environmental damage $b(0) - b(y)$. The larger the convexity or concavity of the density–yield curve the larger the environmental role of the yield channel relative to the demand channel. The second factor is the short-term (for fixed farmed area) elasticity of supply. Note that the first factor is bounded below 1 for a convex density–yield curve, while it is not bounded for a concave density–yield curve. And

[19] For a quadratic density–yield curve: $b(y) = b_0 + b_1 y + b_2 y^2/2$ we have $b'(y)y + b(0) - b(y) = (b_1 + b_2 y)y - (b_1 y + b_2 y^2/2) = b_2 y^2/2$ so that the β-elasticity of the optimal yield is then $\beta b''/(c'' - \beta b'')$.

that the second factor is likely to be relatively small. Even if the price elasticity of demand is low, the demand channel can play a non negligible role if the supply is inelastic on the short-term, or the density–yield curve is highly convex (land-sparing).

The case of jointness of production can be incorporated within the framework by considering a function b that is first increasing. If this function is eventually decreasing it can only be concave, given our assumption of a function that is either everywhere convex or concave. However, in theory, it might also be convex and increasing everywhere. The consequences are the following: The quantity of food consumed is increasing with respect to β, as the environmental cost $b(0) - b(y)$, in equation (8), is actually a benefit since $b(y) > b(0)$. The optimal yield monotonicity is not affected; it is decreasing with respect to β if b is concave and increasing if b is convex.[20] However, the intuition is changed since an increasing and concave function means that most environmental benefits are obtained when land is converted, which justifies yield reduction, even though it is associated with environmental loss in farms.

Corollary 1 is still true when jointness of production is considered; a concave density–yield curve is associated with land sharing: a reduction of the yield and increase of farmed area, whereas a convex density–yield curve is associated with land-sparing.

The influence of the demand elasticity on the total farmed area is modified. Since food consumption increases, the total farmed area increases if the density–yield curve is convex (land-sharing), and it also increases if the density–yield curve is concave and the demand sufficiently elastic. It is then possible, for an elastic demand function, to have an increase of the yield together with an expansion of farmed area. A situation that might be relevant for some European cultural landscapes.

4 Second-best policy

Having described the optimal solution, let us consider the policy implications.

The optimal allocation has been described by a yield, which could be interpreted as a farming technique, and the farmed area. This allocation can be implemented by a Pigouvian subsidy on each species specimen equal to β and received by the land-owner even in the absence of farming. The Pigouvian solution has an informational advantage if the farmer is more able to determine practices that would increase the species abundance at low cost. This feature is outside the scope of the present model. Within the present model, the optimal allocation can be implemented by directly setting these two quantities via technical standards and natural reserves. For instance, well designed agri-environmental schemes coupled with natural reserves could implement the optimal policy.

Current agri-environmental schemes as described and analyzed by Kleijn et al. (2006) rarely target a specific species, and only concern farmed land. In addition to mixed environmental results (Kleijn et al., 2006) they still constitute a subsidy to farming, even if result-based. In Europe, the "direct payments" of the Common Agricultural Policy (among which 30% are "green")[21] even though "decoupled" from production are conditioned on be-

[20] The left-hand side of equation (9) is then better written as $(b(y) - b(0)) - b'(y)y$, increasing the yield reduce the area farmed which costs $b(y) - b(0)$ but increases biodiversity in the fields ($b'(y) > 0$).

[21] Most of the direct payments of the CAP consists in a basic income for farmers, complemented by

ing an active farmer: "As a general rule, only land suitable for agricultural production is considered as agricultural area (e.g. forests are in principle not eligible)[...]farmers must also show that this land is used for some form of agricultural activity[...][or] ensure that the land is maintained in good agricultural condition, i.e., suitable for grazing or cultivation." (European Commission, 2018) We will explore under which circumstances such a policy can improve efficiency.

In this section, several situations are considered in which the regulator cannot implement the optimal policy, for whatever reason. Several reasons could be proposed to explain why it is not feasible to implement the subsidy on the species specimen or to directly set the optimal yield and optimal farmed area. For example, it might be impractical to estimate the density of a species, if property rights are not well defined on unfarmed land, and it is not possible to remunerate an owner to create an incentive for land conservation. This situation is more likely to occur in developing countries.

4.1 A general assessment

The regulation is represented by a variable r. The regulation influences the incentive to farm and the choice of the yield; it does not have other effects. For instance, public funds are assumed to be costless, and, if the regulation is either a tax or a subsidy, the associated monetary transfers are welfare-neutral. With a regulation r , the cost of farming for a landowner is $\gamma(y, r)$ \$ per ton. The profit of the representative land-owner is

$$\pi(y, F, P) = PF - \gamma(y, r)F. \tag{11}$$

The equilibrium yield minimizes the production cost, and the quantity of food produced is such that the price is equal to the marginal cost $\gamma(y, r)$. The two equilibrium quantities $y(r)$ and $F(r)$ satisfy

$$P(F) = \gamma(y, r) \text{ and } \frac{\partial \gamma}{\partial y}(y, r) = 0. \tag{12}$$

The situation $r = 0$ corresponds to a no-regulation situation with $\gamma(y, 0) = c(y)/y$ so that $y(0) = y_0$ and $P(F(0)) = c(y_0)/y_0$. The quantity of food produced and the yield can be either increasing or decreasing with respect to the regulatory variable. Before considering some particular regulations, we first provide an analysis without further specifying the regulation.

At the laissez-faire equilibrium the difference between consumers surplus and production cost is maximized, and it is not affected by small changes of the yield and production by an envelope argument. For a small change in r, from $r = 0$, only the environmental effect matters, so that the derivative of welfare is

$$\left.\frac{dW}{dr}\right|_{r=0} = \beta \frac{F}{y} \left\{ [b'(y)y + b(0) - b(y)] \frac{y'}{y} + [b(0) - b(y)] \frac{-F'}{F} \right\} \tag{13}$$

The first term in equation (13) casts the influence of the yield change, the sign of which depends on the shape of the density–yield curve. If the density–yield curve is concave (resp.

other payments. Green payments are obtained for crop diversification, maintenance of permanent grassland, ecological focus area (e.g. afforested area within a farm). In addition to the direct payments there are also "rural development programmes" that includes Agri-environment measures.

convex) a decrease (resp. increase) of the yield is welfare enhancing. And, the second term represents the unambiguous benefit from any reduction in the quantity of food produced. The amplitude of which depends upon the price elasticity. If both the changes of the yield and the food production increase welfare, a small increase in the regulatory variable is beneficial. For instance, if the density–yield curve is concave, a regulation that both reduces the yield and food production is beneficial. Otherwise, the comparison of the two terms is needed, and demand elasticity will play a crucial role.

A general expression of the threshold price-elasticity of demand is

$$\tilde{\epsilon} = \left| \left(1 + \frac{b'(y_0) y_0}{b(0) - b(y_0)} \right) \times \frac{\partial^2 \gamma / \partial r \partial y}{y_0 \, \partial^2 \gamma / \partial y^2} \times \frac{\gamma}{\partial \gamma / \partial r} \right| \tag{14}$$

The first factor represents the gain from an increase in the yield relative to the gain from a reduction in food consumption. It is related to the shape of the density–yield curve. It is null if the density–yield curve is linear, positive if it is concave, and negative if it is convex. The second factor is the rate of change of the yield with respect to the regulatory variable it is notably determined by the curvature of the cost function. And the last factor is the inverse of the rate of change of the production cost with respect to the regulatory variable. In case of ambiguity about the merit of a small positive regulation, the demand elasticity should be compared with this ratio. The following table summarizes the possible cases.[22]

Proposition 2 *The sign of the welfare effect of a small increase of the regulatory variable depends on the shape of the density–yield curve, as follows:*

		b concave ($y^* < y_0$)	*b convex ($y^* > y_0$)*
$y' < 0$	$F' < 0$	*A:* +	*C: + if $\epsilon \geq \tilde{\epsilon}$, - otherwise*
	$F' > 0$	*B: + if $\epsilon \leq \tilde{\epsilon}$, - otherwise*	*D:* -
$y' > 0$	$F' < 0$	*C: + if $\epsilon \geq \tilde{\epsilon}$, - otherwise*	*A:* +
	$F' > 0$	*D:* -	*B: + if $\epsilon \leq \tilde{\epsilon}$, - otherwise*

The Proof is in Appendix C. A regulatory small change that modifies the yield in accordance with the optimal solution is welfare-enhancing either if the quantity produced decreases (the two shaded A boxes) or if the elasticity of the demand is sufficiently close to zero ($\epsilon \leq \tilde{\epsilon}$ in B Boxes). If production increases and the yield diverges from the optimal one, a small regulatory change is indeed detrimental (D boxes). In the two last cases (C boxes), even though the yield moves in the wrong direction (compared to the optimal yield) the regulation can be welfare-enhancing if the demand is sufficiently elastic.

A focus on the yield can be misguided when the demand elasticity is large in two types of situations: First, if the yield moves closer to the optimal one, the regulation is detrimental if production increases (B boxes), as would be the case if a subsidy is implemented. Second,

[22]Note that the product of the last two factors is the ratio between the r elasticity of the yield and the r elasticity of the cost, it corresponds to the threshold identifies in Corollary 2 for $r = \beta$ a subsidy per specimen on both farmed and unfarmed land.

if the yield moves away from the optimal one, the regulation can still be welfare enhancing if production decreases sufficiently (C boxes).

Another way to look at the trade-off is to write the derivative of welfare at $r = 0$ as a function of the yield and the farmed area, and since only the effect on the environment matters at $r = 0$ we have:

$$\left.\frac{dW}{dr}\right|_{r=0} = \beta\Big[b'(y_0)Ly' - \beta\left(b(0) - b(y_0)\right)L'\Big] \tag{15}$$

This expression emphasizes the trade-off between yield and farmed area, even though it masks the role played by the demand price elasticity. However, this expression still allows us to obtain the following intuitive and reassuring result.

Proposition 3 *A sufficient condition for a small regulation to improve welfare is that both the yield and the farmed area decrease.*

Armed with these results, we can now consider several particular regulations.

4.2 Subsidizing wildlife-friendly farming

We begin by considering the consequences of a subsidy that would only apply on farmed land. Let us denote by s a subsidy on species specimen for farmland. This subsidy is constrained to be positive; that is, a tax on the specimen is not feasible. The profit of the representative land-owner is

$$\pi = PF - c(y)L + sb(y)L = PF - c(y)F/y + sb(y)F/y. \tag{16}$$

When choosing the yield for farmland, the land-owner does not consider the environmental effect of land substitution between farmed and unfarmed land. He sets $y(s)$ and produces $F(s)$ so that

$$P(F) = \left[c(y) - sb(y)\right]/y \text{ and } c(y) - c'(y)y = s\left[b(y) - b'(y)y\right] \tag{17}$$

When the subsidy increases, the representative farmer reduces the yield and increases his production of food, increasing the total farmed area. Therefore, land-sparing is not feasible with this policy.

Corollary 3 *With a subsidy per specimen on farmland (and not on unfarmed area),*

- *If the density–yield curve is convex, the optimal subsidy is null.*
- *If the density–yield curve is concave, the optimal subsidy is null if $\epsilon > \tilde{\epsilon}$ and positive otherwise. The expression of the threshold is*

$$\tilde{\epsilon} = \left(\frac{-b'(y_0)y_0}{b(0) - b(y_0)} - 1\right)\left(1 - \frac{b'(y_0)y_0}{b(y_0)}\right)\frac{c'(y_0)}{c''y_0} \tag{18}$$

Proof. From the two equations (17), F is increasing and y is decreasing with respect to s. The situation corresponds to the third line of the table in proposition 3.

The expression of the threshold is obtained by inserting into the general expression (14) the relation $\partial\gamma/\partial s = -b(y)/y$ and the derivative of the yield at $s = 0$ (obtained from eq. (17)):

$$y'(0) = \frac{b - b'(y_0)y_0}{-c''y_0}.$$

■

Subsidizing environmental quality of farmland has the adverse consequence of increasing the incentive to farm! If the species under consideration is very sensitive to the initial increase of the yield (b convex), it is clearly detrimental to subsidize in-farm environmental quality. The gains in abundance on the existing farmland cannot compensate the loss due to the increase in farmland. However, if the species is resistant to the implementation of farming (b concave), the gains from the reduction of the yield in farms are not fully negated by farms' expansion if the demand for food is sufficiently inelastic. In that case, the food consumed does not increase much following the reduction of food price.

4.3 Implementation of natural reserves

Let us now consider the implementation of a natural reserve. This regulation would consist of setting $L = F/y$ as the total farmed area. It is formally equivalent to consider a tax on farmland (the shadow price of the farmland constraint), which better suits our general approach. The regulatory variable is then the tax t.[23] The profit of the representative land-owner is then

$$\pi = PF - (c(y) + t)L = \left[P - \frac{c(y) + t}{y}\right] F. \tag{19}$$

The effect of an increase of the tax t, is equivalent to an increase of the fixed cost, explored in Lemma 1. Following an increase in fixed cost, the price of food increases, farmers intensify farming (y increases) and less land is farmed.

Corollary 4 *When the regulator envisions taxing or subsidizing farmland,*

- *If the density–yield curve is convex, farming should be taxed.*
- *If the density–yield curve is concave, farming should be taxed if $\epsilon > \tilde{\epsilon}$ and subsidized otherwise. The expression of the threshold is*

$$\tilde{\epsilon} = \left(\frac{-b'(y_0)y_0}{b(0) - b(y_0)} - 1\right) \frac{c'(y_0)}{c''y_0} \tag{20}$$

Proof.

With a tax on farmland, the marginal production cost of food is $\gamma(y, t) = (c(y) + t)/y$, the two quantities $y(t)$ and $F(t)$ satisfy

$$c'(y)y - c(y) = t \text{ and } P(F) = (c(y) + t)/y.$$

[23]The tax is equivalent to a payments for land, it is as if the regulator buys land at a price t per ha and set it aside.

The quantity of food is decreasing and the yield is increasing with the regulatory variable. This case corresponds to the second line of the table in Proposition 3.

The particular expression of the threshold is obtained from equation (14) and the two following derivatives at $t = 0$:

$$\partial\gamma/\partial t = 1/y_0 \text{ and } y' = 1/c''$$

. ■

If the density–yield curve is convex, either taxing farmland or implementing natural reserves is unambiguously good because it both reduces food consumption and increases the yield. Land is effectively spared, and the species gains more from this than it loses from the increased yield.

If the density–yield curve is concave, in a first-best setting, it would be optimal to reduce the yield, which suggests that farmland should be subsidized. With an inelastic demand function, farming should indeed be subsidized. However, if the demand function is sufficiently elastic, farming should be taxed, because the loss of environmental quality within farms is compensated by the overall reduction of food consumption. The expression of the threshold elasticity is a product of two factors: the first is the negative relative loss of environmental quality from the increased yield, and the second is the convexity of the cost function, which determines the sensitivity of the yield to an increase of the tax.

4.4 Taxing a dirty input

A last possibility considered would be to tax the input responsible for environmental degradation. Here, we do not consider substitution among inputs; we consider only that the yield is determined by a quantity q of a dirty input. The function $q(y)$ is the quantity of inputs required to obtain a yield y It is null at zero, positive, increasing and convex. The function $b(y)$ is then an observed indirect relationship between the yield and the density that occurs via the quantity q.

If the regulator envisions taxing the input, the regulatory variable r is the tax, and the profit of farmers is

$$\pi = \left[P - \frac{c(y) + rq(y)}{y}\right] F$$

The yield decreases with the tax on the input, and the food produced is reduced. Farmed area decreases unambiguously with the input tax.

Corollary 5 *If the regulator can only either tax or subsidize a pollutant input,*

- *If the density–yield curve is concave, the dirty input should be taxed.*
- *If the density–yield curve is convex, the dirty input should be taxed if $\epsilon > \tilde{\epsilon}$ and subsidized otherwise.*

The expression of the threshold elasticity is

$$\tilde{\epsilon} = \left(\frac{-b'(y_0)y_0}{b(0) - b(y_0)} - 1\right)\left(1 - \frac{q'(y_0)y_0}{q(y_0)}\right)\frac{c(y_0)}{y_0^2 c''} \tag{21}$$

Proof. The two first-order conditions are

$$c(y) + rq(y) - (c' + rq')y = 0 \text{ and } P(F) = (c + rq)/y \tag{22}$$

The derivative of the yield with respect to r at $r = 0$ is $y' = \frac{1}{y_0}\frac{q - q'y_0}{c''}$, which is negative. The derivative of the cost is $\partial\gamma/\partial r = q(y_0)/y_0$. Therefore, the situation corresponds to the first (resp. third) line of the table in Proposition 3 for a tax (resp. a subsidy). Injecting these two derivatives into the general expression of the thresholds (14) gives the particular threshold (22).

■

If the density–yield curve is concave, both the reduction of the food consumed and the reduction of the yield go in the right direction, from a welfare perspective.

If the density–yield curve is convex, it would be optimal in a first-best setting to increase the yield and reduce the area farmed. However, if the demand for food is sufficiently elastic, it is optimal in a second-best setting to tax the dirty input. Such a tax induces a reduction of the food consumed which ensures that the area farmed does not increase too much and may decrease. However, and somehow paradoxically, if the demand is inelastic it is worth subsidizing the dirty input to increase the yield and reduce the area farmed.

5 Discussions

Several issues are discussed: first the scale at which the model should be interpreted and the associated question of the coordination of policies; second, the substitution among inputs; and third dynamic aspects.

5.1 Scale and trade

The question of the scale at which the model should be interpreted is related to trade among heterogeneous locations and the coordination of policies. In the model developed there is one type of land or eco-system, one good/market, and one valuable species. It can be interpreted on a broad scale (e.g., wheat market), assuming that environmental quality over an hectare could be described through an indicator $b(y)$, summable over hectares,[24] the value of which is β. Such an interpretation erases the heterogeneity and complexity of multiple species interacting over heterogeneous lands used to produce diversified goods sold on imperfectly integrated world markets.

With a more palatable interpretation relating one particular good and one species, the relevant biological scale is relatively small; a region within a country. If the good considered is also produced elsewhere, then any change of local food production would be accompanied by an adjustment in that other location and the associated environmental consequences. Two issues arise : whether a valuable species is present in the other location, and whether policies are coordinated or set unilaterally.

Multiple locations

[24] The key assumption being that this indicator can be summed up over land areas, and, for instance, a biodiversity indicator à la Weitzman (1992) does not fulfill such a condition.

Let us first ignore coordination issues and focus on the interaction between multiple production sites. Two extreme cases are easily handled: if there are no environmental consequences (valued by the decision maker) in the other location, and if the exact same good–species interaction occurs in multiple locations. In both cases, the framework developed can be applied directly.

A second supply source without environmental consequences can be incorporated within the surplus $S(F)$, the demand function considered is then a residual demand. Any reduction of the quantity produced locally is then, implicitly, accompanied by an increase of production elsewhere. The elasticity of the residual demand may then be large if either the second supply is elastic or the region considered is small.[25] It is worth redistributing production to non-polluting sites.

The second extreme case is a direct scaling of the model. If the same good–species couple appears in N different 'regions', and F is local production, then total production is NF and total consumers surplus $S(NF)$, all the results are still valid using the elasticity of the total demand derived from the surplus $S(.)$, and the number of locations N does not have any qualitative impact on the analysis.

In between these two extremes, the allocation of production among heterogeneous sites becomes a central issue. The allocation of production among sites of various agronomic quality is at the heart of the analysis of Martinet (2013), it is also stressed in the empirical work of Ruijs et al. (2017). The extension of Martinet (2013) analysis within a welfare framework is a path for future research.

Coordination of policies

Whether good–species sites are distributed over either a country or several countries does not matter as long as one focuses on the optimal strategy from the point of view of a central regulator. If several countries are involved the coordination of policies is questionable.

If a country values the species abroad, but cannot regulate production there, there is a possibility of "leakage": any attempt to reduce the environmental cost at home is compensated by an environmental degradation abroad because of the substitution between home and foreign production.[26] Leakage is an issue by itself, and it is studied extensively, notably with respect to climatic change and trade of carbon intensive good (e.g. Meunier and Ponssard, 2014; Meunier et al., 2017). Some lessons can be deduced from this literature. The regulating country is willing to pay foreign land owners to protect the environment there, which is true whether food is imported or not, and, if it can do so, it can implement the first best. If it cannot do so, a border tax (or quotas) on imports is justified, and, finally, if a border tax is not feasible, home food regulation should be modified to take into account the sensitivity of foreign food production to home food quantity change. The higher the leakage

[25] Formally, let us denote $G(p)$ the second supply curve and $D_w(p)$ the world demand. Residual demand is $D(p) = D_w(p) - G(p)$, denoting ϵ_w the price-elasticity of world demand and ϵ_G the price-elasticity of foreign supply, we have that

$$\frac{F}{F+G}\epsilon = \epsilon_w + \frac{G}{F+G}\epsilon_G$$

the local elasticity is larger than the world elasticity, and it is larger the smaller the market share of local producers and the larger the foreign supply elasticity.

[26] Setting aside protectionism motives (the incentive of countries to distort terms of trade), if each country only values home located species, unilateral implementation of optimal policies leads to a global optimum.

rate, the larger the home food optimal quantity.

5.2 Inputs' substitution

The density–yield curve observed is the result of a complex interaction between farming practices and the ecosystem. Various farming practices can induce similar yields at different environmental costs. Indeed, organic farming can have a high yield, but it might require more work and knowledge than does intensive farming. From a micro-economic perspective, this would mean that it is possible to substitute environment degrading inputs (e.g., pesticides and fertilizer) with less damaging ones (e.g., labor and knowledge).

The model should be extended by writing the yield and the density of the species as functions of a vector of input quantities. The optimal input combination would depend on the value of the species. The environmental effect of an input would be its direct effect on in-farm density plus its indirect effect via land use. The latter is related to the productivity of the input, so it would likely exhibit decreasing returns to scale. Environment preserving inputs have clear environmental benefits because they increase yield while preserving in-farm environmental quality. Whether environment damaging inputs should be more intensively used would depend on whether their influence on the yield is sufficient to compensate for their in-farm environmental cost.

Substitution can be difficult to manage and can have surprising consequences. For instance, if increasing the quantity of clean inputs increases the productivity of dirty ones, this can reinforce the case for their use. The analysis of policy would be affected by such substitution patterns, because policies usually target some inputs and not others.

5.3 Dynamics

It is often argued that technical progress is a necessary ingredient to decouple economic growth from its environmental footprints, and, in particular, to increase food production while reducing the environmental externalities of farming. An interesting question related to the issue of input substitution is the direction of technical change and the orientation of agronomic research toward the productivity of certain inputs (e.g. Bommarco et al., 2013, on "ecological intensification").

Finally, the ecological dynamic of the model should be developed. The long history of farming in Europe is partly responsible for the current environmental situation, and the currently observed density–yield curve is the result of past choices. It would be helpful to obtain dynamic trajectories of farming practices associated with the evolution of the species density. It would also help to understand the impact of the irreversibility of some habitat destruction on the trade-off between land-sparing and land-sharing. Whether the quasi-option value associated with this irreversibility (Henry, 1974; Arrow and Fisher, 1974) either reinforces or reduces the case for land-sparing is an important research question.

6 Conclusion

This article has analyzed the trade-off between food production and environmental conservation. The growing human population raises concern about the difficulty of ensuring food security and protecting the environment. It seems that highly productive techniques (e.g., modern industrial farming) can ensure the former but may sacrifice the latter. This is not necessarily the case if these techniques allow land to be spared for nature. This issue has been debated hotly within conservation biology, framed as a choice between land-sparing and land-sharing.

This article has revisited the land-sparing vs land-sharing debate from an economic perspective. The density–yield curve was incorporated within an economic framework, in which a species is negatively affected by farming, and land can either be farmed or not. The analysis stressed the role of demand together with the role of the agricultural technique (choice of the yield) to internalize the value of a threatened species.

The total quantity produced and consumed should be reduced to internalize the value of an endangered species. Whether farming should be intensified and yield increased depends on the shape (concave vs convex) of the density–yield curve. The optimal yield can be increasing with respect to the value of a threatened species if this species is highly sensitive to the first increase of the yield (convex curve). It is then optimal to protect it by sparing land and increasing yield. Otherwise, even though it is optimal to reduce the yield, such reduction is not necessarily associated with expanded farmland if the demand for food is sufficiently elastic. A combination of agri-environmental schemes and natural reserves can be the optimal solution, and not either land-sparing or land-sharing, but both, as argued — on both side — in the conservation literature.

The optimal policy consists of subsidizing species density on both farmed and unfarmed land. Such policy decentralizes the reduction in food consumption and the choice of the optimal yield. Second-best policies were analyzed both to consider actual policies and to stress that the type of agriculture that should be promoted depends on the policy implemented.

Second-best policies are welfare enhancing under certain conditions on the density–yield curve and the demand elasticity. For instance, if the density of the species is decreasing with respect to a dirty input, it is optimal to tax this input and reduce the yield even in cases in which it would be optimal in a first-best setting to increase the yield. This is because the decrease of the yield is compensated by a decrease of the food consumed, which ensures that farmed area does not increase significantly. However, if the demand function is inelastic, then it may be optimal to subsidize a dirty input to spare land.

The analysis of the second-best setting, although highly stylized, shows that policy recommendations that are a priori true in a first-best setting are not necessarily true in second-best settings. People inspired by conservation purposes should not jump to the conclusion that a certain type of agriculture should be promoted because this type of agriculture is part of a first-best strategy. Since the reduction of total food production is an important element of the optimal strategy, any subsidy has adverse consequences and should be considered with caution.

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7 Proof of Lemma 1

The laissez-faireyield minimizes $c(y)/y$ it solves $c'(y)y = c(y)$, with eq. (5), $(c_1 + c_2 y)y = c_0 + c_1 y + c_2 y^2/2$, the solution of which is given by equation (6). The average cost is then: $c(y_0)/y_0 = c'(y_0) = c_1 + c_2 y_0 = c_1 + \sqrt{2c_0 c_2}$.

If $\bar{L}$ is sufficiently large, the equilibrium, is such that equation (7) is satisfied. Taking the derivative with respect to c_i, and injecting equation (1), gives

$$\frac{\partial y_0}{\partial c_i} L_0 + y_0 \frac{\partial L_0}{\partial c_i} = \frac{1}{P'(L_0)} \frac{\partial c'(y_0)}{\partial c_i} = -|\epsilon| \frac{y_0 L_0}{P} \frac{\partial c'(y_0)}{\partial c_i}$$

so

$$\frac{1}{L_0} \frac{\partial L_0}{\partial c_i} = -\frac{1}{y_0} \frac{\partial y_0}{\partial c_i} - |\epsilon| \frac{1}{c'(y_0)} \frac{\partial c'(y_0)}{\partial c_i} \tag{23}$$

- For c_0: y_0 is increasing with respect to c_0 and

$$\frac{1}{L_0} \frac{\partial L_0}{\partial c_0} = -\frac{1}{y_0} \frac{1}{\sqrt{2c_0 c_2}} - |\epsilon| \frac{1}{c_1 + \sqrt{2c_0 c_2}} \frac{\sqrt{c_2}}{\sqrt{2c_0}} < 0$$

- For c_1: y_0 does not depend on c_1 so

$$\frac{1}{L_0} \frac{\partial L_0}{\partial c_1} = -|\epsilon| \frac{1}{c_1 + \sqrt{2c_0 c_2}}$$

- For c_2: y_0 is decreasing with respect to c_2 with

$$\frac{\partial y_0}{\partial c_2} = -\frac{\sqrt{2c_0}}{2c_2 \sqrt{c_2}} = -\frac{y_0}{2c_2}$$

  injecting into equation (23) gives:

$$\frac{1}{L_0} \frac{\partial L_0}{\partial c_0} = \frac{1}{2c_2} - |\epsilon| \frac{1}{c_1 + \sqrt{2c_0 c_2}} \frac{\sqrt{c_0}}{\sqrt{2c_2}}.$$

  which is positive if and only if

$$|\epsilon| < \frac{c_1 + \sqrt{2c_0 c_2}}{\sqrt{2c_0 c_2}}.$$

8 Proofs of Proposition 1 and Corollary 2

Proof of Proposition 1

1. The optimal quantity of food satisfies

$$P(F^*) = \frac{c(y^*)}{y^*} + \beta\frac{b(0) - b(y^*)}{y^*}$$

The derivative of the right-hand side with respect to β is, by the envelop theorem, $[b(0) - b(y^*)]/y^*$, which is positive. Because the price function is a decreasing function, the optimal quantity of food is decreasing w.r.t. β.

2. The derivative of the right-hand side of equation (9) is $-b''(y)$, and the right-hand side is null at $y = 0$.

- If $b(.)$ is concave,

  the marginal environmental damage is increasing ($-b'' > 0$). Because it is null at $y = 0$, it is positive. At the optimum, a marginal change of β would increase the marginal environmental damage and subsequently decrease the optimal yield. (at the interior optimum, the second-order condition is satisfied, and the effect of a change of β on the optimal yield is the opposite of its effect on the right-hand side of (9)).

- If $b(.)$ is convex,

  the right-hand side of (9) is decreasing and null at zero; therefore, it is negative. The optimal yield is increasing with respect to β.

Proof of Corollary 2

The optimal farmed area is $L^*(\beta) = F^*(\beta)/y^*(\beta)$.

If the density–yield curve is convex, L^* is decreasing w.r.t. β because F^* is decreasing and y^* is increasing w.r.t. β.

If the density–yield curve is concave, let us write the β elasticity of the farmed area

$$\frac{\beta L^{*\prime}}{L^*} = \frac{\beta F^{*\prime}}{F^*} - \frac{\beta y^{*\prime}}{y^*}. \tag{24}$$

Using equation (8) and (9) gives

$$\frac{\beta F^{*\prime}}{F^*} = -\epsilon\frac{\beta(b(0) - b(y))}{c(y) + \beta(b(0) - b(y))} \text{ and } \frac{\beta y^{*\prime}}{y^*} = \frac{\beta}{y^*}\frac{b'y^* + (b(0) - b(y^*)}{(c'' - \beta b'')y^{*2}}.$$

Inserting these two equations into the expression (24) gives the result.

9 Proof of Proposition 2

The price of food is equal to the marginal cost. Taking the derivative of the first equation in (12) gives (by the envelop theorem) $P'F' = \partial\gamma/\partial r$, so

$$F' = \epsilon F \frac{\partial\gamma/\partial r}{\gamma(y,r)}.$$

Then, inserting the above equation into equation (15), the derivative of welfare is

$$\begin{aligned} \frac{dW}{dr} &= \beta\frac{F}{y}\left(b(0)-b(y)\right)\left[-\epsilon\frac{\partial\gamma/\partial r}{\gamma(y,r)} + \left(1+\frac{b'(y)y}{b(0)-b(y)}\right)\frac{y'}{y}\right] \\ &= \beta\frac{F}{y}\left(b(0)-b(y)\right)\frac{\partial\gamma/\partial r}{\gamma(y,r)}\left(\tilde{\epsilon}-\epsilon\right) \qquad \text{using (14).} \end{aligned} \tag{25}$$

Let us consider that $\partial\gamma/\partial r$ is positive; therefore, F' is negative.

- If $b(.)$ is concave, the effect of the yield $b(0) - b(y) + b'(y)y$ is negative.

  If y' is negative, then the two terms in the expression (15) of the derivative of welfare are positive, and a small increase of r has a positive effect.

  If y' is positive, the threshold $\tilde{\epsilon}$ is negative, and from (25), the derivative of welfare is positive if $\epsilon < \tilde{\epsilon}$ and negative otherwise.

- If $b(.)$ is convex, the effect of the yield $b(0)-b(y)+b'(y)y$ is positive. A similar reasoning gives the results for the second row of the table.

If $\partial\gamma/\partial r$ is negative, F' is positive. Symmetrical reasoning could be applied to obtain the last two lines of the table.