Designing conditional schemes for green industrial policy under different information structures

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Abstract

We assume that a project requires an initial outlay and may either succeed or fail. The probability of success depends on its type and on the effort of the firm. Only in the case of success do private and external benefits appear. The paper analyzes the optimal design of subsidies under different information structures the state agency and the firm may have over the characteristics of the project. It is proved that under symmetric information structures rewarding success is optimal while, ordinarily, under asymmetric ones, rewarding failure is optimal. While reward success encourages effort, rewarding failure mitigates windfall profit. In asymmetric structures, the second feature dominates. These results emphasize the crucial significance of properly identifying the underlying structure in designing an efficient incentive scheme. The policy relevance of our analysis is discussed in the context of risky programs such as those for the energy transition associated with COVID recovery plans.

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1 Introduction

We analyze the optimal way to subsidize a risky innovative project with scarce public funds. A project can generate both private and (external) social benefits, but it requires an initial funding. The regulatory intervention is justified by the external social benefits. Indeed, it is common to consider that innovation activities have positive spillovers of multiple sorts that justify subsidies at the various stages of the innovation process. Notably, pilot and demonstration plants are a key step between the lab and the industrial scaling, but they are risky and capital-intensive activities. Even though the analysis developed is general, the present work is motivated by the transition to a low-carbon economy. The recent COVID pandemics has triggered recovery plans in many countries. These plans are seen as an opportunity to promote the energy transition through a green industrial policy as it was the case after the 2007-2008 financial crisis (Rodrik; 2014).

Subsidies to low-carbon innovative projects are justified by both a lack of carbon pricing and knowledge externalities, likely to be large because of the youth and future growth of green technologies in the energy, mobility, and agricultural sectors (Greaker et al.; 2018; Hepburn et al.; 2020). Private companies do finance innovative projects, and scarce public funds should be targeted toward projects that would lack private funding but asymmetric information constraints the ability of a public agency to optimally screen projects. Public loans with a payback conditional on the project success are a common tool (e.g. Rodrik; 2014) that might help reduces rents. Rodrik (2014) revisits the traditional criticisms raised by economists (poor selection by government and rent-seeking) and illustrates its argumentation with several case studies. He notably stresses that failures and bankruptcies, far from being a sign of policy failure, are likely events given the intrinsic uncertainty of many projects. He notes that a public agency should give due attention to the information available to both parties along the deployment process, a point largely ignored in most papers.

We develop a partial equilibrium model in which a public agency acting on behalf of the state subsidizes risky projects carried out by a firm. The firm invests in a project that may succeed or fail, the probability of success depends upon the type of the project and the effort of the firm. Only successful projects generate private and social benefits. Without subsidies, some (low-type) projects would not be launched while others (high-type) would be. The agency can propose a couple of nonnegative subsidies conditional on success or failure.

Our aim is to investigate how the information structure influences the design of incentive schemes. We consider symmetric structures, where both the agency and the firm have identical information on the probability of success. Two extreme cases are studied with either imperfect or perfect information. We also consider an asymmetric information structure in which the firm is better informed than the agency. In the latter case an adverse selection issue arises on top of moral hazard.

Under perfect information, moral hazard coupled with the non-negativity of subsidies prevent the agency from implementing the first-best scheme. In that case, the agency should only subsidize success in order to incentivize the firm to make an effort (high-powered incentives). The success subsidy plays the dual role of both increasing the profitability of projects and the effort made by the firm. If the latter role dominates, the firm obtains a rent. Whereas the effort is distorted away from the first-best, the selection of projects is not, and all projects that would be deployed in a first-best situation are deployed in that configuration. With imperfect information, the situation is similar, both the firm and the agency ignoring the type of the project, and given the linearity of our model, it is *as if* they were facing a project with an average type.

Under asymmetric information, adverse selection explains the existence of windfall profit: some projects that would be financed even without subsidy are subsidized. A subsidy conditional on failure, which is similar to a loan, reduces these rents. This is so because high-type projects are more likely to succeed and thus receive a lower expected subsidy. The effort exerted by the firm is suboptimal and fewer projects are financed than under symmetric information. With adverse selection only, subsidy should only be conditional on failure and success not rewarded. When both adverse selection and moral hazard are at play, a delicate balance between adverse selection and the power of incentives determines the optimal scheme. The shape of the distribution of types plays an important role that we further investigate with two polar cases: a uniform and a binomial distribution. Strikingly, with a uniform distribution, under fairly general conditions, only failure should be subsidized and the incentives are low powered. With a binomial distribution, the structure of the scheme depends on the frequency of the high types, and the more frequent they are the lower the power of incentives.

Based on the previous results, we investigate the respective values of information for the agency and the firm to move from one information structure to another one. Through the detailed analysis of an illustrative example, we show that the firm benefits from acquiring more information, i.e., for the firm to move from a symmetric imperfect information environment to an asymmetric one, but then the agency should move from the asymmetric case to the symmetric perfect information case. The value of information along this process may be altogether negative for the agency but it is always an incremental best response. It would be positive if the social benefit is high, and then calibrating the scheme for all possible types to induce the effort of the firm is beneficial to the agency, in spite of the fact that the constraint for inducing the firm to launch the project would be more stringent than when the high-type rents are averaged with low-type losses.

Several articles in environmental economics discuss the issue of financing green projects under asymmetric information. Fischer (2005) provides an insightful analysis of the issue of "additionality" in the design of CDM.[1] Mason and Plantinga (2013) consider the optimal design of contracts for carbon offsets with asymmetric information.[2] To our knowledge, the issue of windfall profits from innovative risky green projects has not been studied.[3]

[1]Zhang and Wang (2011) empirical analysis does cast some doubt about the additionality of Chinese CDM.

[2]Within the agricultural sectors, the design of agri-environmental schemes raises similar issues (e.g. Wu and Babcock; 1996; Engel et al.; 2008), and a related concerns is the "stacking" of green payments: a farmer maybe rewarded twice, for biodiversity and greenhouse gases reduction for the same action (Woodward; 2011; Lankoski et al.; 2015).

[3]There is a large literature on the coordination between environmental and innovation policy in dynamic models, most notably endogenous growth models, Smulders et al. (2014) provide a survey, and Greaker et al.

From a more theoretical perspective, our analysis is related to the literature on mechanism design with both adverse selection and moral hazard.[4] In this respect our model features a risk-neutral principal and a risk-neutral agent, and constrained incentive schemes. The principal (the agency) is constrained to propose a single couple of non-negative subsidies. A key element of our analysis is that some projects would be financed without the scheme, making the participation constraint dependent on the type. Some papers have introduced some of these restrictions (e.g. Lewis and Sappington; 2000a,b; Quérou et al.; 2015). The article by Ollier and Thomas (2013) is the closest to the present work; it introduces ex-post participation constraint (the firm should recover its cost even if the project fails) in a mixed model relatively similar to ours.[5] They notably show that because of countervailing incentives pooling is optimal and the principal should only reward success. This is the case in our setting when moral hazard issues dominate, but we also show that there are situations in which subsidies are used in both cases or there is a reward only in the case of failure. The key difference is the absence of a fixed cost in Ollier and Thomas (2013), which limits adverse selection issues: there is no need to finance low-profitability projects but only to motivate efforts. Finally, rewarding failure could also be justified by the risk-aversion of the agent, as in the work of Gary-Bobo and Trannoy (2015) on students loans, but this justification is absent from our model since we consider risk-neutral actors (both the public agency and firms).

The rest of the paper is organized as follows: In Section 2 the general model and the various information structures are introduced. In Section 3 we study the optimal scheme for each information structure, and a complete resolution for two specific probability distributions of success, namely uniform and binomial. An illustrative example is studied at length in Section 4 to discuss the respective values of information. Section 5 generalizes

(2018) is a recent contribution that highlights that even with a long-lived patent green research subsidies are justified.

[4]Mixed models are covered in Chapter 7 in Laffont and Martimort (2002).

[5]In Subsection 5.2. they replace the ex-post participation constraint by a limited liability constraint, making their model closer to ours.

our results to two simple extensions: the introduction of an uncertain market phase in the deployment process of the project, and imperfect observability of the outcome of the project by the agency. Policy implications and extensions are discussed in the last section.

2 The model

2.1 The general setting

Consider the following situation. A given innovative project may or may not be initiated by a firm. The decision to initiate the project is represented by a binary variable $\delta \in 0, 1$. If $\delta = 1$, the project is initiated and the firm incurs a fixed cost F. The project either succeeds or fails. In case of success, the firm gets a private revenue R, and a social external benefit b is generated. In case of failure neither private nor external benefits are created. If a project is not initiated, $\delta = 0$, the reference payoffs are zero, and no fixed cost is incurred. The probability of success depends on the type of the project θ and the effort of the firm e with θ and $e \in [0, 1]$: $p(e, \theta) \in [0, 1]$, and $p(0, \theta) = \theta$. Types are distributed according to the cumulative distribution function $G(\theta)$, continuously differentiable with $G'(\theta) = g(\theta)$. The effort e induces a cost $f(e, \theta)$. The cost function is assumed to be increasing with respect to e and decreasing with respect to θ, and the cross derivative is negative (the marginal cost to increase the probability of success is decreasing with the type).

The firm and the regulatory agency (henceforth the agency) know F, R, b, $G(\theta)$, and $f(e, \theta)$. Both entities observes whether a project is initiated and its outcome, i.e., failure or success. The agency does not observe the effort of the firm. We will consider several cases regarding the information about the type θ of the project.

The question at stake for the agency is to select an incentive scheme (s_1, s_2). The firm may either accept or reject the incentive scheme. If it accepts the scheme it initiates the project ($\delta = 1$), makes an effort e, and gets s_1 in case of success (probability $p(e, \theta)$) and s_2 in case of failure (probability $1 - p(e, \theta)$. Therefore, we refer to s_1 as rewarding success and to s_2 as rewarding failure. For the sake of realism, both s_1 and s_2 are nonnegative. And without loss of generality, we certainly have:

$$0 \leq s_1 \leq b \text{ and } 0 \leq s_2 \leq F.$$

For convenience we also define the bonus as $s = s_1 - s_2$ so that: $0 \leq s \leq b$.

The profit of the firm if the project is of type θ is

$$\pi(\delta, e, \theta, s_1, s_2) = \delta[p(e, \theta)(R + s_1) + (1 - p(e, \theta))s_2 - (F + f(e, \theta))]. \tag{1}$$

Under similar conditions, the surplus of the agency is written as:

$$v(\delta, e, \theta, s_1, s_2) = \delta[p(e, \theta)(b - s_1) - (1 - p(e, \theta))s_2]. \tag{2}$$

We also introduce the welfare, which is the sum of the agency surplus and firm profits:

$$w(\delta, e, \theta) = v + \pi = \delta[p(e, \theta)(R + b) - F - f(e, \theta)]. \tag{3}$$

The non-negativity constraint on subsidies implies that in most cases considered the agency will only be able to implement a second-best solution. For the sake of comparison, we shall also identify the first-best solution.

2.2 The information structures and the respective incentive programs

An information structure specifies the information of the agency and the firm about the type of the project, i.e., its probability of success. We consider symmetric and asymmetric structures. For symmetric ones, either both players have perfect information about the type (Case 1) or they do not know the type, and their information is called imperfect (Case 2).

For an asymmetric structure the firm knows its type but the agency does not (Case 3). For Cases 1 and 2 there is a moral hazard issue. For Case 3 there are both moral hazard and adverse selection issues. In all cases the firm chooses to initiate the project or not, $\delta = 1, 0$, and its effort e by maximizing its profit. The agency maximizes its expected surplus.

We now formalize the program to be solved for each information structure.

- **Case 1**: Perfect information.

  For each θ the agency selects $(s_1(\theta), s_2(\theta))$ in order to maximize

$$v(\delta, e, \theta, s_1, s_2)$$

  subject to

$$(\delta, e) = \text{argmax} \pi(\delta, e, \theta, s_1, s_2).$$

- **Case 2**: Imperfect information

  The agency selects (s_1, s_2) to maximize

$$\int_0^1 v(\delta, e, \theta, s_1, s_2) g(\theta) d\theta$$

  subject to

$$(\delta, e) = \text{argmax} \int_0^1 \pi(\delta, e, \theta, s_1, s_2) g(\theta) d\theta.$$

- **Case 3**: Asymmetric information

  The agency selects (s_1, s_2) to maximize

$$\int_0^1 v(\delta, e, \theta, s_1, s_2) g(\theta) d\theta$$

  subject to

$$(\delta, e) = \text{argmax } \pi(\delta, e, \theta, s_1, s_2), \ \forall \theta \in [0, 1].$$

2.3 The specification for the effort function $f(e, \theta)$

We will make use of the following linear quadratic specification to be defined for $0 \leq e \leq 1$:

$$p(e, \theta) = \theta + e(1 - \theta) \tag{4}$$

$$f(e, \theta) = (1 - \theta)\frac{\gamma}{2}e^2 \tag{5}$$

This formulation can be motivated by considering that a project is constituted of a continuum of technical steps: for a project of type θ a share θ of steps have already been completed (in the lab) and $(1-\theta)$ steps remain to be completed (with the pilot) to guarantee success. A given level of effort has a larger impact on projects with an initially low probability of success, but it is more costly.[6]

The profit of a firm could be rewritten as:

$$\pi(1, e, \theta, s_1, s_2) = [\theta(R + s_1) + (1 - \theta)s_2 - F] + (1 - \theta)\left[e(R + s_1 - s_2) - \frac{\gamma e^2}{2}\right].$$

The net benefit from effort is encompassed in the last bracketed term. The effort exerted by a firm does not depend on the type θ but only on the bonus s. It is:

$$e(s) = \min\{\frac{R + s}{\gamma}, 1\}. \tag{6}$$

This property greatly facilitates the analysis because the effort of the firm will not depend on the information available to the firm. Observe that the property is also true for the effort that maximizes welfare: it is simply equal to $\min\{\frac{R+b}{\gamma}, 1\}$.

[6] Rewriting cost as a function of the probability of success gives:

$$\phi(p, \theta) = \frac{\gamma}{2}\frac{(p - \theta)^2}{1 - \theta}, \text{ for } p > \theta, = 0 \text{ otherwise.}$$

The function $\phi(p, \theta)$ satisfies the technical assumptions in Ollier and Thomas (2013). It is increasing with respect to p and decreasing with respect to θ, and the cross derivative is negative (the marginal cost to increase the probability of success is decreasing with the type), which ensures that the probability of success is decreasing with the type, for a given bonus s.

The following technical assumption will be needed in some results. It ensures that all efforts considered are strictly lower than 1.

Assumption 1. *The slope of the marginal cost of effort is larger than the marginal social benefit: $\gamma > R + b$.*

2.4 Business As Usual, First Best, Pigouvian Solution

Business As Usual refers to the situation in which there is no subsidy and firms are informed about their types, while First Best refers to the allocation that maximizes welfare. For each project there are two choices: whether to initiate the project and the level of effort. Business As Usual and First Best could be described by threshold types, respectively denoted θ^{BAU} and θ^{FB}, such that projects with a larger type are initiated, and by effort levels e^{BAU} and e^{FB}.

The threshold θ^{BAU} is the lowest type, such that a project is initiated by an informed firm without any regulation $\pi(1, e(0), \theta, 0, 0) \geq 0$. With our specification, $e^{BAU} = R/\gamma$ and the profit is $\pi(1, e, \theta, 0, 0) = [\theta R - F] + (1 - \theta)R^2/2\gamma$ so that:

$$\theta^{BAU} = \frac{1}{R}\frac{2F\gamma - R^2}{2\gamma - R}. \tag{7}$$

The following assumptions make the problem interesting. They ensure that some projects are initiated without subsidy while some are not.

Assumption 2. *Some projects are profitable without subsidies: $F < R$.*

Assumption 3. *Not all projects are initiated without subsidies: $R^2 < 2\gamma F$.*

As regards First Best, $e^{FB} = (R + b)/\gamma$, and all projects with $p(e^{FB}, \theta)(R + b) \geq F + f(e^{FB}, \theta)$ are initiated. The threshold θ^{FB}, if positive, solves $p(e^{FB}, \theta^{FB})(R + b) = F + f(e^{FB}, \theta^{FB})$. We have:

$$\theta^{FB} = max\left\{\frac{1}{R + b}\frac{2F\gamma - (R + b)^2}{2\gamma - (R + b)}, 0\right\} \tag{8}$$

Note that $e^{FB} > e^{BAU}$ and $\theta^{FB} \leq \theta^{BAU}$. Furthermore $\theta^{FB} \geq 0$ if and only if $(R+b)^2 \leq 2F\gamma$ and, as b increases, θ^{FB} decreases from θ^{BAU} to 0.

Subsidies will have the dual role of triggering the initiation of projects and motivating effort. In each information structure, the optimal second best scheme relative to the first best may induce a selection bias–projects that should be socially implemented are not; a suboptimal effort by the firm; and, therefore, welfare may not be maximized.

In order to clarify the difference between the schemes analyzed here and standard Pigouvian regulation, it is worth considering the optimal regulation with an unlimited set of instruments. With informed firms (Cases 1 and 3), the first best can be decentralized with $s_1 = b$ and $s_2 = 0$, which corresponds to a Pigouvian subsidy.[7] However, the agency surplus is not maximized with such a scheme and firms get a rent. If the agency were able to tax profits with a proportional tax then a 100% profit tax realigns the agency objective with social welfare ($V = W$ and $\Pi = 0$), and $s_1 = b$ both implements the first best and maximizes the agency surplus ($V = W^{FB}$); information asymmetry is then irrelevant. This result resonates with the fact that in a Ramsey optimal taxation framework the optimal corporate tax on pure profit is 100% (Munk; 1978). And the present framework can be interpreted as an optimal taxation exercise with fairness concerns but a limited set of instruments.

3 Solving the incentive programs for the different information structures

3.1 Case 1: Perfect information

In this pure moral hazard setting the type of the project θ is known by the firm and the agency, but the agency can observe neither the effort nor its cost. We shall show that rewarding success only is the second best solution and detail the corresponding scheme. The

[7]The fixed cost F does not by itself justify the implementation of a subsidy because projects are infinitesimally small.

first best is not achieved.

We now describe the optimal second best scheme (s_1, s_2) as a function of the type θ of the project. The agency should decide whether to ensure the deployment of a project as soon as $\theta \geq \theta^{FB}$ and whether to further motivate effort. First, if the agency ensures the deployment of a project, it is optimal to do so by rewarding success and not failure because it maximizes the effort of the firm. Second, three cases may occur: the optimal subsidy could be null for $\theta \geq \theta^{BAU}$, it could ensure a null profit to the firm for $\theta < \theta^{BAU}$, or it could be larger to further increase the effort. Let us denote $s_{1B}(\theta)$ as the subsidy that ensures a null profit, and $s_{1A}(\theta)$ as the subsidy that maximizes the agency surplus without the profit positivity constraint (note that $v(1, e(s_1), \theta, s_1, 0)$ is a quadratic function of s_1). It is easily seen that:

$$s_{1B}(\theta) = \gamma \frac{\theta}{1-\theta} \left[\sqrt{1 + \frac{2F}{\gamma} \frac{1-\theta}{\theta^2}} - 1 \right] - R \tag{9}$$

and

$$s_{1A}(\theta) = \frac{b-R}{2} - \gamma \frac{\theta}{2(1-\theta)} \tag{10}$$

Note that $s_{1A}(\theta)$ is relevant only if $s_{1A}(\theta) \geq \max\{s_{1B}(\theta), 0\}$; otherwise the project is not initiated. The best of the three occurrences depends on the values of the parameters b, γ, R, and F. Since $s_{1A}(\theta)$ is increasing with b while $s_{1B}(\theta)$ is independent of b, for given values of γ, R and F, there will be a critical value of b, denoted as $b^*(\theta)$, such that $s_{1A}(\theta)$ should be preferred for $b \geq b^*(\theta)$ while $s_{1B}(\theta)$ should be preferred if $b \leq b^*(\theta)$.

The following proposition holds.

Proposition 1. *At the optimal scheme $(s_1^*(\theta), s_2^*(\theta))$ only success is rewarded ($s_2^* = 0$), and a project is initiated if and only if $\theta \geq \theta^{FB}$.*

For all $\theta \geq \theta^{FB}$, the optimal subsidy is conditional on success and equal to $\max\{s_{1A}, s_{1B}, 0\}$, *with s_{1A}, s_{1B} given by equations (10) and (9).*

If b is sufficiently large and F small, $s_1^* = s_{1A} > 0$ and the firm gets a positive profit. Otherwise, the firm gets its BAU profit which is null for $\theta < \theta^{BAU}$.

The proof is in Appendix A.1. We can interpret the choice of the agency as follows. If $\theta^{BAU} \leq \theta$, it may be worthwhile to induce more effort by selecting $s_{1A}(\theta)$ rather than $s_1 = 0$. If $\theta \leq \theta^{BAU}$, $s_{1B}(\theta)$ compensates the firm for its private loss while $s_{1A}(\theta)$ induces a larger effort and gives some profit to the firm. Both $s_{1A}(\theta)$ and $s_{1B}(\theta)$ are decreasing functions of θ but their ranking may not be monotonous with respect to θ. Figure 1 illustrates a situation in which the best scheme is to choose s_{1A} for low values of θ then s_{1B} up to $\theta = \theta^{BAU}$ and then $s_1 = 0$. As b decreases, it may be that the optimal sequence is s_{1B}, s_{1A}, s_{1B}, $s_1 = 0$. For low b it will be s_{1B}, $s_1 = 0$.

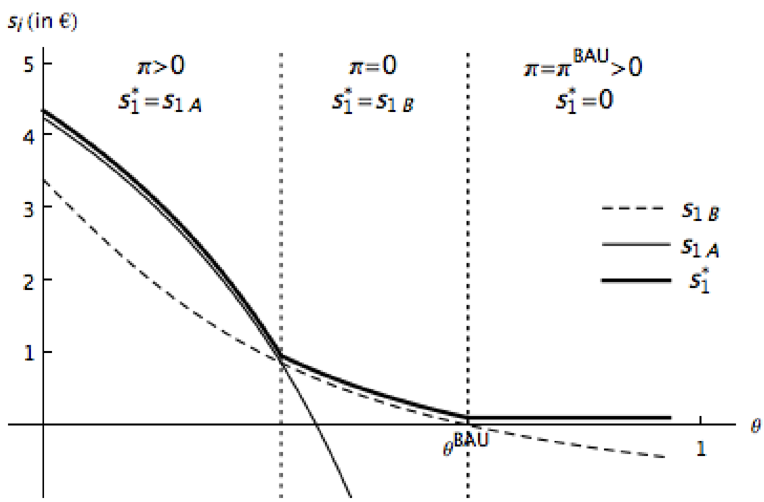

Figure 1: The optimal subsidy s_1^* ($s_2^* = 0$) with respect to θ for $R = 1.5$, $F = 1$, $\gamma = 12$ and $b = 10$ ($\theta^{FB} = 0$ so that all types should be initiated).

The first best cannot be achieved with an incentive scheme that respect the non negativity constraints but it can be with a negative subsidy s_2. At the first best welfare is maximized and the firm gets its BAU profit, otherwise it does not adhere to the scheme. The proof of this lemma is straightforward.

Lemma 1. *The first best is obtained with a scheme such that:*

- *if $\theta \leq \theta^{FB}$, no subsidy is proposed and the project is not initiated,*

- *if $\theta \geq \theta^{FB}$, the optimal scheme is such that $s_1 - s_2 = b$ and $\pi = \pi^{BAU}$.*

The agency surplus is then $v = p(e, \theta)(b - (s_1 - s_2)) - s_2 = -s_2$, and the subsidy s_2 is *negative and could be interpreted as a tax on undue profit.*

For completeness, note that a flat subsidy has no influence on the effort, which remains at e^{BAU}. It can only be optimal if the agency would not observe the outcome of the project.

3.2 Case 2: Imperfect information for both

Because both π and v are linear with respect to θ this case is a simple replication of Case 1 in which θ is replaced by its expected value.

Let us denote $\bar{\theta} = \int_0^1 \theta g(\theta) d\theta$, the expected type. For an uninformed observer, either the firm or the agency, the probability of success of a project for an effort e is :

$$\int_0^1 p(\theta, e) g(\theta) d\theta = p(\bar{\theta}, e).$$

And the expected profit of a firm is

$$\begin{aligned}\int_0^1 \pi(\delta, e, \theta, s_1, s_2) g(\theta) d\theta &= \int_0^1 \delta\Big[p(\theta, e)(R + s_1) + (1 - p(\theta, e))s_2 - F\Big] g(\theta) d\theta \\ &= \pi(\delta, e, \bar{\theta}, s_1, s_2).\end{aligned}$$

And similarly for the agency, so that the situation is *as if* information is perfect and the type equal to $\bar{\theta}$, its expected value. Proposition 1 applies, replacing θ with $\bar{\theta}$: only success is subsidized, the project is initiated if and only if $\bar{\theta} \geq \theta^{FB}$, and the firm gets a rent (a profit larger than its BAU value) if the external benefit b is sufficiently large.

3.3 Case 3: Asymmetric information

This case is clearly the most complex one. We cannot completely solve it in full generality but we can get some important insights into the solution. First, we consider a “pure” adverse selection situation, without any effort, and demonstrate that the optimal scheme is to only reward failure.

Second, we derive the solution under two specific probability distributions for θ: either a uniform distribution or a binomial one over two possible θ, a low probability of success or a high probability of success. We show that rewarding failure is a robust solution with a uniform distribution, whereas, with a binomial distribution, the optimal solution would be to reward success if the probability of success is low while it would be to reward failure if the probability of success is high. For intermediate situations both subsidies would be used.

3.3.1 Pure adverse selection

The problem of additionality and windfall profit appears in its simplest form in a pure adverse selection problem, in which no efforts is exerted.[8] We shall assume in that section that $\gamma = +\infty$, so $e = 0$ and $p = \theta$.

The agency does not know the ex-ante probability of success $p = \theta$ of a given project. The sole remaining purpose of the incentive scheme is to select projects to be initiated. The optimal second best scheme consists in rewarding failure and it does not get the first best. We shall further discuss how it departs from a flat subsidy $s_1 = s_2$.

The threshold type $\tilde{\theta}$ at which expected profit is null is (from eq 1 with $e = 0$) :

$$\tilde{\theta}(s_1, s_2) = \frac{F - s_2}{R + s_1 - s_2}. \tag{11}$$

If $R + s_1 > s_2$, as will be the case at relevant schemes, all $\theta \geq \tilde{\theta}$ will be initiated. The agency surplus can be rewritten:

$$V(s_1, s_2) = \int_{\tilde{\theta}(s_1,s_2)}^{1} [\theta(b - s_1) - (1 - \theta)s_2]g(\theta)d\theta. \tag{12}$$

[8] To be fully rigorous, in a standard adverse selection model effort would be exerted and contractible, the agency would propose a contract e, s_1, s_2 to firms, it would be relatively similar to the case without effort since all firms would make the same effort, but with an additional regulatory variable.

The first best threshold type is:

$$\theta^{FB} = \frac{F}{R+b}, \tag{13}$$

and the threshold type without any subsidy is:

$$\theta^{BAU} = \tilde{\theta}(0,0) = \frac{F}{R} \tag{14}$$

Decompose the problem of the agency in two steps.

Step 1: given a targeted threshold probability θ^t the agency minimizes the expected cost of the subsidy:

$$C(\theta^t) = \min_{s_1,s_2} \int_{\theta^t}^1 \max\{\theta s_1 + (1-\theta)s_2, 0\} dG(\theta),\ s.t.\ \tilde{\theta}(s_1,s_2) = \theta^t.$$

It will be enlightening to make explicit the impact of non negativities for the subsidies. If the subsidy could be negative, then firms might be better off investing without subscribing to the scheme, and they do so if the expected subsidy is negative. This possibility explains the maximum function in the integrand. Indeed, if the subsidies are constrained to be non negative then an investing firm subscribes to the scheme. This will give rise to Lemma 2.

Step 2: the optimal choice of θ^t maximizes $V = b\int_{\theta^t}^1 pg(p)dp - C(p^t)$. This will give the proposition that follows.

Lemma 2. *Whatever the targeted threshold type θ^t, the scheme that minimizes the expected cost of the subsidy is:*

- *For non negative subsidies, $s_1 = 0$ and $s_2 = (F - \theta^t R)/(1-\theta^t)$. The profit of a firm of type $\theta \in (\theta^t, 1]$ is positive, and the agency surplus is lower than welfare minus BAU profit.*

- *For unconstrained subsidies, $s_2 = F$ and $s_1 = F - R + \epsilon$ with ϵ infinitely small. Then* *the profits of firms that subscribe to the subsidy are null, and the surplus of the agency is equal to welfare minus the BAU profit:* $W(\theta^t) - \Pi^{BAU}$.

The proof is in Appendix A.2.

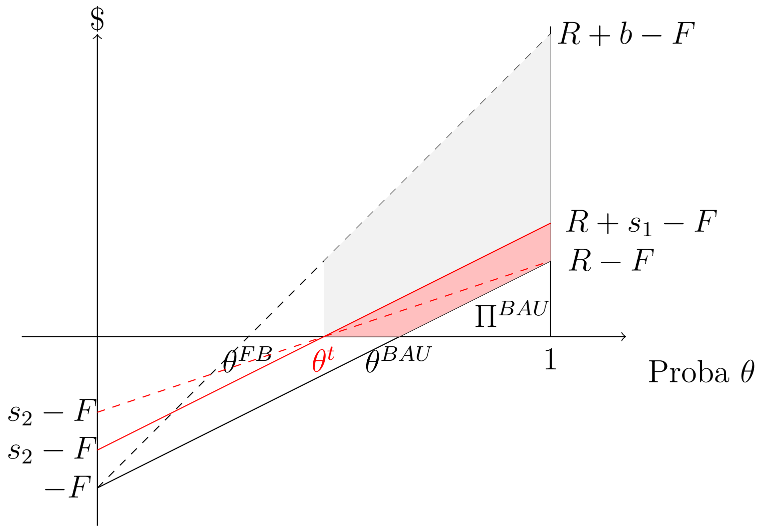

Figure 2: Expected subsidy as a function of the firm type: the red area is equal to the total expected subsidy (weighted by $g(\theta)$).

The result of Lemma 2 is illustrated in Figure 2. Given a couple s_1, s_2 the red area corresponds to the total expected subsidy, and the dashed line depicts a change of the subsidy line associated with an increase of s_2 and a reduction of s_1 that leaves the threshold firm unchanged. As can be seen such a change reduces the total expected subsidy by reducing the expected subsidy obtained by high-type firms. High-type firms succeed more frequently than the threshold type. They more frequently get the subsidy in case of success, and less frequently the subsidy in case of failure; the expected subsidy is then reduced by rewarding more failure and less success. At the extreme it is optimal to reward only failure in order to limit windfall profit.

We shall now show that without positivity constraints, the optimal value of θ^t is θ^{FB} and the first best is achieved, while $\theta^{FB} \leq \theta^t \leq \theta^{BAU}$ with constraints. Let us denote θ^{SB} as the optimal value of $\theta^t(s_1, s_2)$ in the second best approach. Indeed the following proposition holds:

Proposition 2. *At the optimal scheme*

- *For non negative subsidies, the first best is not achieved, the optimal scheme rewards failure only with $s_1 = 0$, and $s_2 \geq 0$ is such that:*
  - *(i) $s_2 = 0$ and $\theta^{SB} = \theta^{BAU}$ if*

$$b \leq \frac{R^3}{F(R-F)} \int_{F/R}^{1} (1-\theta) g(\theta) d\theta \tag{15}$$

  - *(ii) otherwise $s_2 > 0$ and $\theta^{FB} \leq \theta^{SB} \leq \theta^{BAU}$ with θ^{SB} defined by the following implicit equation:*

$$\theta^{SB} = \theta^{FB} + \frac{1}{g(\theta^{SB})} \frac{R-F}{b+R} \int_{\theta^{SB}}^{1} \frac{1-\theta}{(1-\theta^{SB})^2} dG \tag{16}$$

- *For unconstrained subsidies, the optimal scheme is such that $\theta^t = \theta^{FB}$, and the first best is achieved. The profit of firms that subscribe to the scheme is null, and the agency surplus is equal to $W^{FB} - \Pi^{BAU}$.*

See Appendix A.3 for the proof. It is relatively straightforward to establish that a menu of subsidies cannot improve the situation whenever Assumption 1 holds. Whatever the initial subsidy couple proposed (s_1, s_2), there is no room for maneuver: the agency cannot propose another couple (s_1', s_2') that would be both more interesting to a firm of type $\theta > \tilde{\theta}(s_1, s_2)$ and less costly to the agency. The first condition is equivalent to $\theta s_1' + (1-\theta)s_2' > \theta s_1 + (1-\theta)s_2$ and the second to $\theta s_1' + (1-\theta)s_2' < \theta s_1 + (1-\theta)s_2$. Note that the above reasoning does not rest on the positivity constraints but on the risk neutrality of the principal and the agent.

3.3.2 Some general results

If the cost of effort is finite, both adverse selection and moral hazard are at work, and the optimal scheme cannot be characterized without additional assumptions on the distribution of types. The analysis of the general situation helps us to understand which mechanisms justify the subsidization of success or failure.

Again, we denote $\tilde{\theta}(s_1, s_2)$, the threshold type such that a project is initiated if and only if its type is above that threshold.[9] The effort is $e(s_1 - s_2)$, given by equation (6), and $\tilde{\theta}$, if positive, is the solution of $\pi(1, e, \tilde{\theta}, s_1, s_2) = 0$, that is:

$$p(e, \theta)(R + s_1 - s_2) + s_2 - (F + f(e, \theta)) = 0 \tag{17}$$

The agency surplus is:

$$V = \int_{\tilde{\theta}}^{1} [p(e, \theta)(b - s_1) - (1 - p(e, \theta))s_2]g(\theta)d\theta. \tag{18}$$

The choice of either of the subsidies s_1 and s_2 has three effects: (i) on the selection of projects via its influence on $\tilde{\theta}$, (ii) on the effort via $s_1 - s_2$, and (iii) on the total expected transfer to firms. As is usual in agency problems, the agency trades off efficiency for rents.

It is illuminating to isolate the selection of projects from the precise design of subsidies. Instead of considering the two variables s_1 and s_2, we rewrite the profit of the firm and agency surplus as functions of s and $\tilde{\theta}$. Injecting equation (17) into the expression (1) gives the profit of a firm as a function of s and $\tilde{\theta}$:

$$\pi = [p(e, \theta) - p(e, \tilde{\theta})](R + s) - [f(e, \theta) - f(e, \tilde{\theta})] \tag{19}$$

And, with a slight abuse of notation, the agency surplus can be rewritten:

$$\begin{aligned} V(s, \tilde{\theta}) = \int_{\tilde{\theta}}^{1} \Big\{ &[p(e, \theta)(R + b) - (F + f(e, \theta))] \\ &- [p(e, \theta) - p(e, \tilde{\theta})](R + s) + [f(e, \theta) - f(e, \tilde{\theta})] \Big\} g(\theta)d\theta. \end{aligned} \tag{20}$$

[9] For any subsidy couple (s_1, s_2) we have $0 < R + s_1 - s_2$ since $s_2 \leq F < R \leq R + s_1$. It follows that $\pi(1, e, \theta, s_1, s_2)$ is increasing with respect to θ so that projects with a type above a threshold are initiated, and those below are not.

For a given $\tilde{\theta}$, a change of s has the following effect on the agency surplus:[10]

$$\begin{aligned}\frac{\partial V}{\partial s} &= \int_{\tilde{\theta}}^{1}[p_e(R+b)-f_e]e'(s)dG(\theta)-\int_{\tilde{\theta}}^{1}[p(e,\theta)-p(e,\tilde{\theta})]dG(\theta)\\ &= \int_{\tilde{\theta}}^{1}[p_e(b-s)]e'(s)dG(\theta)-\int_{\tilde{\theta}}^{1}[\theta-\tilde{\theta}]dG(\theta)(1-e)\\ &= \frac{1}{\gamma}\int_{\tilde{\theta}}^{1}\Big\{(1-\theta)(b-s)-(\theta-\tilde{\theta})(\gamma-R-s)\Big\}g(\theta)d\theta. \end{aligned} \tag{21}$$

The first line makes use of $p_e(R+s)=f_e$ for all θ, which cancels the influence of s via e on the second line of eq. (20). In the second line, f_e is replaced by $p_e(R+s)$ for all θ, and the third line makes use of equation (6).

There are two effects: effort is increased (first term) and the expected subsidy transferred to firms is increased (second term). The expected subsidy is increased because a high-type firm is more likely to succeed $p(e,\theta)>p(e,\tilde{\theta})$ and get the s_1 subsidy. So any change of the scheme that transfers subsidy from failure to success while keeping constant the expected subsidy of the threshold firm has a positive effect on the expected subsidy of initiated projects. This gives the following proposition, which characterizes the optimal second best threshold θ^{SB}.

Proposition 3. *The optimal couple (s_1^*, s_2^*) is such that the bonus $s^*=s_1^*-s_2^*$ is lower than b, the effort exerted by firms is then suboptimal and fewer projects are selected than in the first-best $\theta^{SB}>\theta^{FB}$.*

Furthermore if both subsidies are positive, they satisfy:

$$\int_{\tilde{\theta}}^{1}(1-\theta)dG(\theta)(b-s^*)=\int_{\tilde{\theta}}^{1}[\theta-\tilde{\theta}]dG(\theta)(\gamma-(R+s^*)), \tag{22}$$

[10] A change of s that keeps $\tilde{\theta}$ fixed is equivalent to a change of s_1 and a corresponding change of s_2 with $ds=ds_1-ds_2$, and from eq. (17) we get $p(e,\tilde{\theta})ds_1+(1-p(e,\tilde{\theta}))ds_2=0$. A change of $\tilde{\theta}$ for a given s only necessitates a change of s_2 exactly offset by a change of $ds_1=ds_2$.

and θ^{SB} *solves*

$$p(e,\theta)(R+b) - [F + f(e,\theta)] = [1 - G(\theta)][p_\theta(e,\theta)(R+s) - f_\theta(e,\theta)]. \tag{23}$$

The proof is in Appendix A.4. In Appendix A.10, we characterize the optimal menu when subsidies are not constrained and firms need the regulator's consent to initiate a project. Equation (22) exhibits the trade-off between efficiency (left-hand side) and rent extraction (right-hand-side), and it is reminiscent of the equation satisfied by the optimal menu (cf Appendix A.10).

Several comments are in order that point out the significance of restricting subsidies to be non negative. First, if subsidies are restricted to be non-negative, the expected subsidy received by a firm is positive whatever its type, which is not necessarily true if subsidies can be negative. If $s_1 < 0$, then high-type firms will not subscribe to the scheme and their projects will be initiated with purely private funding. A second threshold should then be introduced for projects that do not subscribe to the scheme.

Second, the disentangling between the choice of $\tilde{\theta}$ and the bonus s is feasible as long as neither non-negativity constraints on subsidies is binding. If one of these two constraints is binding, either $s_1 = 0$ or $s_2 = 0$, then the choices of the bonus and the threshold type can no longer be made independently. This will be further illustrated in the next section.

Third, the surplus of the agency $V(s,\tilde{\theta})$ is not necessarily concave with respect to s without further assumptions on the distribution of types. This non-concavity arises because a larger bonus induces more efforts which reduces the gap between the probabilities of success of high-type and low-type projects and thus the rent to high-types. Formally, from equation (21), the agency surplus is quadratic with respect to s with a second-order coefficient:

$$\int_{\tilde{\theta}}^{1} [(\theta - \tilde{\theta}) - (1-\theta)] g(\theta) d\theta = \int_{\tilde{\theta}}^{1} (\theta - \tilde{\theta}) \Big[g(\theta) - g(1 + \tilde{\theta} - \theta) \Big] d\theta \tag{24}$$

the sign of which depends on the shape of G and the threshold $\tilde{\theta}$.[11]

3.3.3 Moral hazard and adverse selection with a uniform distribution on θ

The following two propositions provide a first remarkable insight on the qualitative structure of the solution. With a uniform distribution of types over $[0, 1]$, the coefficient of s in $\frac{\partial V}{\partial s}$ is null (eq.21), and the surplus of the agency V is either everywhere increasing or decreasing with respect to s, whatever the threshold $\tilde{\theta}$. The following proposition can then be deduced.

Proposition 4. *With a uniform distribution of types θ over $[0, 1]$, $s_1^* = 0$ and $s_2 > 0$, only failure only should be subsidized.*

The proof is in Appendix A.5. This proposition may be interpreted as follows. On the one hand, a positive subsidy s_1 encourages effort, which is the more valuable the lower the θ and the higher the b. On the other hand, it opens the way for windfall profit for high-type projects. With a uniform distribution on θ, these two effects annihilate each other and a positive subsidy s_2 is good enough. The agency is better off rewarding only failure.

The robustness of this result depends on two features. First, as long as the probability distribution over θ remains sufficiently flat and large, we may expect that the result will remain true. Second, the importance of Assumption 1: $\gamma > (R + b)$ should be stressed in the reasoning. The cost of effort should be sufficiently large or the social benefit should be sufficiently low to ensure that it is not optimal that all projects succeed with probability 1. Otherwise, the optimal bonus should be high enough to ensure that the probability of success equals 1 and the selection of projects is only a matter of cost comparison. The following proposition may be seen as a counterpoint to Proposition 4: if encouraging effort is not too costly, or if the social benefit is quite large, all projects should be encouraged and will succeed, and the adverse selection problem evaporates. Interestingly, in that case the

[11] In general, multiple local maxima might arise, and a side consequence is that even if the optimal couple of subsidy within S^+ is composed of positive subsidies, the optimal couple of subsidies within the broader class S can be different and have a negative component.

subsidy in case of failure is never paid, since all projects succeed, but still it is necessary to incentivize firms to exert a proper effort.

Proposition 5. *If Assumption 1 is not satisfied, i.e. $\gamma < R + b$, there is a local maximum for unconstrained subsidies such that all projects succeed with probability* 1*:*

$$s = \gamma - R \Rightarrow e(s) = 1,$$

and

$$R + b - (F + f(1, \tilde{\theta})) = \frac{1 - G(\tilde{\theta})}{g(\tilde{\theta})} \frac{\gamma}{2}. \tag{25}$$

This local optimum can be achieved through constrained subsidies if γ is below a threshold.

See Appendix A.6 for the proof.

3.3.4 Moral hazard and adverse selection with a binomial distribution on θ

As a counterpart to a uniform distribution of types θ over the whole interval $[0, 1]$, Case 1 with full information could be seen as an extreme case of a distribution centered around a particular type, it is then optimal to reward success. We shall introduce a binomial distribution and formally show that the optimal scheme continuously shifts from rewarding success to rewarding failure as the weight of distribution moves from low types to high types.

More specifically, we consider two types: θ_L and θ_H with $\theta_L < \theta_H$. The probability of type θ_H is denoted as λ. We analyze the influence of λ over the optimal scheme. The following assumption is introduced to get the results.

Assumption 4. *We take $(R + b) < \sqrt{2F\gamma}$ (i.e. $\theta^{FB} > 0$) and θ_L and θ_H such that $\theta^{FB} < \theta_L < \theta^{BAU}$ and $\theta^{BAU} < \theta_H$.*

To get insight into the structure of the optimal second best scheme, start with a situation in which the cost of effort γ is very high. Rewarding failure only is optimal. As γ decreases, for low values of λ it may become worthwhile to induce a low-type firm to make an effort through rewarding success, the incremental rent for the high-type firm being more than compensated. How do these two situations of rewarding success and rewarding failure combine together? As λ increases the balance between the benefit accruing from a higher effort from a low-type firm should exactly balance the increase in the rent of the high-type firm. The following lemma precisely defines the relationship between s_1 and s_2 for these intermediary situations.[12]

Lemma 3. *If both subsidies are strictly positive the optimal scheme* (s_1^*, s_2^*) *satisfy:*

$$s_1^* - s_2^* = b - \frac{\gamma\lambda(\theta_H - \theta_L)}{(1-\theta_L) - 2\lambda(\theta_H - \theta_L)}(1 - e^{FB}), \tag{26}$$

and s_2^* *is such that the profit of the low-type firm is null, it solves:*

$$s_2^* = F - \theta_L(R + (s_1^* - s_2^*)) - (1-\theta_L)\frac{(R + (s_1^* - s_2^*))^2}{2\gamma} \tag{27}$$

The proof is in Appendix A.9. We now characterize the optimal second best scheme for all values of λ.

Proposition 6. *The optimal scheme* (s_1^*, s_2^*) *depends on three thresholds* λ_1, λ_2 *and* λ_3 *as follows:*

– *for* $0 < \lambda \leq \lambda_1$: $s_1^* > 0$ *and* $s_2^* = 0; s_1^* = s_{1B}(\theta_L)$ *given by equation (9)*

– *for* $\lambda_1 < \lambda < \lambda_2$: $s_1^* > 0$ *and* $s_2^* > 0$ *given by Lemma 3* ;

– *for* $\lambda_2 < \lambda < \lambda_3$: $s_1^* = 0$ *and* $s_2^* > 0$ *such that* $\pi(1, e, \theta_L, 0, s_2^*) = 0$:

$$s_2^* = R - \gamma + \gamma\left[1 - \frac{2}{\gamma}\frac{R-F}{1-\theta_L}\right]^{1/2}$$

– *for* $\lambda_3 < \lambda \leq 1$: $s_1^* = 0$ *and* $s_2^* = 0$.

The profit of a low-type firm is always null, a high-type firm gets a windfall profit as long as $\lambda < \lambda_3$.

[12] In Appendix we show that in such a situation there are two potential benefits to using a menu with unconstrained subsidies, i.e. inducing different effort levels depending on the type of the firm and taxing profits. Still the optimal menu leaves a gap as compared with the first best: asymmetry of information generates some inefficiency independently of constraints on the incentive schemes.

Proof in Appendix A.9.

4 An illustrative example and the values of information for the firm and the agency

In this section we discuss the role of the institutional design in the relationship between the agency and the firm. More precisely we compare the agency surplus and the firm profit for each one of the information structures. We also compare the benefit of using the optimal second best constrained incentive scheme relative to a simple flat subsidy, i.e. using the best scheme in which $s_1 = s_2$.

We derive the value of information as the players moves from one information structure to another one.[13] More precisely, suppose that neither the agency nor the firm have information on θ, and suppose further that the agency does not observe the outcome of the project, it can only use a flat scheme. Does the agency have an incentive to acquire information on the outcome of the project to use the conditional scheme associated with Case 2? Then, does the firm have an incentive to acquire information on θ to be in Case 3? At this point does the agency have an incentive to also get information on θ to be in Case 1?

Using an illustrative example we compute these values and show that, ordinarily, all along this process the value is positive for the respective player. However we also show that the global value for the agency when moving from Case 2 to Case 1 may be negative. In such a case, more information is not beneficial for the agency but, incrementally, the best response is to pursue the acquisition process.

For this illustrative example we take $F = 1$, $R = 1.5$, $b = 2$ and $\gamma = 12$. We have $\theta^{FB} = .16$ and $\theta^{BAU} = .64$. To highlight the impact of a high social benefit we also consider $b = 10$ for a uniform distribution: then $\theta^{FB} = 0$, and the agency induces a large effort for

[13] In this analysis we assume that both players know in which information structure they are. Both private and public information structures are considered, secret information is not (Levine and Ponssard; 1977).

small values of θ by choosing s_{1A}. The optimal schemes for each case are given in Appendix B.1.

4.1 Uniform distribution

4.1.1 Payoffs

We calculate the agency surplus and the firm profit for each case and each set of subsidies, second best and flat. The numerical results are reported in Table 4.1.1. For all information structures, the payoffs refer to the expected value over θ.

Uniform (b=2)		Imperfect	Asymmetric	Perfect
SB	Agency	**.949**	.786	.932
	Firm	0	**.167**	.097
	Welfare	.949	.953	**1.029**
Flat	Agency	**.922**	.712	.906
	Firm	0	**.221**	.097
	Welfare	.922	.933	**1.003**

Uniform (b=10)		Imperfect	Asymmetric	Perfect
SB	Agency	5.568	4.801	**5.896**
	Firm	0	**.387**	.235
	Welfare	5.568	5.188	**6.131**
Flat	Agency	5.422	4.353	**5.625**
	Firm	0	**.261**	097
	Welfare	5.422	4.614	**5.722**

Table 1: Surplus of the agency and firm's profit with a uniform distribution for $b = 2$ and $b = 10$. For each agent, for each scheme, the maximum payoff with respect to the information structure is in bold.

4.1.2 The values of information

Start with the case of imperfect information with a flat scheme. The agency surplus is .922. Getting information about the outcome of the project allows the agency to use a conditional scheme, here rewarding success. Its surplus reaches its maximum (.949). The firm's profit remains at zero, its BAU level. While the distribution of profit depends on the scheme, its expectation does not. With both schemes (flat and SB) the expected gain from profitable projects (high types) is compensated by the expected losses from unprofitable ones (low types).

The firm would then like to acquire information on θ, and its profit would increase from zero to .167 while the agency surplus would decline from .949 to .786. At this stage the agency would like to also acquire this information, and its surplus would increase to .932. Note that the firm profit would decrease from .167 to .097.

Altogether comparing imperfect and perfect information (with second best schemes) we see that the agency surplus decreases from .949 to .932 while the firm profit increases from 0 to .097. This illustrates that the two players have antagonistic incentives regarding the information acquisition process: an offensive view versus a defensive view. The reason for the latter is because the zero profit constraint is more stringent knowing θ than taken in expectation over θ. The benefit of calibrating the scheme using $s_{1A}(\theta)$ and $s_{1B}(\theta)$ (cf. Proposition 1) to elicit the effort is not worthwhile for the agency, while it is for the firm (.097 instead of 0).

The analysis is different with a large social benefit. With $b = 10$, the perfect information case is better than the imperfect one: 5.896 versus 5.568 respectively. The fine calibration of the reward is now also beneficial for the agency.

It is interesting to see that while for the agency conditional schemes dominate flat ones whatever the information structure, this is not true for the firm for $b = 2$ but it is for $b = 10$. Note also that welfare is maximal for perfect information whatever the scheme, while we saw that the agency would prefer imperfect information if $b = 2$.

4.2 Binomial distribution

Consider now a binomial distribution over θ. Two types are introduced: $\theta^{FB} \leq \theta_L = .3 < \theta^{BAU}$ and $\theta_H = .75 > \theta^{BAU}$. A low-type firm would not implement the project, but it would be socially valuable to do so. A high-type firm would implement the project without subsidy. The probability of type θ_H is denoted as λ, and it is known to both the agency and the firm.

It will be useful to denote $\bar{\theta}(\lambda) = (1-\lambda)\theta_L + \lambda\theta_H$ and λ^{BAU} the solution of $\bar{\theta}(\lambda) = \theta^{BAU}$. With $\theta^{BAU} = 0.64$ we get $\lambda^{BAU} = .76$.

4.2.1 Payoffs

The numerical results depend on λ. We take $\lambda = .3$ and report the corresponding payoffs in Table 4.2.1.

It is worth recalling the meaning of the information structure. Imperfect means that neither the agency nor the firm knows the true value of θ. They play a game of averages, applying Proposition 1 for the expected value for $\bar{\theta}(\lambda = .3) = .435$. Asymmetric means that the firm knows the true value of θ but the agency does not. We use Proposition 6. It turns out that with $\lambda = .3$ we are precisely at the point where the second best scheme is to reward failure. Appendix B.1 details the optimal schemes (both second best and flat) for all values of λ. Perfect information means that both the agency and the firm know the true value of θ. We again use Proposition 1 for θ_L and θ_H and take the expectation over λ.

Binomial		Imperfect	Asymmetric	Perfect
SB	Agency	**.763**	.535	.739
	Firm	0	**.096**	.045
	Welfare	.763	.631	**.784**
Flat	Agency	**.717**	.527	.672
	Firm	0	**.190**	.045
	Welfare	.717	.717	.717

Table 2: Surplus of agency and firm profit with a binomial distribution for $b = 2$.

4.2.2 The values of information

Comparing Tables 4.1.1 and 4.2.1, we see that they provide similar rankings. Again the agency would prefer to remain in an imperfect situation rather than in a perfect one (getting .763 instead of .739). The firm has an incentive to move from imperfect to asymmetric (getting .096 instead of 0). If it does so the agency is induced to acquire more information as well to get .739 instead of .535. This provides some robustness for our analysis of the incentives to acquire information.

With asymmetric information, the agency rewards failure with the uniform or binomial distributions. Interestingly the incremental gain from using reward failure rather than a flat subsidy for the agency may or may not be larger than the corresponding incremental loss for the firm. Consider the change of welfare: for a uniform distribution it goes from .933 to .953 (gains for the agency exceed losses for the firm), while for the binomial distribution it goes from .717 to .631 (losses for the firm exceed gains for the agency). Rewarding failure maximizes the surplus for the agency, and reduces the windfall profit of the firm, but it needs not increase the welfare.

5 Simple extensions

5.1 The final outcome is not known ex-ante

In practice, R&D projects go through several stages from the lab to the market, and each of these stages could be subsidized and subject to informational asymmetry. As a first extension to model that dynamic, each project could be decomposed into two stages: a technical and a market stage. In the technical stage, a pilot plant is built to prove the concept of the innovation, a fixed cost is incurred, and the project might succeed or fail. In the market stage, in the case of technical success, the product might be commercialized or not. The revenue generated in the market stage R, conditional on technical success, is a random variable that is eventually observed by both the agency and the firm when realized, but it might be better anticipated by the firm than the agency. The probability distribution of R is $H(R)$, which is known ex ante to both the firm and the agency. Since one would expect that the environmental benefit of an innovation depends on the quantity produced and sold, we consider that it is a non-decreasing function of R: $b(R)$.

There is now an interim stage, between the technical and market stages, at which the outcome of the technical stage (success or failure) is observed by both parties. Moreover, the firm learns at the interim stage the future revenue R of the market stage while the agency does not. Indeed it is likely that in knowing the details of the technical stage the firm will be in a good position to forecast its future market and the associated revenue. We assume that R may include some set up cost so that its minimal value may be negative; denote R_{max} as its maximal value.The firm decides whether to commercialize the good, and it could be subsidized to do so.

Our analysis can be extended to such situations using backward induction.[14] At the interim stage the fixed cost F is sunk so that if $R < 0$ the firm would not market the product on a business as usual basis while the agency would as long as $R + b(R) > 0$. Let R^{FB} be the threshold revenue at which the agency would like to market the product: $R^{FB} + b(R^{FB}) = 0$. The interim stage is easily solved through a conditional reward s_3 such that:

$$s_3(R) = \begin{cases} 0 & \text{if } R < R^{FB} \\ -R & \text{if } R^{FB} < R < 0 \\ 0 & \text{if } 0 < R \end{cases}$$

Define $\bar{R}$ and $\bar{b}$ as the expected outcomes seen from ex ante conditional on using the optimal interim solution. That is:

$$\bar{R} = \int_{R^{FB}}^{R_{max}} (R + s_3(R))dH(R) \text{ and } \bar{b} = \int_{R^{FB}}^{R_{max}} (b(R) - s_3(R))dH(R)$$

. Our analysis of the one stage game can now be applied substituting R and b by $\bar{R}$ and $\bar{b}$ respectively.The optimal scheme then consists of a couple of subsidies conditional on the technical outcome (s_1 and s_2) together with $s_3(R)$ conditional on the market outcome.

[14] See Kirneva (2018) for a more complete study of this extension.

5.2 The outcome is imperfectly observable

There may be situations in which the agency does not perfectly observe the outcome of the project. This opens an opportunity for manipulation from the firm. If the agency were to reward failure, the firm may pretend that a success is a failure. In the following we limit ourselves to an extension of Section 3.3.1 (pure adverse selection) and show that as long as the agency receives an informative signal the solution is not qualitatively affected, though its efficiency is deteriorated (the threshold project is higher).

Let α_1 be the probability of observing a signal of failure if the project is a success and α_2 the probability of observing a signal of failure if the project fails. We assume that $\alpha_2 \geq \alpha_1$, a perfect signal corresponds to $\alpha_2 = 1$ and $\alpha_1 = 0$, and an uninformative signal corresponds to $\alpha_2 = \alpha_1$. The subsidy obtained by a firm is $\alpha_1 s_2 + (1-\alpha_1)s_1$ in case of success and $\alpha_2 s_2 + (1-\alpha_2)s_1$ in case of failure. The threshold project is then:

$$\tilde{\theta}(\alpha_1 s_2 + (1-\alpha_1)s_1, \alpha_2 s_2 + (1-\alpha_2)s_1),$$

and the expected total subsidy is

$$\int_{\tilde{\theta}}^{1} \Big\{\theta\Big[(1-\alpha_1)s_1 + \alpha_1 s_2\Big] + (1-\theta)\Big[(1-\alpha_2)s_1 + \alpha_2 s_2\Big]\Big\} dG(\theta)$$

Corollary 1. *If the success and failure of a project are not perfectly observable, the optimal scheme remains of the form $s_1 = 0$ and $s_2 > 0$. The second best threshold type, the expected subsidy, the agency surplus, the welfare, and the profit of firms only depend on the ratio α_1/α_2.*

- *If $\alpha_1 = 0$ (success is perfectly observed), then, whatever α_2, at the optimal second best scheme, the threshold probability, welfare and s_2 do not depend on α_2 and correspond to the perfect observability situation.*

- *Otherwise, with a uniform distribution, the threshold probability is higher and welfare* *and the agency surplus are lower than in the case with a perfect signal.*

See Appendix A.7 for the proof.

6 Policy recommendations and extensions

This article is concerned with public financing of risky R&D projects for the energy transition. Public financing is justified because of the discrepancy between socially and privately profitable projects. In such a context the question of what information is available to each party is crucial for the design of the relevant incentive scheme because moral hazard and adverse selection may be at work. The schemes to be considered are non negative subsidies, i.e., rewarding success and/or rewarding failure.

We show that rewarding success is a good strategy in cases of symmetric information structures while, ordinarily, rewarding failure is a good strategy for asymmetric ones. Such a drastic qualitative change emphasizes the major role of a proper identification of the underlying information structure in a real context. This is particularly so because of the high value of information for the firm. Clearly the firm is in a better situation than the agency to acquire information, and to keep it private.

These findings justify the importance of the empirical recommendations made by Rodrik (2014) for an agency in charge of monitoring a green policy. Let us review three of them briefly. Embeddedness: rather than considering the contracting process as a given arm's length relationship, make the informational design part of the issue to discuss its role in the efficiency of the contracting process. Discipline: clarify ex-ante objectives, build an evaluation protocol on what is to be observed, and introduce a sunset clause for renewing support while avoiding a not credible threat to stop funding, i.e., design appropriate conditional incentives. Gaming: beware that private investors are likely to seek informational advantages and manipulate outcomes. We think that our formalization provides helpful guidelines for implementing these recommendations in a relevant way.

As a matter of fact the motivation for this article comes from numerous discussions we had with the state agency in charge of the program launched in France in 2010 known as the Investments for the Future Programme. It covers a period of 10 years (2010–2020), for a total budget of 57 B€, and several types of activities,[15] among which there are innovative activities for the energy transition. This last part, of a total budget of 4 B€, is monitored by ADEME.[16] Each year, ADEME opens calls for innovative projects on some predefined areas. Each project is examined on its own merit, and a selection is made. Then ADEME proposes a contract to each eligible project and the firm accepts or rejects the contract. Over the 2010–2015 period, ADEME financed more than 250 projects in areas such as renewable energy, zero emission vehicles, and green chemistry. Similar programs exist in other countries, notably the SunShot initiative in the US, launched in 2011 with the aim of driving down the cost of solar energy.[17]

Initially ADEME only used flat subsidies that would simply compensate the firm for launching an unprofitable but socially valuable project (based on a reference scenario, which is similar to our Case 1, perfect symmetric information). However, evidence of windfall profits appeared quite clearly in some projects. This led the agency to introduce repayable advances, that is subsidies to be paid back in case of success. This recall our analysis of Case 2, asymmetric information. In some instances, the empirical difficulty to clearly observe success led the agency to define intermediary technical steps and have repayable advances paid partially back along the way to avoid being manipulated. These empirical observations indirectly suggested the issues to be studied in the current investigation.

From a theoretical standpoint more work would be worthwhile. First, from a pure technical point, the robustness of our results should be tested with more general functional forms. It would also be interesting to incorporate the dynamic aspects of innovation and decompose a project into several technical steps that need to be completed. The timing of these steps

[15]https://www.gouvernement.fr/secretariat-general-pour-l-investissement-sgpi

[16]http://www.ademe.fr/en/investments-for-the-future

[17]https://www.energy.gov/eere/solar/sunshot-initiative

and the determination of a stopping point, a time at which a project is abandoned, would be worth analyzing. To such end the burgeoning literature on experimentation with new ideas under asymmetric information could be inspiring (e.g. Bergemann and Hege; 1998, 2005). Second, in terms of the architecture of the formalization, the information structure may involve another party. Quite often the state agency plays the role of a middleman between the firm and the banking system. Indeed, at first, the asymmetry of information is much more acute between the firm and the banking system (which induces a capital market failure) than between the firm and the state agency (which has much higher technical expertise than a bank). The formalization should explicitly analyze how the contractual arrangement between the state agency and the firm should evolve as the asymmetry between the bank and the firm reduces over time.

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Appendix

A Proofs

A.1 Proof of Proposition 1

- First step: The project is initiated if and only if $\theta \geq \theta^{FB}$

If $\theta < \theta^{FB}$, the joint surplus of the agency and the firm is negative whatever the subsidy scheme that triggers initiation, therefore the agency is not willing to make the firm initiate the project and no subsidy are required.

If $\theta \geq \theta^{FB}$, the agency can set $s_1 = b$ and $s_2 = 0$, the project is initiated and the agency surplus is null. Therefore, the agency can obtain a positive surplus for $\theta > \theta^{FB}$ with an optimal scheme that triggers the initiation of the project.

• Second step: $s_2^* = 0$:

Let us consider that $\theta > \theta^{FB}$. The regulator maximizes its surplus (eq. 2) subject to the non-negativity constraints on profit (eq.1) and subsidy s_1 and s_2. The Lagrangien is:

$$\mathcal{L} = v(1, \theta, e(s_1 - s_2), s_1, s_2) + \mu_0 \pi + \mu_1 s_1 + \mu_2 s_2$$

With μ_0 the Lagrange multiplier associated to the initiation constraint, μ_i the multiplier associated with the non-negativity constraint of s_i, $i = 1, 2$. At the optimum:[18]

$$p_e(e, \theta)[b - (s_1 - s_2)]e' - (1 - \mu_0)p + \mu_1 = 0 \tag{28}$$

$$-p_e[b - (s_1 - s_2)]e' - (1 - \mu_0)(1 - p) + \mu_2 = 0 \tag{29}$$

And the corresponding slackness conditions. Summing the two equations gives

$$\mu_1 + \mu_2 + \mu_0 - 1 = 0 \tag{30}$$

At least one of the μ_i is positive, otherwise $\mu_0 = 1$ and $s_1^* - s_2^* = b$. The agency surplus is then $-(1 - p)s_2^* < 0$, which cannot be optimal. Consequently, $\mu_1 + \mu_2 > 0$ and $\mu_0 < 1$ (from eq. (30)). Then, from equation (29)

$$\mu_2 = (1 - \mu_0)(1 - p) + p_e[b - (s_1^* - s_2^*)]e' > 0$$

and $s_2^* = 0$.

[18] The derivative of the profit of the firm with respect to s_i does not directly involve e' by an envelop argument.

• <u>Third step</u>: Expressions of the optimal subsidy

There are four possible cases: i) $s_1^* = 0$ and $\pi > 0$, ii) $s_1^* = 0$ and $\pi = 0$, iii) $s_1^* > 0$ and $\pi > 0$, or, iv) $s_1^* > 0$ and $\pi = 0$.

Case i) corresponds to "business as usual" no subsidy is used and the project is implemented with suboptimal effort. Case ii) corresponds to a situation in which the project is not profitable and it is not worth subsidizing it.

In case iii) $s_1^* > 0$ and $\pi > 0$ then $p_e[b - s_1^*]e' = p$, and in case iv) $s_1^* > 0$ and $\pi = 0$ then $p_e[b - s_1^*]e' - p = -\mu_0 p \leq 0$.

The subsidy $s_{1A}(\theta)$ defined by equation (10) is the solution of $p_e[b - s_1^*]e' = p$, if positive. And $s_{1B}(\theta)$ is the solution of $\pi(1, \theta, e(s_1), s_1, 0) = 0$, if positive. Replacing e by $(R + s_1)/\gamma$ in eq. (1) gives a second order equation in $(R + s_1)$ with one positive root given by equation (9).

If $s_1^* > 0$ and $\pi > 0$ then $s_1^* = s_{1A}$, and if $s_1^* > 0$ and $\pi = 0$ then $s_1^* = s_{1B}$. Furthermore, by concavity of v, if both expressions s_{1A} and s_{1B} are positive the optimal subsidy is the larger of the two, otherwise it is null.

Ranking of the subsidies:

Some properties of the subsidies should be noted: $s_{1B}(\theta^{FB}) = b$ if $\theta^{FB} > 0$ and $s_{1B}(\theta^{BAU}) = 0$, while $s_{1A} < b$ for all θ.

From the two expressions (10) and (9) the sign of the difference $s_{1B} - s_{1A}$ is the sign of:

$$\frac{\theta}{1-\theta}\left[\sqrt{1 + \frac{2F}{\gamma}\frac{1-\theta}{\theta^2}} - 1\right] - \frac{R}{\gamma} - \left[\frac{b-R}{2\gamma} - \frac{1}{2}\frac{\theta}{1-\theta}\right]$$
$$= \sqrt{x^2 + \frac{2F}{\gamma}(1+x)} - \frac{b+R}{2\gamma} - \frac{1}{2}x \qquad \text{with } x = \frac{\theta}{1-\theta}$$

the sign of which is the sign of the second degree polynomial:

$$P(x) =_{def} \left[4x^2 + 8\phi(1+x)\right] - [\beta + x]^2 = 3x^2 - (2\beta - 8\phi)x + (8\phi - \beta^2) \tag{31}$$

in which $\beta = (b+R)/\gamma < 1$ (by Assumption 1) and $\phi = F/\gamma$ which is lower than R/γ (by Assumption 2) and larger than $(R/\gamma)^2/2$ (by Assumption 3). And we have $\theta^{FB} > 0$ if and only if $2\phi > \beta^2$ (from 8). The full characterization of the possible cases is cumbersome.However, despite our three assumptions, there is still a lot of room for maneuver in the choice of parameters. The two following corollary provide conditions on parameters F, b, R and γ such that only s_{1B} is used for all θ.The third provide conditions so that s_{1A} is used.

Corollary A1. *If $F > (b+R)^2/2$, that is, $\theta^{FB} > 0$, then $s_{1A} < s_{1B}$ for all θ so that the optimal subsidy is* $\max\{s_{1B}(\theta), 0\}$ *for all $\theta \in [0,1]$, and the firm gets no extra profit.*

Proof. The discriminant of $P(x)$ (eq. 31) is $16[\beta^2 - 2\beta\phi - 6\phi + 4\phi^2]$ which is negative if $\phi > \beta^2/2$, and $P(0) = 8\phi - \beta^2 > 0$ so that $P(x) > 0$ for all x and $s_{1B} > s_{1A}$. □

Corollary A2. *If $F > \gamma(1 - \sqrt{3}/2)(\simeq 0.134\gamma)$, then for all b such that $(R+b) < \gamma$ and all types $\theta > \theta^{FB}$, the optimal subsidy is* $\max\{s_{1B}(\theta), 0\}$ *and the firm gets no extra profit.*

Proof. The discriminant of $P(x)$ has the sign of $\beta^2 - 2\beta\phi - 6\phi + 4\phi^2$ which is increasing with respect to β (since $\phi < 1$) and therefore lower than $1 - 8\phi + 4\phi^2$ which is equal to $4(1-\phi)^2 - 3$ which is negative if and only if $\phi > 1 - \sqrt{3}/2$ □

Corollary A3. *If $F < (R+b)^2/(8\gamma)$, then, $\theta^{FB} = 0$ and the optimal subsidy is equal to s_{1A} for small θ, for larger θ the optimal subsidy switches to s_{1B} and eventually to* 0.

If F is slightly larger than $(R+b)^2/(8\gamma)$, then $\theta^{FB} = 0$ and the optimal subsidy is equal to s_{1B} for small θ, for intermediate θ the optimal subsidy switches to s_{1A}, then switches back to s_{1B} and eventually to 0.

Proof. First, $F < (R+b)^2/(8\gamma) < (b+R)^2/2$ so that $\theta^{FB} = 0$ for F below or slightly larger than $(R+b)^2/(8\gamma)$. Second, we have

$$P(0) = 8\frac{F}{\gamma} - \frac{(R+b)^2}{\gamma^2} < 0$$

So $P(0) < 0$ gives that $s_{1B}(0) < s_{1A}(0)$ and so $s_1^* = s_{1A}(0)$ for small θ. Then, we know that $P(.)$ has a unique positive root (the other is negative), so that for θ below that root (actually $x = \theta/(1+\theta)$ is the argument of P) s_{1A} is used, then $s_{1B}(\theta)$ until it is null.

If F is slightly larger than the threshold, then $P(0) > 0$, but one can show that $P'(0) < 0$ so that the first root of $P(.)$ is close to zero, the optimal subsidy switches from s_{1B} to s_{1A} from there, and then back to s_{1B} after the second root of $P(.)$. □

A.2 Proof of Lemma 2

Consider a change of the subsidy couple that keeps θ^t unchanged: $\theta^t ds_1 + (1-\theta^t)ds_2 = 0$. For $\theta > \theta^t$ the effect of this change on the expected subsidy received by the firm of type θ is: $(\theta - \theta^t)(ds_1 - ds_2)$ which is negative if $ds_2 > 0$. Therefore, to reduce $C(\theta^t)$ the agency should increase s_2 and reduce s_1.

A.3 Proof of Proposition 2

The threshold probability as a function of s_2 is $\tilde{\theta}(0, s_2)$, the derivative of welfare with respect to s_2 is:

$$-[\tilde{\theta}b - (1-\tilde{\theta})s_2]g(\tilde{\theta})\frac{\partial\tilde{\theta}}{\partial s_2} - \int_{\tilde{\theta}}^{1}(1-\theta)g(\theta)d\theta \tag{32}$$

the first term is the benefit from the marginal project, the second term is the increased subsidy to all more profitable projects. the derivative of the threshold probability is

$$\frac{\partial\tilde{\theta}}{\partial s_2} = \frac{1-\tilde{\theta}}{R-s_2} = \frac{(1-\tilde{\theta})^2}{R-F}$$

the derivative of welfare could then be rewritten:

$$[\tilde{\theta}(R+b) - F]g(\tilde{\theta})\frac{(1-\tilde{\theta})^2}{R-F} - \int_{\tilde{\theta}}^{1}(1-\theta)g(\theta)d\theta \tag{33}$$

At $s_2 = 0$ $\tilde{\theta} = F/R$ and the derivative of welfare is negative if

$$[F(R+b) - FR]g(F/R)\frac{1}{R}\frac{(1-F/R)^2}{R-F} \leq \int_{\tilde{\theta}}^{1}(1-\theta)g(\theta)d\theta$$

point (i) follows. Otherwise, the optimal subsidy cancels the derivative of welfare and point (ii) describes the first order condition.

A.4 Proof of Proposition 3

The agency surplus is positive for $s_1 = s_2 = 0$, and if $s_1 - s_2 \geq b$ it is non-positive, therefore, at the optimum scheme $s_1 - s_2 < b$.

Concerning the selection of projects, several cases should be distinguished according to the sign of the two subsidies at the optimum:

i If $s_1 > 0$ and $s_2 > 0$: the derivative of V with respect to s, expressed in eq. (21), is null and eq. (22) is satisfied.

  θ^{SB} cancels the derivative of V, given by eq. (20), with respect to $\tilde{\theta}$ which gives (23).

ii If $s_1 \geq 0$ and $s_2 = 0$: then $s_1 < b$ and at θ^{SB}

$$0 = p(e, \theta^{SB})(R + s_1) - [F + f(e, \theta^{SB})] \text{ from eq. (17)} \tag{34}$$

$$< p(e, \theta^{SB})(R + b) - [F + f(e, \theta^{SB})] \tag{35}$$

$$< p(e^{FB}, \theta^{SB})(R + b) - [F + f(e^{FB}, \theta^{SB})] \tag{36}$$

  therefore, $\theta^{SB} > \theta^{FB}$ less projects are selected than at the first best.

iii If $s_1 = 0$ and $s_2 > 0$: the above method cannot be applied, the first order condition should be considered. The threshold $\tilde{\theta}$ cannot be chosen independently from the bonus

$s = -s_2$, and s_2 cancels the derivative of V given by eq. (18) so

$$p(e,\tilde{\theta})(b+s_2) - s_2 = \int_{\tilde{\theta}}^{1} \left[p_e.(b-s_2)e' + (1-p)\right] dG(\theta) \Big/ \left[-\partial\tilde{\theta}/\partial s_2\right]$$

and injecting eq. (17) the left hand side is $p(e,\tilde{\theta})(R+b) - [F+f]$ which is then strictly positive, since $\partial\tilde{\theta}/\partial s_2 < 0$, and together with the fact that $e < e^{FB} =$ implies that $\theta^{SB} < \theta^{FB}$.

A.5 Proof of Proposition 4

The second order coefficient of s is given by eq. (24), and with a uniform distribution over $[0,1]$ it is null.

Therefore, the derivative of V with respect to s is (from eq. (21)):

$$\begin{aligned}
\frac{\partial V}{\partial s} &= \frac{1}{\gamma}\int_{\tilde{\theta}}^{1} \left[(1-\theta)b - (\theta-\tilde{\theta})(\gamma-R)\right] d\theta \\
&< \frac{b}{\gamma}\int_{\tilde{\theta}}^{1} \left[(1-\theta) - (\theta-\tilde{\theta})\right] d\theta \text{ since } \gamma - R > b \\
&= 0
\end{aligned}$$

V is strictly decreasing with respect to s so $s_1 = 0$ (otherwise s can be decreased while keeping $\tilde{\theta}$ constant).

A.6 Proof of Proposition 5

From equation (6), $e(s) = 1$ for $s \geq \gamma - R$ so that from equation (21), V is flat for $s \geq \gamma - R$, and by continuity it is increasing for s slightly below. It is then locally optimal for any targeted threshold to set $s = \gamma - R$.

With $e = 1$, the cost of a project of type θ is then $F + (1-\theta)\gamma/2$, and the agency surplus can then be written : $V = (1 - G(\tilde{\theta}))[(R+b) - (F + (1-\tilde{\theta})\gamma/2)]$. The maximization of which gives equation (25).

With $s = \gamma - R$, $p(e,\theta) = 1$ and a firm profit is $\pi = (R+s) - [F+f] = \gamma + s_2 - F - (1-\theta)\gamma/2$. The selection of projects is ensured by setting $s_2 = F - (1+\tilde{\theta})\frac{\gamma}{2}$, and $s_1 = s + s_2 = F + (1-\tilde{\theta})\frac{\gamma}{2} - R$.

Both are non-negative for a sufficiently small γ.

A.7 Proof of corollary 1

Let us denote $\sigma_1 = \alpha_1 s_2 + (1-\alpha_1)s_2$ and $\sigma_2 = \alpha_2 s_2 + (1-\alpha_2)s_2$ the subsidies obtained in case of success and failure respectively.

• For unconstrained subsidies: with the couple of subsidy: $s_1 = F - \alpha_2 R/(\alpha_2 - \alpha_1)$ and $s_2 = F + (1-\alpha_2)R/(\alpha_2 - \alpha_1)$, the expected subsidies are $\sigma_1 = F$ and $\sigma_2 = F - R$ which implement the first best.

• For non-negative subsidies::

1. $s_1 = 0$ and $s_2 > 0$: The reasoning of Lemma 1 can be reproduced: an increase of σ_2 coupled with a reduction of σ_1 that leaves $\tilde{\theta}$ unchanged reduces the total expected subsidy. Consequently it is optimal to set $s_1 = 0$ and $s_2 > 0$.

2. Then, with $s_1 = 0$, $\sigma_1 = x\sigma_2$ with $x = \alpha_1/\alpha_2$ and the threshold probability is $\tilde{\theta}(x\sigma_2, \sigma_2)$, the regulator surplus is

$$V(x\sigma_2, \sigma_2) = \int_{\tilde{\theta}}^{1} \Big[\theta(b - x\sigma_2) - (1-\theta)\sigma_2\Big] dG(\theta)$$

and welfare is $W(\tilde{\theta}(x\sigma_2, \sigma_2))$.

3. If $\alpha_1 = 0$: then $x = 0$ and the surplus of the regulator, the profit of firms, and total welfare could all be written as functions of σ_2 without any other dependence on α_2. The optimum second best scheme is then similar to the scheme described by Proposition 2 with $\alpha_2 s_2$ being independent of α_2.

4. Otherwise, for $\alpha_1 > 0$: then $x > 0$,

4.1. Let us prove that θ^{SB} is increasing with respect to x, to do so we first write the first order condition:

- the total derivative of the threshold type w.r.t. σ_2 is:

$$\frac{d\tilde{\theta}}{d\sigma_2} = -\frac{1-(1-x)\tilde{\theta}}{R-(1-x)\sigma_2}$$

the first order condition satisfied at the optimal scheme is

$$\left[\tilde{\theta} - \theta^{FB}\right] g(\tilde{\theta}) \frac{1-(1-x)\tilde{\theta}}{R-(1-x)\sigma_2} = \int_{\tilde{\theta}}^{1} \left[\theta x + (1-\theta)\right] dG(\theta)$$

and with a homogeneous distribution it gives:

$$\tilde{\theta} = \theta^{FB} + \frac{R-(1-x)F}{R+b} \frac{1}{2(1-x)} \left[1 - \frac{x^2}{\left(1-(1-x)\tilde{\theta}\right)^2}\right]$$

- θ^{SB} increases with respect to x (brutal calculations): The right hand side of the first order condition above side is a decreasing function of $\tilde{\theta}$, and it is increasing with respect to x: Its derivative is

$$\frac{(1-\tilde{\theta})}{2(R+b)} \frac{2Fx + R[(1-\tilde{\theta})^2 - x\tilde{\theta}(1+\tilde{\theta})]}{\left(1-(1-x)\tilde{\theta}\right)^3}$$

the sign of which is the sign of $2Fx + R[(1-\tilde{\theta})^2 - x\tilde{\theta}(1+\tilde{\theta})]$ which is positive (using that $\tilde{\theta} < F/R$).

4.2. The effect of x on the regulator surplus at the optimal scheme, by an envelop argument, it is

$$\frac{\partial V}{\partial s_1}(x\sigma_2, \sigma_2)\sigma_2$$

which is negative.

Welfare is decreasing with respect to $\tilde{\theta}$ as long as $\tilde{\theta} > \theta^{FB}$, so it is decreasing with respect to x.

A.8 Proof of Lemma 3

To alleviate notation the probability $p(e, \theta_L)$ and $p(e, \theta_H)$ are denoted with subscripts: $p_L(e)$ and $p_H(e)$, and the profits π_L and π_H.

If both s_1^* and s_2^* are positive, then low type projects are implemented (otherwise one would apply Proposition 1 to high type) and their profits are null (otherwise the subsidies could be reduced). The regulator surplus is then

$$v(s_1, s_2) = (1-\lambda)\Big[p_L(b - s_1) - (1-p_L)s_2\Big] + \lambda\Big[p_H(b - s_1) - (1-p_H)s_2\Big] \tag{37}$$

and the optimal scheme satisfies the following equation

$$\frac{\partial v}{\partial s_1}\frac{\partial \pi_L}{\partial s_2} - \frac{\partial v}{\partial s_2}\frac{\partial \pi_L}{\partial s_1} = 0$$

that is

$$\frac{\partial v}{\partial s_1}(1-p_L) - \frac{\partial v}{\partial s_2}p_L = 0$$

which gives, denoting $s^* = s_1^* - s_2^*$:

$$\begin{aligned}
\lambda[p_H(1-p_L) + (1-p_H)p_L] &= \Big[(1-\lambda)\frac{\partial p}{\partial e}(e, \theta_L) + \lambda\frac{\partial p}{\partial e}(e, \theta_H)\Big](b - s^*)e' \\
\lambda[p_H - p_L] &= \Big[(1-\lambda)(1-\theta_L) + \lambda(1-\theta_H)\Big](b - s^*)\frac{1}{\gamma} \\
\lambda(\theta_H - \theta_L)(\gamma - (R + s^*)) &= \Big[(1-\lambda)(1-\theta_L) + \lambda(1-\theta_H)\Big](b - s^*)
\end{aligned}$$

which then gives equation (26). Equation (27) corresponds to $\pi_L = 0$.

A.9 Proof of Proposition 6

The solution $s_1^* = s_2^* = 0$ corresponds to the situation in which L firms do not enter. The regulator surplus in that situation is:

$$V_1(\lambda) = \lambda p_H b$$

In all other situations, if one of the optimal subsidy is positive, L firms do enter (from Proposition 1, if only H firms enter then it is optimal to set $s_1 = s_2 = 0$). The regulator surplus when L firms enter is

$$V_2 = (1-\lambda)[p_L(b-s_1) - (1-p_L)s_2] + \lambda[p_H(b-s_1) - (1-p_H)s_2]$$

that can be equivalently defined as a function of $s = s_1 - s_2$ and s_2:

$$V_2(\lambda, s, s_2) = (1-\lambda)[p_L(b-s) - s_2] + \lambda[p_H(b-s) - s_2]$$

and the constraint $s_1 \geq 0$ is then $s + s_2 \geq 0$.

The problem of the regulator can be decomposed in two steps: first maximize V_2 and then compare the maximum obtained with V_1.

Let us consider the maximization of V_2 subject to $\pi_L \geq 0$, $s_2 \geq 0$ and $s + s_2 \geq 0$ and denote $s^{**}(\lambda)$ and $s_2^{**}(\lambda)$ the solution, and $s_1^{**} = s^{**} + s_2^{**}$. The problem can be simplified by transforming the three constraints $\pi_L \geq 0$, $s_2 \geq 0$ and $s + s_2 \geq 0$ into two constraints on s, by parameterizing everything by s.

- At the maximum $\pi_L = 0$: by contradiction, if $\pi_L > 0$ then $s_2^{**} = 0$ and s_1^{**} is larger than s_{1B} (which cancels π_L, it is defined by eq. 9) and solves

$$[(1-\lambda)\frac{\partial p_L}{\partial e} + \lambda\frac{\partial p_H}{\partial e}](b-s_1)e' = (1-\lambda)p_L + \lambda p_H$$

then $\partial p_L/\partial e(b-s_1^{**})e' > p_L$ that is $s_1^{**} < s_{1A}(\theta)$ (given by eq. 10) which is lower than $s_{1B}(\theta)$ when $(R+b) \geq 2\sqrt{2F\gamma}$ (proof of Proposition 1), a contradiction.

- We can then define $s_2(s)$:

$$s_2(s) = F - \max_e \left[p(e,\theta_L)(R+s) - f(e,\theta_L)\right]$$

it is decreasing with respect to s with $s_2'(s) = -p_L$. And $s_1(s) = s + s_2(s)$ is strictly increasing with respect to s ($s_1' = 1-p$).

- For $s = -R$, $s_2(-R) = F$ and the associated s_1 is $F - R < 0$.

- At $s = s_{1B}$, the profit $\pi_L(e, s_{1B}, 0)$ is null so that $s_2(s_{1B}) = 0$, and $s > s_{1B} \Leftrightarrow s_2(s) < 0$. Note also that $s_{1B} < b$.

- At $s = 0$, $s_2(0)$ is positive equal to $-\pi_L(e,0,0)$.

- Define $\underline{s}$ the solution of $s + s_2(s) = 0$, it is between $-R$ and 0. The corresponding s_2 is such that $\pi_L(e, 0, s_2) = 0$.

The regulator's objective is then equivalent to the maximization of

$$\max_s V_2(\lambda, s, s_2(s)) \text{ s.t. } \underline{s} \leq s \leq s_{1B}$$

The derivative of the objective function with respect to s is:

$$\begin{aligned}
\mathcal{V}(\lambda, s) &= \left[(1-\lambda)\frac{\partial p_L}{\partial e} + \lambda\frac{\partial p_H}{\partial e}\right](b-s)\frac{1}{\gamma} - \lambda[p_H - p_L] \\
&= \Big[(1-\lambda)(1-\theta_L) + \lambda(1-\theta_H)\Big](b-s)\frac{1}{\gamma} - \lambda(\theta_H - \theta_L)(1 - \frac{R+s}{\gamma}) \\
&= \Big[(1-\theta_L) - 2\lambda(\theta_H - \theta_L)\Big](b-s)\frac{1}{\gamma} - \lambda(\theta_H - \theta_L)(1 - e^{FB}) \text{ using } \mathbf{??} \\
&= (\theta_H - \theta_L)\Big[(\underline{\lambda} - \lambda)(b-s)/\gamma - \lambda(1 - e^{FB})\Big]
\end{aligned}$$

in which

$$\underline{\lambda} = \frac{1-\theta_L}{2(\theta_H-\theta_L)}$$

This derivative is strictly decreasing with respect to s as long as $\lambda < \underline{\lambda}$. It is also decreasing with respect to λ for $s < s_{1B}$.

For all $s \in [\underline{s}, s_{1B}]$ we have $\mathcal{V}(0,s) = (1-\theta_L)(b-s)/\gamma > 0$ and $\mathcal{V}(\underline{\lambda}, s) < 0$.

So we already know that $s^{**}(0) = s_1^{**}(0) = s_{1B}$ and $s_2^{**}(0) = 0$, and that, $\forall \lambda > \underline{\lambda}$, $s^{**}(\lambda) = \underline{s}$: $s_1^{**}(\lambda) = 0$ and $s_2^{**}(\lambda) = s_2(\underline{s})$ the solution of

$$p_L(e)R + (1-p_L)s_2 = F + f_L(e)$$

And we can define :

- λ_1 the solution of $\mathcal{V}(\lambda, s_{1B}) = 0$
- λ_2 the solution of $\mathcal{V}(\lambda, \underline{s}) = 0$

Then the optimal solution as a function of λ is such that

- $0 \leq \lambda < \lambda_1$: $s^{**}(\lambda) = s_1^{**}(\lambda) = s_{1B}$ and $s_2^{**}(\lambda) = 0$
- $\lambda_1 \leq \lambda < \lambda_2$: $s^{**}(\lambda) \in (\underline{s}, s_{1B})$, $s_1^{**}(\lambda) > 0$ and $s_2^{**}(\lambda) > 0$
- $\lambda_2 \leq \lambda \leq 1$: $s^{**}(\lambda) = \underline{s}$, $s_1^{**}(\lambda) = 0$ and $s_2^{**}(\lambda) = s_2(\underline{s}) > 0$

Then, the regulator should compare V_2 and V_1, the difference $V_2 - V_1$ is decreasing with respect to λ and positive for $\lambda = 0$ and negative for $\lambda = 1$ (by Proposition 1). There is then a λ_3 so that $\lambda > \lambda_3$ implies $s_1^{**}(\lambda) = s_2^{**}(\lambda) = 0$.

A.10 Optimal menu without constraints

We provide a description of what would be the structure of a menu, with a general distribution of types, if the subsidies are not constrained to be positive, and firms cannot initiate the project without the regulator consent.

It is easier to work with the bonus $s(\theta)$ and consider $s_2(\theta)$ as a fixed transfer. The agency proposes a structured menu $(s(\theta), s_2(\theta))_{\theta\in(0,1)}$, a firm of type θ selecting the item $(s(\eta), s_2(\eta))$ has a profit $\pi(\theta,\eta) = p(e,\theta)(R + s(\eta)) + s_2(\eta) - (F + f(e,\theta))$, and the first order condition necessary for self selection is $p(e,\theta)s'(\theta) + s_2'(\theta) = 0$.

Using the standard methodology in contract design, denoting $\pi^m = \pi(\theta,\theta)$, its total derivative is $d\pi^m/d\theta = p_\theta(R + s) - f_\theta(e,\theta)$ which only depends on the bonus $s(\theta)$ and not $s_2(\theta)$ because e does. Thanks to this relationship and an integration by part the surplus of the agency can be written:

$$V = \int_{\tilde{\theta}}^{1} \left\{ \Big[p(e,\theta)(R+b) - (F - f(e,\theta))\Big] g(\theta) - \frac{d\pi^m}{d\theta}[1 - G(\theta)] \right\} d\theta$$

The optimal bonus $s(\theta)$ should be such that

$$p_e(b-s)\frac{de}{ds} = \frac{1-G}{g}\frac{d}{ds}\Big[p_\theta(e,\theta)(R+s) - f_\theta(e,\theta)\Big]$$

and with our quadratic specification $s(\theta)$ solves:

$$(1-\theta)(b-s)\frac{1}{\gamma} = \frac{1-G(\theta)}{g(\theta)}(1-e(s)) = \frac{1-G(\theta)}{g(\theta)}(\gamma - R - b)\frac{1}{\gamma} \tag{38}$$

which looks like the equation (22) satisfied by a simple scheme (s, s_2). We recover the usual result that $s = b$ for high types. The selection of projects is done with the choice of $s_2(\tilde{\theta})$ the profit of the $\tilde{\theta}$ firm being nul.

B The illustrating example

B.1 Optimal schemes for the illustrating example

B.1.1 Uniform distribution with $b = 2$

Case 1: Perfect information

• The optimal second best scheme: From Proposition 1, with our numerical values if $b \geq 7.7$ then $s_{1A}(\theta) \leq s_{1B}(\theta)$ for all θ. Hence for $b = 2$ the optimal second best scheme is:

- if $\theta \leq \theta^{FB}$ the optimal subsidy is null, the project is not initiated;

- if $\theta^{FB} \leq \theta < \theta^{BAU}$ the optimal subsidy is $s_{1B}(\theta)$, the project is initiated and the firm gets no windfall profit;

- if $\theta \geq \theta^{BAU}$ the optimal subsidy is null, it is business as usual, the project is initiated and the firm gets no windfall profit.

• The optimal flat scheme: with a flat scheme $s_1 = s_2$ and the effort does not depend on the subsidy.The subsidy is then only used to incentivize the firm to initiate the project. We first derive the subsidy which gets a zero profit for the firm. It is $F - \theta R - (1-\theta)R^2/(2\gamma)$. The corresponding surplus is positive if $\theta > .21$. Altogether we get:

- if $\theta < .21$, no subsidy and the project is not initiated;

- if $.21 < \theta < \theta^{BAU} = .64$ use $s_1(\theta) = s_2(\theta) = F - \theta R - (1-\theta)R^2/(2\gamma)$, the project is initiated with zero profit for the firm;

- if $\theta > \theta^{BAU} = .64$ the project is initiated without subsidy and the firm gets its BAU profit.

Case 2: Imperfect information

We do as before with the expected value of type $\bar{\theta} = 0.5$. For both the optimal second best scheme and the optimal flat subsidy the firm gets a zero expected profit.More precisely:

- for the optimal second best scheme $s_1 = s_{1B}(\bar{\theta}) = 0.36$;

- for the optimal flat scheme $s_1 = s_2 = 0.2$.

Case 3: Asymmetric information

From Proposition 4 we know that the optimal second best scheme is such that $s_1 = 0$. The optimal value of s_2 is obtained numerically, it is $s_2 = .704$. The optimal flat scheme is also obtained numerically: $s_1 = s_2 = .27$.

B.1.2 Uniform distribution with $b = 10$

Case 1: Perfect information

• Optimal second best scheme:

We now have $\theta^{FB} = 0$ and the ranking between $s_{1A}(\theta)$ and $s_{1B}(\theta)$ depends on θ (cf Figure 1). The optimal second best scheme is again obtained using Proposition 1 and it is:

- if $\theta^{FB} = 0 \leq \theta \leq .36$ the optimal subsidy is $s_{1A}(\theta)$, the project is initiated and the firm gets some rent;

- if $.36 \leq \theta < \theta^{BAU}$ the optimal subsidy is $s_{1B}(\theta)$, the project is initiated and the firm gets zero profit;

- if $\theta \geq \theta^{BAU}$ the optimal subsidy is null, it is business as usual, the project is implemented and the firm gets its BAU profit.

• Optimal flat scheme:

The flat subsidy which gets a zero profit for the firm is: $F - \theta R - (1-\theta)R^2/(2\gamma)$. The corresponding surplus is always positive. Altogether we get:

- if $\theta^{FB} = 0 < \theta < \theta^{BAU} = .64$ use $s_1(\theta) = s_2(\theta) = F - \theta R - (1-\theta)R^2/(2\gamma)$, the project is initiated with zero profit for the firm;

- if $\theta > \theta^{BAU} = .64$ the project is initiated with no subsidy, either flat or second best, and the firm gets its BAU profit.

Case 2: Imperfect information

While the optimal schemes differ whether $b = 2$ or $b = 10$ for perfect information, they do not for imperfect one. It never pays for the agency to induce a larger effort than the one needed to compensate the firm.

Case 3: Asymmetric information

From Proposition 4 we know that the optimal second best scheme is such that $s_1 = 0$. The optimal value of s_2 is obtained numerically as $s_2 = .864$. The optimal flat scheme is obtained numerically. It is $s_1 = s_2 = .344$.

B.1.3 Binomial distribution

We detail the optimal schemes for all values of λ from 0 to 1.

Case 1: Perfect information

If both the agency and the firm know that θ is θ_L, since $\theta^{FB} \leq \theta_L < \theta^{BAU}$ the optimal second best subsidy is $s_{1B}(\theta_L)$ (cf Proposition 1); the optimal flat subsidy is $s_1(\theta_L) = s_2(\theta_L) = F - \theta_L R - (1-\theta_L)R^2/(2\gamma)$; the project is implemented, the agency gets the associated surplus and the firm zero profit;

If both the agency and the firm know that θ is θ_H, since $\theta_H \geq \theta^{BAU}$ the optimal subsidy is null, it is business as usual; the project is implemented, the agency gets the BAU surplus and the firm the BAU profit.

Case 2: Imperfect information

Both the agency and the firm know that θ is either θ_L or θ_H with respective probability $1 - \lambda$ and λ.

For a given λ, to derive the optimal second best scheme we compare the agency surplus either with BAU or with a scheme that triggers initiation (and $\pi = 0$) if necessary ($\lambda < \lambda^{BAU}$).

The firm initiates the project through BAU for $\lambda > \lambda^{BAU}$, in that case the agency is better off with no subsidy $s_1 = s_2 = 0$. For λ between 0 and λ^{BAU} we have $s_1 = s_{1B}(\bar{\theta}(\lambda))$ (cf Proposition 1) and $s_2 = 0$.

For the optimal flat scheme one needs to compare the surplus obtained with a scheme such that $s_1 = s_2 = -\pi(1, e^{BAU}, \theta(\lambda), 0, 0)$, with its BAU surplus; The critical value of λ is lower than λ^{BAU}, that is .64.

Case 3: Asymmetric information

In this case, the agency knows λ and the firm knows the true value of θ. We obtain the second best scheme using Proposition 6. We can derive numerically the thresholds for λ to approximately be $\lambda_1 = .1$, $\lambda_2 = .3$ and $\lambda_3 = .6$. We recall the strucure of the solution:

– for $0 < \lambda \leq \lambda_1$: $s_1^* > 0$ and $s_2^* = 0; s_1^* = s_{1B}(\theta_L)$ given by equation (9)

– for $\lambda_1 < \lambda < \lambda_2$: $s_1^* > 0$ and $s_2^* > 0$ given by Lemma 3 ;

– for $\lambda_2 < \lambda < \lambda_3$: $s_1^* = 0$ and $s_2^* > 0$ such that $\pi(\theta_L, e, 0, s_2^*) = 0$:

$$s_2^* = R - \gamma + \gamma \left[1 - \frac{2}{\gamma}\frac{R-F}{1-\theta_L}\right]^{1/2}$$

– for $\lambda_3 < \lambda \leq 1$: $s_1^* = 0$ and $s_2^* = 0$.

The profit of a low type firm is always null, a high type firm gets a rent as long as $\lambda < \lambda_3$.

The optimal flat subsidy induces the BAU effort from the firm. For low values of λ it is calibrated so that the firm gets zero profit with θ_L and a rent with θ_H. We get $s_1 = s_2 = F - \theta_L R - (1-\theta_L)R^2/2\gamma = .43$. For high values of λ there is no subsidy the firm only launch the project iff $\theta = \theta_H$. The threshold value for λ can be derived to be .375.