Directed technical change and the resource curse

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Abstract

The "resource curse" is a threat to all countries relying on export income from abundant natural resources such as fossil fuels. The early literature hypothesized that easily accessible natural resources would lead to lack of technological progress. In this article we instead propose that abundance of fossil fuels can lead to the *wrong type* of technological progress.

In order to inquire into our research question, we build a model of a small, open economy having specialized in export of fossil fuels. R&D in fossil fuel extraction technology competes with R&D in clean energy technologies. Moreover, technological progress is path dependent as current R&D within a technology type depends on past R&D within the same type. Finally, global climate policy may reduce the future value of fossil fuel export.

Surprisingly, we find that global climate policy may both lead to a resource curse or help the country escaping a potential resource curse. The ripeness of the clean energy technologies is essential for the outcomes: If the clean technology level is not too far beyond the fossil fuel technology, a shift to exporting clean energy technologies is optimal independent of global climate policy and climate policy can accelerate this shift. While if the clean technologies is far behind, a shift should only happen as a responce to global climate policy, and the government must intervene to make it happen.

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1 Introduction

Many countries with abundant natural resources experience lower economic growth than countries with little or no natural resources. In the economics literature this is referred to as the "resource curse". Recent articles in this literature attribute the curse to political failures (Robinson et al. 2014), while previous literature hypothesized that easily accessible natural resources providing export income would lead to lack of technological progress (van der Ploeg, 2011). In this article we revisit the early explanations for the resource curse, but instead of explaining the curse by lack of technological progress, we investigate whether resource abundance can lead to the *wrong type* of technological progress.

Our focus is on countries having abundant fossil fuel resources providing them with a generous export income. Using fossil fuels to produce energy in any form implies greenhouse gas (GHG) emissions. The Paris agreement sets an ambitious target for the global reduction of GHG emissions, and a large share of discovered fossil fuel resources needs to be left in the ground (Welsby et al, 2021). Most countries, apart from the EU block, have so far been reluctant to implement ambitious climate policies, and hence, we have not yet seen a climate policy induced slump in global fossil fuel prices. This could, however, occur later in this or the next decade as the evidence for climate change caused by GHG emissions accumulates.

Although the current postponement of more stringent climate policies implies that fossil fuel exporting countries can continue to profit from their fossil fuel extraction, their prosperity may actually be hampered in the longer run. Recent economic theory has hypothesized that technological change within energy technologies may be path dependent (Acemoglu et al., 2012). Path dependency can only happen if there are separate innovation systems for clean and dirty energy technologies, see e.g. Greaker et al. (2018) which interprets "dirty technologies" as all technologies related to the extraction and usage of fossil fuels. A natural question thus arises: Will innovation markets induce a switch away from fossil fuel technologies to clean technologies in time, or will innovators' choices be locked in by history and result in a resource curse?

In order to shed light on this question, we build a model of a small, open economy having specialized in fossil fuel exports. In our model, even though fossil fuels are non-renewable resources, the country can continue to export fossil fuels as long as it devotes enough resources to research and development (R&D) making fossil fuel extraction from more costly deposits profitable. In the business as usual case (BaU), the government only enacts a neutral R&D policy and do not tax fossil fuel extraction. On the contrary, in the optimal policy case, the government may use both an extraction tax and a directed subsidy to either clean or dirty innovation.

We first solve the model analytically. A fall in the price of fossil fuels will reduce the value of dirty R&D both in the decentralized market and the planner's solution. The relevant question when analyzing optimal subsides towards either of the two research sectors is, however, how the market's valuation of clean versus dirty innovation changes relative to the planner's valuation of clean versus dirty innovation. We show that this relative valuation, and thus the direction of subsidies, depends on a *technology effect* and a *price effect.*

The technology effect reflects that the optimal subsidy to clean innovation today increases when clean technology improves at a higher rate. The reason is that scientists do not factor in the positive knowledge externality from their R&D on future R&D. Since the social value of the positive knowledge externality is larger when a certain technology is being more used in the future, R&D within this technology should be subsidized more.

The price effect is the isolated effect of fossil fuel prices on optimal R&D subsides. As scientists discount the future harder than the planner, future changes in fossil fuel prices will not properly be taken into account by the scientists (in our model patent life-time is finite). Thus, if fossil fuel prices fall with a constant rate or a higher rate in the future than today, the price and the technology effect together push R&D subsidies to clean technologies upwards.

As shown by Heal (1976), extraction paths are likely to be forward biased when extraction costs depend positively on accumulated extraction, i.e., there is too much extraction in early periods of the lifetime of the resource. Our model shares this feature, and correcting for the cost-externality increases the present value of the resource sector, which through the technology effect lowers the need for subsidies to clean innovation. Moreover, the extraction tax shifts market value of fossil fuel production from today to the future, which may further lower the need for subsidies to clean innovation.

To complement our theoretical findings, we provide several numerical illustrations based on our model comparing the BaU case with both first and second best policy cases. Depending on the future development in fossil fuel prices (influenced by global climate policy) and the current state of the clean technology alternative, the small, open economy may evolve according to the following four scenarios:

I Steady course: There is no need for intervention in redirecting R&D since global climate policy stays weak and fossil fuel prices do not decline.

II Resource curse due to global climate policy: R&D should be redirected since researchers do not sufficiently take into account the effects of future climate policy on fossil fuel prices.

III Resource curse with no global climate policy: R&D should be redirected even if fossil fuel prices stay constant, due to better prospects for clean technology.

IV Induced change in course due to climate policy: Even without policy, the private sector shifts R&D effort to the clean energy sector due to declining fossil fuel prices caused by global climate policy.

Note that global climate policy may both lead to a resource curse (Scenario II) or help the country escaping a potential resource curse (Scenario IV). Furthermore, in the three scenarios II to IV, the government can improve the outcome by offering directed subsidies to clean R&D. Moreover, since extraction paths are forward biased, the R&D subsidy to clean R&D in Scenario II to IV should be higher without an extraction tax. On the other hand, in our model simulations the extraction tax alone does little to redirect R&D towards the clean sector.

The rest of the paper is laid out as follows: In the next section we briefly review the relevant literature. Then in Section 3 we present and solve the model for the *laissez fair* (BaU) solution. This solution is compared to the socially optimal solution in Section 4. Our results are then illustrated by a numerical example in Section 5. Finally, in Section 6 we conclude.

2 Literature

Our paper relates to the so-called "resource curse" literature which informs us that when a country is endowed with a valuable and tradable resource, the country may experience low or even negative economic growth. As mentioned in the introduction, several explanations for the resource curse have been discussed in the literature (e.g., van der Ploeg (2011); Robinson et al (2014)). In particular, the earlier literature emphasized lack of technological progress due to a down scaling of the traded good sector in which most of the technological progress happened (van der Ploeg, 2011). In our paper, harvesting the resource requires a high level of technological skills, however, since technological change is directed, the country may be locked into developing a technology with poor possibilities in the future.

The literature on directed technological change and the environment is steadily increasing in size. Several papers modify and simulate the model from Acemoglu et al. (2012), though in different directions and analyzing other problems than in the present paper: Hourcade, Pottier, and Espagne (2011) discuss parameter choices related to the climate part of the model; Mattauch, Creutzig, and Edenhofer (2015) add learning-by-doing effects to the framework; Durmaz and Schroyen (2014) extend the model by adding abatement technology (carbon capture and storage); André and Smulders (2014) have either energy-saving R&D or labor-saving R&D; Hémous (2013) and van den Bijgaart (2015) extend the model to include more than one country and analyze unilateral environmental policies in a global context, and finally Hart (2019) introduces deadweight losses from R&D subsidies. Apart from Acemoglu et al (2012), none of the above mentioned papers include extraction of a non-renewable resource. And even more importantly for our contribution, none of the papers explore the implication of directed technical change for a small, open economy having specialized in export of a non-renewable resource.

Hassler et al (2021) look at directed technical change in a model with a non-renewable resource, and show that R&D effort will be allocated by the market improving the effciency of resource use when the resource becomes scarce. The same happens in our model; R&D can make up for higher prices of the resource, however, as there exists an alternative technolgical development path in our model economy, devoting more and more R&D effort to a dwindling non-renewable resource may be sub-optimal.

Lemoine (2020) also looks at directed technical change, but studies whether a technological shift will happen by itself or whether the economy is locked-in to a certain technology for ever. Lemoine finds that this depends on the elasticity of substitution between resources and capital in the intermediate energy services production function. We consider the same question, but focus on whether global climate policy can induce a shift in due time.

Acemoglu, Akcigit, Hanley, and Kerr (2016) develop a model of endogenous growth with clean and dirty R&D. They model the innovation process differently from us. In their model clean and dirty machines within a product line are perfect substitutes, and hence, in order to have a market, a new clean machine must in most cases outcompete the dirty machine within the same product line. This only happens rarely, and thus, innovators may not get any profits from clean R&D at all even if they improve upon the clean technology.

A key assumption in models of directed technical change is that innovation is path (state) dependent. A new innovation builds on past quality within a field and increases the productivity of future innovations within the same field. Aghion, Dechezlepretre, Hemous, Martin, and Van Reenen (2016) analyze clean and dirty technologies in the automotive industry. By using patent citations they find evidence for separate innovation systems within clean and dirty technologies.Further, they find that the productivity of innovation within a field depends positively on the accumulated innovation within the same field.

In most models of directed technical changem research opportunities in dirty and clean technologies are identical. Thus, the standing on shoulders effect dominates such that research in one area becomes ever more potent when knowledge accumulates. Popp et al. (2013), however, find that research opportunities in any one area of research may dry out. That is, the best ideas are taken first, and later ideas improve the state of the technology to a lesser degree. In the literature this is referred to as *fishing out*, see, e.g., Greaker and Pade (2009). Clearly, if *fishing out* occurs, path dependency may be less likely. Dechezleprêtre, Martin, and Mohnen (2013) find based on a patent citation analysis that spillovers are larger in clean than dirty technologies. The driving force behind the result seems to be that clean technologies are newer technologies than dirty, and that a new technology field has larger spillovers than an old technology field.

3 The model

3.1 Preliminaries

The model is an infinite-horizon, discrete-time open economy with directed technical change and natural resources. We focus on the energy sector, and do not model other sectors of the economy. In each period t, the country in question produces dirty energy Y_{dt} based on extraction of fossil fuels and clean energy Y_{ct} from different kinds of renewable resources. Furthermore, we assume that the two types of energy (denoted by $j = d, c$) are freely traded, and that the country faces exogenous world market prices on the dirty and the clean energy good, P_{dt} and P_{ct}, respectively. Consequently, the decision on how much to consume and how much to produce of the two energy goods are effectively separated. Furthermore, this implies that national wealth in our stripped down model is synonymous with the current net value of future energy production. First, we start out by describing the *laissez faire* solution.

3.2 Production of the energy goods

In order to produce the two types of energy goods j, the open economy at time t uses machine variants i of different qualities A_{jit} in the amount x_{jit}, and natural resources R_{jt}. The production function is given by:[1]

$$Y_{jt} = R_{jt}^{\alpha_2} \int_0^1 A_{jit}^{1-\alpha_1} x_{jit}^{\alpha_1} di, \tag{1}$$

where α_1 and α_2 are parameters. Within each time period, we assume that there is decreasing returns to scale in energy goods production, and hence, $\alpha_1 + \alpha_2 < 1$. Every time a new innovation is made in one of the sectors, one particular machine type i is replaced by a better machine of the same type. The innovation is drastic, implying the older version of the machine type is no longer used in the market.

Production of dirty energy is based on fossil fuel resources of different accessibility. We assume that the most accessible resources are developed first, and hence, the unit extraction cost c_{dt} will tend to increase in accumulated extraction. Production of clean energy on the other hand, is based on setting aside dedicated areas (onshore, offshore and/or below ground), which is assumed to be in abundant supply and have constant unit costs $c_{ct} = \bar{c}$. The profits π_{jt} in each of the sectors are then given by:

$$\pi_{jt} = P_{jt} R_{jt}^{\alpha_2} \int_0^1 A_{jit}^{1-\alpha_1} x_{jit}^{\alpha_1} di - \int_0^1 p_{jit} x_{jit} di - (c_{jt} + \tau_{jt}) R_{jt}, \tag{2}$$

where p_{jit} is the price of machine type i , c_{jt} is the unit resource costs, and τ_{jt} is an extraction tax

[1] Without loss of generality, we disregard labour input in energy production.

reflecting the fossil resource constraint with the corresponding scarcity rent ${\mu_{jt}}^2$. The representative producer maximizes profits π_{jt} with respect to R_{jt} and x_{jit}. The first order condition with respect to the optimal use of the resources is given by:

$$\alpha_2 P_{jt} R_{jt}^{\alpha_2-1} \int_0^1 A_{jit}^{1-\alpha_1} x_{jit}^{\alpha_1} di - c_{jt} - \tau_{jt} = 0, \tag{3}$$

yielding the resource use on reduced form:

$$R_{jt} = \left(\frac{\alpha_2 P_{jt} \int_0^1 A_{jit}^{1-\alpha_1} x_{jit}^{\alpha_1} di}{c_{jt} + \tau_{jt}} \right)^{\frac{1}{1-\alpha_2}}. \tag{4}$$

Or alternatively, using the definition of Y_{jt}, the resorce use can be written:

$$R_{jt} = \frac{\alpha_2 P_{jt} Y_{jt}}{c_{jt} + \tau_{jt}}. \tag{5}$$

In the business as usual case we have $\tau_{jt} = 0 \; \forall t$. Note that increased unit extraction costs c_t will decrease the resource use, while higher average machine quality A_{jt} will increase the resource use. That is, new innovations increase how much energy can be produced from a resource.

As mentioned above, we assume that the unit extraction cost for the fossil resources c_{dt} is increasing in the accumulated amount of resources already extracted (there is no physical limit to the amount that can be extracted):

$$c_{dt} = c(Q_t),$$

where Q_t is accumulated extraction at time t ($Q_0 = 0$), and $c' > 0$, $c'' > 0$.[3] We assume that firms in the dirty sector do not take into account the development in c_{dt} when maximizing profits, for instance, because they do not have property rights of the resources but can apply for concessions issued by the government and have to start their activity within a certain time.

In an alternative case, we assume that firms face an extraction tax τ_{dt}. The effective private cost per unit extracted is then $c_{dt} + \tau_{dt}$. The extraction tax is set equal to the scarcity rent μ_{dt}, which is given by:

[2] Note that since $c_{ct} = \bar{c}$ and $\mu_{ct} = 0$ for all t, we set $\tau_{ct} = 0$ for all t.

[3] In the simulations later we use $c_{dt} = c_0(1 + \phi Q_t^2)$ where ϕ is a parameter.

$$\mu_{dt} = (1+r)\mu_{dt-1} - c'(Q_t)R_{dt},$$

with $lim_{t\to\inf}(1+r)^{-t}\mu_{dt}Q_t = 0$, where r is the per period discount rate.The extraction tax τ_{jt} reflects that higher extraction today increases extraction costs in future periods.

Next, the first order condition with respect to the optimal use of machines given by:

$$\alpha_1 P_{jt} R_{jt}^{\alpha_2} A_{jit}^{1-\alpha_1} x_{jit}^{\alpha_1 - 1} - p_{jit} = 0, \tag{6}$$

Rearranging (6) yields the demand function for machines in both sectors:

$$x_{jit} = \left(\frac{\alpha_1 P_{jt} R_{jt}^{\alpha_2}}{p_{jit}}\right)^{\frac{1}{1-\alpha_1}} A_{jit.} \tag{7}$$

The machine producers have a monpoly on their machine type i and maxmize profit with point of departure in (7). Lastly, note that the government may tax π_{jt} without changing the first order conditions (3) and (6). Resource rent taxation is however beyond the scope of the paper.

3.3 Supply of machines

A domestic producer with the highest quality machine type ji is in effect a monopolist and solves:

$$\max_{p_{jit}}[(p_{jit} - \psi(1-\sigma))x_{jit}], \tag{8}$$

where demand x_{jit} is given by (7) above, ψ is the unit cost of a machine, and σ is a subsidy to correct for the static monopoly distortion. The problem (8) yields:

$$p_{jit} = \frac{\psi(1-\sigma)}{\alpha_1} \tag{9}$$

Without loss of generality, costs are normalized to $\psi = \alpha_1^2$ (cf. Acemoglu et al., 2012), and the efficient subsidy rate that gives price equal to marginal cost is then $\sigma = 1 - \alpha_1$, which we assume is implemented for both machine types. The profit maximizing price on machines is then $p_{jit} = \psi = \alpha_1^2$. Inserting back into (8), we have: $\pi_{jit} = \alpha_1^2(1-\alpha_1)x_{jit}$. Further, using (7), we obtain for the per period profit π_{jit} of a machine producer:

$$\pi_{jit} = \bar{\alpha} \left(P_{jt} R_{jt}^{\alpha_2} \right)^{\frac{1}{1-\alpha_1}} A_{jit.}, \tag{10}$$

where $\bar{\alpha} = (1 - \alpha_1) \, \alpha_1^{\frac{1-2\alpha_1}{1-\alpha_1}}$.

3.4 Innovation and allocation of scientists

Average machine quality increases both due to successful innovation by domestic scientists and the arrival of foreign innovations. When a new innovation is made or imported of machine type i, A_{jit} bumps up to $(1+\gamma)A_{jit}$, where γ is the quality step.

We normalize the number of domestic scientists to one:

$$\ell_{ct} + \ell_{dt} = 1 \tag{11}$$

where the mass of scientists in one sector is given by ℓ_{jt}. A scientist can choose sector, but not target a specific machine type; instead a scientist is randomly allocated to a machine type in the specific sector. Thus, the scientist makes her decision based on the average machine quality in sector A_{jt} which is given by:

$$A_{jt} \equiv \int_0^1 A_{jit} di. \tag{12}$$

A scientist engaged in innovation in sector j then expects a quality $(1+\gamma)A_{jt}$ upon successful innovation. Further, we assume that there may be duplication by other scientists, i.e. more than one scientist may have the same successful innovation in a given period. We let the duplication effect be represented by decreasing returns to labor input on aggregate sector innovation given by the function $\ell_{jt}^{\varpi}$where $\varpi \in (0,1)$. The probability of a successful domestic innovation in sector j is then given by $\eta_j \ell_{jt}^{\varpi}$, where η_j is a parameter.

Foreign innovations in sector j arrive with probability ν_j. The average quality of the machine types then develops according to:

$$A_{j,t} = \left(1 + (\eta_j \ell_{jt}^{\varpi} + \nu_j)\gamma\right) A_{jt-1}. \tag{13}$$

We will not analyze the effect of forreign innovation v_j in the theory section, but return to it in the numerical simulations. Moreover, we follow Acemoglu et al. (2012) and assume that scientists only earn profits on an innovation in the same period as they innovate. Using (10), the expected profit Π_{jt} of a single scientist entering sector j at time t is then given by:

$$\Pi_{jt} = (1 + s_{jt})\eta_j \ell_{jt}^{(\varpi-1)} \bar{\alpha} \left(P_{jt} R_{jt}^{\alpha_2}\right)^{\frac{1}{1-\alpha_1}} (1+\gamma) A_{jt-1} \tag{14}$$

where $\ell_{jt}^{(\varpi-1)}$ is the average productivity of a scientist in sector j and s_{jt} is a subsidy to research in sector j. The allocation of scientists is then decided by the following arbitrage condition: $\Pi_{ct} = \Pi_{dt}$. By solving for ℓ_{ct}/ℓ_{dt} we obtain:

$$\frac{\ell_{ct}}{\ell_{dt}} = \left(\frac{(1 + s_{ct})\eta_c \left(P_{ct} R_{ct}^{\alpha_2}\right)^{\frac{1}{1-\alpha_1}} A_{ct-1}}{(1 + s_{dt})\eta_d \left(P_{dt} R_{dt}^{\alpha_2}\right)^{\frac{1}{1-\alpha_1}} A_{dt-1}} \right)^{\frac{1}{1-\varpi}} . \tag{15}$$

First, remember from (4) that, *ceteris paribus*, resource use will be higher, the lower the resource price. Second, implementing a neutral R&D policy, e.g., setting $s_{ct} = s_{dt}$ and assuming $\eta_c = \eta_d$, we observe from (15) that the allocation of scientist in the economy under study is governed by three factors:

Proposition 1 *More researchers will be allocated to a sector j i) the higher is the final product price P_{jt}, ii) the lower is the private resource cost $c_{jt} + \tau_{jt}$, and iii) the higher is the level of technology A_{jt-1}.*

In case resource use is regulated by the state e.g. through concessions such that (4) may not hold, the state will also (indirectly) influence the allocation of researchers by its concession policy.

The decentralized *laissez fair* equilibrium is now characterized. That is, for each period t (1), (3), (11). (13) and (15) constitute eigth equations which together determine the eight variables $Y_{dt}, Y_{ct}, R_{dt}, R_{ct}, A_{dt}, A_{ct}, l_{ct}$ and l_{dt} given initial values A_{d0} and A_{c0}. Finally, the wealth W^0 of the open economy in the *laissez fair* is given by:

$$W^0 = \sum_{t=0}^{\infty} \frac{1}{(1+r)^t} \left[\sum\nolimits_j P_{jt} Y_{jt} - \psi \left(\int_0^1 x_{cit} di + \int_0^1 x_{dit} di \right) - c_{dt} R_{dt} - \bar{c} R_{ct} \right] \tag{16}$$

where the terms in the brackets are the per period net value of the energy production. In the next section we compare the *laissez fair* with the first best.

4 First best and innovation subsides

4.1 The socially optimal allocation of researchers

The planner's problem is to maximize W^0 in (16), that is, the present value of production, with technologies A_j and resource Q as the stock variables:

$$\begin{aligned} \max_{L_{jt},\ell_{jt},R_{jt},Y_{jt}} \quad & \sum_{t=0}^{\infty} \tfrac{1}{(1+r)^t} \left[\sum_j P_{jt}Y_{jt} - \psi\left(\int_0^1 x_{cit}di + \int_0^1 x_{dit}di\right) - c_{dt}R_{dt} - \bar{c}R_{ct}\right] \\ s.t \quad & Y_{jt} = R_{jt}^{\alpha_2}\int_0^1 A_{jit}^{1-\alpha_1} x_{jit}^{\alpha_1} di \\ & A_{jt} = [1 + (\eta_j \ell_{jt}^{\varpi} + \nu_j)\gamma]A_{jt-1} \\ & Q_t = Q_{t-1} + R_{d,t-1} \\ & \ell_{ct} + \ell_{dt} \leq 1, \end{aligned} \tag{17}$$

given $A_{c0} < A_{d0}$, Q_0, and $c_{dt} = c(Q_t)$.

Solving the problem (17) gives the following expression for the optimal allocation of scientists (where we have set $\eta_c = \eta_d$):

$$\frac{\ell_{ct}^S}{\ell_{dt}^S} = \left(\frac{\frac{A_{ct-1}}{A_{ct}} \sum_{k=0}^{\infty} (\frac{1}{1+r})^k P_{c,t+k} Y_{c,t+k}}{\frac{A_{dt-1}}{A_{dt}} \sum_{k=0}^{\infty} (\frac{1}{1+r})^k P_{d,t+k} Y_{d,t+k}} \right)^{\frac{1}{1-\varpi}}, \tag{18}$$

where ℓ_{jt}^S denotes the first best allocation of researchers at time t to innovation in sector j (see Appendix A.1 for derivation).

To compare the socially optimal allocation with the market allocation of researchers, we rewrite (15) and set $\eta_c = \eta_d$ and $s_{ct} = s_{dt}$. We then get the following expression for the decentralized allocation of scientists[4]:

$$\frac{\ell_{ct}^M}{\ell_{dt}^M} = \left(\frac{\frac{A_{ct-1}}{A_{ct}} P_{ct} Y_{ct}}{\frac{A_{dt-1}}{A_{dt}} P_{dt} Y_{dt}} \right)^{\frac{1}{1-\varpi}} \tag{19}$$

[4] Inserting (7) and $p_{ijt} = \alpha_1^2$ into (1), we can substitute R_{jt} with Y_{jt} by using $R_{jt}^{\frac{\alpha_2}{1-\alpha_1}} = Y_{jt} A_{jt}^{-1} \left(\frac{P_{jt}}{\alpha_1}\right)^{\frac{-\alpha_1}{1-\alpha_1}}$. This is then inserted into (15).

where ℓ_{jt}^M denotes the market allocation of researchers at time t to innovation in sector j.

Not surprisingly, the first best allocation of researchers given by (18) may depart from the market allocation of researchers given by (19). Along the socially optimal growth path, the social planner allocates more scientists to the innovation sector in which the net present value of the total future production is greater, while in the market allocation researchers are allocated to the sector in which today's value of production is greater.[5] This difference in allocation may lead to a (technology) resource curse:

Definition 1 *The economy is in a state of (technology) resource curse if along the* laissez fair *growth path, more researchers are allocated to dirty innovation than clean innovation in each period, i.e.,* $\ell_{dt}^M > \ell_{ct}^M \; \forall t$*, while there exist alternative development paths that give higher wealth as given by (16) in which more researchers are allocated to clean innovation than dirty innovation from a period* T *onwards, i.e.,* $\ell_{dt} < \ell_{ct} \; \forall t > T$.

Our numerical analysis in Sub-section 5.3 shows one example of such a lock-in in technology, which we label as a resource curse. A resource curse is caused by the market misallocating researchers. One obvious reason is that fossil fuel prices in the future may fall relative to clean energy prices due to climate policy. Another reason is that future fossil fuel production will become more costly. As we will see, both factors may in isolation give rise to a resource curse.

4.2 Comparing allocations of scientists

We now want to inquire further into optimal policies for avoiding a resource curse as defined above. Consider the following ratio between the allocation of scientists in the decentralized market solution relative to the planner's solution:

$$\frac{\frac{\ell_{ct}^M}{1-\ell_{ct}^M}}{\frac{\ell_{ct}^S}{1-\ell_{ct}^S}} = \left(\frac{\frac{P_{ct}Y_{ct}}{P_{dt}Y_{dt}}}{\frac{\sum_{k=0}^{\infty}(\frac{1}{1+r})^k P_{c,t+k}Y_{c,t+k}}{\sum_{k=0}^{\infty}(\frac{1}{1+r})^k P_{d,t+k}Y_{d,t+k}}} \right)^{\frac{1}{1-\varpi}}. \qquad (20)$$

[5] Assuming that scientists discount the future completely as in (19) is admittedly a strong assumption, which we have copied from Acemoglu et al (2012). We return to this issue in Section 6.

We pose the following question: What innovation sector must be subsidized more in a given period in order to implement that period's efficient allocation of scientists when either resource market values or resource extraction policies change? Note that a subsidy to an innovation sector j will simply increase the value of doing research in sector j, see (14). For instance, a decline in the price P_{dt} will, ceteris paribus, reduce the allocation of scientists to dirty innovation *both* in the decentralized market and the socially optimal solution. However, the relevant question is not whether dirty innovation is reduced, but whether the relative change in allocations given by (20) is impacted such that a change in R&D policy is warranted.

This relative change in allocations depends on both the direct price effect and the indirect effects of changes in the development of technology $A_{j,t+k}$ and the private resource cost $c_{dt} + \tau_{dt}$. To simplify notation, we define the private resource costs $\chi_{dt} \equiv c_{dt} + \tau_{dt}$. In the following we develop three propositions focusing on the different effects in isolation. That is, in each proposition we fix the paths of two of the three variables $A_{j,t+k}$, $P_{j,t+k}$ and $\chi_{d,t+k}$, and examine the effects on R&D subsidies of a change in the chosen variable.

Holding prices $P_{j,t+k}$ and resource costs $\chi_{d,t+k}$ fixed, we first have a proposition on optimal subsides and the isolated *technology effect*:

Proposition 2 *Along an optimal subsidy path where the socially optimal $A_{j,t+k}$ are induced in the market while the resource cost $\chi_{d,t+k}$ is fixed for $j \in c, d$ for all k, the optimal subsidy s^*_{jt} at time t is increasing in the socially optimal allocation of scientists $\ell^S_{j,t+k}$ for any $k > 0$.*

Proof. See Appendix $A.3$ ■

That is, holding prices and resource costs fixed, if the planner wants to increase the allocation of scientists to clean innovation in the future, the optimal subsides to clean innovation needs to increase today. Thus, even if the fossil fuel price stays constant, the efficient allocation of scientist to dirty innovation may go down due to increasing extraction costs, and this *technology effect* of reduced dirty technology growth tends to push up the need for clean innovation subsides. We demonstrate in the numerical illustrations that along an optimal growth path in which clean technology substitutes for dirty technology, subsidies to clean energy should be higher at the start, but should never completely be removed.

Matters are, however, more complicated as this technology effect may be countervailed by the direct effect of fossil fuel prices on optimal subsides. Suppose the planner sets subsides to induce some given technology paths $A_{j,t+k}$. We can think of these paths as the optimal technology paths prior to a change in fossil fuel prices. Then, taking these paths $A_{j,t+k}$ as given while holding resource costs $\chi_{d,t+k}$ fixed, we have the following proposition on optimal subsides and the isolated *price effect*:

Proposition 3 *Along a subsidy path where some given technology paths $A_{j,t+k}$ are induced in the market while the resource cost $\chi_{d,t+k}$ is fixed for $j \in c, d$ for all k, the subsidy s_{jt} at time t necessary to induce A_{jt} is:*

i) unchanged if the percentage fall in prices $P_{j,t+k}$ is the same for all k;

ii) lower if the percentage fall in prices $P_{j,t+k}$ is larger for at least one k than the price fall at time t (and the price fall is at least equal to the price fall at time t for all other k);

iii) higher if the percentage fall in the price $P_{j,t+k}$ is larger at time t than the price fall for any k.

Proof. See Appendix $A.4$ ■

That is, locking down technology and resource costs paths, whether the optimal subsidy increases or decreases depends on at *when* the price change occurs. Focusing on lower fossil fuel prices, scientists today do not take into account a fall in future prices and thus provide too much dirty innovation today, which tends to push up the need for subsides to clean innovation. Contrary to this, if fossil fuel prices fall relatively more today than in the future, scientists reduce dirty innovation too much. Thus, less subsides to clean innovation is needed.

Proposition 3 together with Proposition 2 imply that optimal subsidies to clean innovation increase today when fossil fuel prices fall with a constant rate or with a higher rate in the future than today. However, when fossil fuel prices fall relatively more today than in the future, the technology effect and the price effect pull in opposite directions, and in sum optimal subsidies to clean innovation today may increase or decrease. In the numerical illustrations we focus on a gradual decline in fossil fuel prices, and show that this induces a large positive shift in the subsidy to clean R&D compared with a scenario in which fossil fuel prices stay constant.

We next turn to changes in the private resource cost $\chi_{d,t+k}$ and how this interacts with innovation incentives and optimal subsides. Recall that resource use in a period is given by (4), so resource use and costs are implicitly contained in (20) through the energy production market values $P_{jt}Y_{jt}$. Consider a market with a resource extraction tax set lower than in an optimally regulated market, i.e. $0 \leq \tau_t < \mu_t$. The problem then is that firms do not sufficiently take into account that future costs are increasing in current resource extraction. The planner should raise the extraction tax τ_t such that $\tau_t = \mu_t$. This shift in policy would increase the net present value of fossil fuel production, which again would increase the socially optimal allocation of scientists to dirty innovation. From Proposition 2 we know that this technology effect in isolation tends to push down the need to subsidize clean innovation today.

However, as with the price effect, this technology effect may be countervailed by the direct effect of the resource extraction costs on optimal subsides. Holding prices $P_{j,t+k}$ fixed and taking some technology paths $A_{j,t+k}$ as given, we have the following proposition on optimal subsides and the isolated *resource costs effect:*

Proposition 4 *Along a subsidy path where some given technology paths $A_{j,t+k}$ are induced in the market for $j \in c, d$ for all k, the subsidy s_{dt} at time t necessary to induce A_{dt} is:*

i) unchanged if the percentage rise in private resource costs $\chi_{d,t+k}$ is the same for all k;

ii) lower if the percentage rise in private resource costs $\chi_{d,t+k}$ is larger for at least one k than the rise at time t (and the rise is at least equal to the rise at time t for all other k);

iii) higher if the percentage rise in the resource costs $\chi_{d,t+k}$ is larger at time t than the rise for any k.

Proof. The proposition follows from Proposition 3 together with the fact that $\frac{\partial Y_{j,t+k}}{\partial(c_{d,,t+k}+\tau_{d,t+k})} < 0$.
■

That is, a change in the resource cost that is the same across all periods does not directly impact on the optimal subsidy. However, as scientists discount future incomes more than the planner, the time profile of a change in resource costs directly impacts on the optimal subsidy.

Perhaps more interestingly, a rise in the extraction tax τ_{dt} today dampens current extraction and thus leads to lower (or unchanged) unit costs $c_{d,t+k}$ in future periods (for given technology paths). Thus, holding prices $P_{j,t+k}$ fixed and taking some technology paths $A_{j,t+k}$ as given, while allowing for changes in the unit costs $\chi_{d,t+k}$, we have the following corollary on optimal subsides and the *scarcity rent effect:*

Corollary 1 *Along a subsidy path where some given technology paths $A_{j,t+k}$ are induced in the market for $j \in c, d$ for all k, the subsidy s_{dt} at time t necessary to induce A_{dt} is higher if the percentage rise in the extraction tax $\tau_{d,t+k}$ is larger at time t than the rise for any $k > 0$.*

That is, a higher extraction tax today increases the net present value of fossil fuel production (given $\tau_{dt} < \mu_{dt}$ before the change) as unit resource costs are lower in the future. This increases both the planner's and the decentralized market's allocation of scientists to dirty innovation in future periods. However, the value of fossil fuel production today is lower, which reduces dirty innovation incentives in the market today, exacerbating the undersupply of dirty innovation that follows from the technology effect given by Proposition 2. Thus, a rise in the extraction tax today and *not* in the future, implies that the optimal subsidy to clean innovation decreases today.

5 Numerical simulations

5.1 Data input

In this section we illustrate our theoretical findings by a numerical simulation of the model. The length of each period is set to 5 years as in Acemoglu et al. (2012). We assume the annual (real) discount rate to be 0.04. Following Acemoglu et al. (2012), we further assume the annual probability of innovation to be 0.02, and the technology quality step to be one. In the main simulations, we disregard arrivals of foreign innovations, and focus on domestic innovations. The parameter ϖ, which determines decreasing returns to scale of scientists in each sector (dirty and clean innovations), is set equal to 0.5 based on Acemoglu et al. (2016). For the parameters in the Cobb-Douglas production function we have $\alpha_1 = 0.3$ and $\alpha_2 = 0.3$ yielding decreasing returns to scale for this small open economy.

We consider four scenarios. The intial level of the dirty technology A_{d0} is normalized to 1 in all scenarios. Moreover, we set the intial price of clean energy P_{c0} and dirty energy P_{d0} to 1. Then in two of the scenarios both prices are kept constant over time, while in the other two scenarios we allow the market price of dirty energy to fall by a constant rate equal to 5 percent each 5 years period. The table below shows the essential parameter combinations in the four scenarios:

Table 1. The four scenarios

Scenario	I	II	III	IV
A_{c0}	0.4	0.4	0.6	0.6
$\frac{\Delta P_{dt}}{P_{dt}}$	0%	-5%	0%	-5%

For all the four scenarios we simulate four policy cases: *laissez fair* (BaU), optimal policy which involves an extraction tax equal to μ_{dt} and a subsidy s_{ct} to clean R&D, a second best subsidy which only involves a subsidy to clean R&D, and finally, a second best tax, which only includes an extraction tax.

In principle we can have four types of outcomes corresponding with our four scenarios:

1. Steady course: It is optimal to keep on extracting fossil fuels since the increasing extraction cost can be counteracted by focusing R&D effort in the dirty energy sector.

2. Resource curse due to global climate policy: Keeping on extracting fossil fuels is not optimal when the market price on fossil fuels decreases, but the private sector does not shift the R&D effort to the the clean energy sector and extraction continues for too long.

3. Resource curse with no global climate policy: Keeping on extracting fossil fuels is not optimal even if the market price on fossil fuels stays constant, but the private sector does not shift the R&D effort to the clean energy sector and extraction continues for too long.

4. Induced change in course due to climate policy: Keeping on extracting fossil fuels is not optimal and the private sector shifts the R&D effort to the clean energy sector without intervention from the government.

Clearly, the state of the clean technology is essential. If this is fairly developed, a shift to this technology is optimal independent of global climate policy, while if this is underdeveloped, a shift should only happen as a responce to global climate policy.

The numerical results are shown in Figures 1-4, one figure for each scenario. In each figure there are four panels, showing respectively fossil fuel resource extraction, the relative contribution to national wealth of the clean technology, allocation of researchers to the dirty technology, and tax and subsidy levels.

5.2 Scenario I: Steady course

Scenario I is the case with constant energy prices, and an underdeveloped clean energy sector. We see from Figure 1a) that fossil fuel resource extraction is intially too high without an extraction tax (the tax is implemented in "optimal" and "2nd best tax"), and that a clean R&D subsidy alone ("2nd best subs") plays no role in changing the extraction path, that is, the BaU path and the path with only a subsidy to clean R&D are almost identical.

Figure 1 to be placed here

From Panel b) we see that profits from dirty energy production (and hence contribution to national wealth) dominates clean energy throughout the whole time horizon, and moreover, that dirty energy's dominance is increasing irrespective of policy. The reason can be found in Panel c): The allocation of researchers is practically the same in all policy cases, with almost all researchers entering the dirty sector. Clearly, there is no resource curse induced by directed technical change in this scenario. That is, reseachers should continue working on reducing the cost of dirty energy production.

Finally, we see from Panel d) that in order to accomplish an optimal extraction of the fossil fuel resource, the government should impose an extraction tax of 50-100% of the resource cost. This should be combined with a slightly negative subsidy to clean R&D. Since the government wants to continue resource extraction, knowledge spillovers from R&D are more valuable for dirty R&D. Without an extraction tax, however, there should be a positive subsidy to clean R&D initially to counteract the boosting effect on dirty R&D from a too high initial extraction.

5.3 Scenario II: Resource curse due to global climate policy

In Scenario II the price on dirty energy declines over time, while we still have an underdeveloped clean energy sector intially. We see from Figure 2a) that resource extraction is continuously too high in BaU, and that, in particular, the optimal extraction path and the extraction path with only an extraction tax involve lower extraction, especially in the beginning.

Figure 2 to be placed here

Panel b) now shows another picture: In all four policy cases clean energy will eventually dominate dirty energy with respect to profits and wealth creation. This, however, happens far later in BaU and with only an extraction tax ("2nd best tax"). In fact, with an optimal policy or with a clean R&D subsidy alone, the value of the clean sector will trumf the value of dirty after 20-30 years, while without the R&D subsidy this will happen after 60-70 years.

From Panel c) we note that the BaU and the second best tax both involve a far too slow redirection of R&D. Both with the opimal policy mix and in the case with only a clean R&D subsidy, researchers are rather quickly moved to clean R&D. Thus, these results suggest that governments cannot look at current profitability for a sector when deciding how to prioritize R&D.

Finally, in Panel d) we see that a redirection of researchers is resolved by substantial subsidies to clean R&D, both in the optimal policy case and in the case without an extraction tax (subsidy rates of 500-700% of the expected private profit initially). Obviously, the clean R&D subsidy needs to be even higher if the government for some reason does not tax extraction. In our opinion, Scenario II qualifies as a resource curse as the economy under study clearly continues extraction for far too long and misses out on the opportunity to develop an alternative sector.

5.4 Scenario III: Resource curse with no global climate policy

In this scenario the price on dirty energy stays constant, but compared to Scenario I and II, we initially have a more developed clean energy sector. As for Scenario II, we see from Figure 3a) that resource extraction is continuously too high compared with the optimal extraction path and the extraction path with only an extraction tax. In Panel b) we see that with an optimal policy or with a clean R&D subsidy alone, the value of the clean sector will trumf the value of dirty after 20 years, while without the R&D subsidy this might never happen (BaU) or happen after more than 80 years ("2nd best tax").

Figure 3 to be placed here

Furthermore, as for Scenario II we see from Panel c) that the BaU involves a far too slow redirection of R&D. Both for the optimal extraction path and the extraction path with only an extraction tax, researchers are even more quickly moved away from dirty R&D than in Scenario II. Thus, we seem to have a resource curse even if fossil fuel prices do not decline relative to clean energy prices. The reason is of course the future increasing cost of extraction, and that clean energy is a relatively low hanging fruit in this scenario, which the private R&D sector misses out on.

Finally, in Panel d) we see that a redirection of researchers is resolved by substantial subsidies to clean R&D in both the policy cases with and without an extraction tax. Again, the clean R&D subsidy needs to be even higher if the government cannot tax extraction.

5.5 Scenario IV: Induced change in course due to climate policy

In the final scenario the price on dirty energy declines over time, and we initially have a more developed clean energy sector (as in Scenario III). Seemingly, the declining dirty energy price partly resolves the resource curse situation we observed in Scenario III. As we can see from Figure 4a), the extraction paths in all four policy scenarios merge over time (as in Scenarios I and II). Thus, in Scenario IV resource extraction is too high only in the first 20-25 years. However, from Panel c), we see that a lack of appropriate policies will still postpone redirection of R&D. While with an optimal policy mix all researchers should be moved to clean R&D initially, in the BaU, more than half of the researchers stay in dirty R&D for 30-35 years. Thus, even with declining fossil fuel prices relative to clean energy prices, we may experience a milder form of resource curse.

Figure 4 to be placed here

As can be seen from Panel b), a redirection of researchers quickly makes the clean energy sector more profitable than dirty; it happens in about 10-15 years time. While in the BaU, dirty energy stays more profitable for another 15 years.

Finally, in Panel d), as in Scenario II and III, we see that a redirection of researchers is resolved by substantial subsidies to clean R&D (whether or not an extraction tax is imposed), with the highest subsidy in the case when the government does not tax extraction.

5.6 Wealth effects

In our model national wealth is given from equation (16). For each of the policy cases in each of the Scenarios I-IV, we can calculate the value of the discounted stream of net profits from energy production over the time horizon of the model. In Table 2 we have the four scenarios in the columns from left to right and the four policy cases in the rows from top to bottom.

Table 2. Wealth comparison

Scenarios	I	II	III	IV
BaU	3.52	2.40	4.17	3.65
Optimal	3.96	2.89	5.00	4.43
2nd best subsidy	3.53	2.82	4.84	4.37
2nd best tax	3.95	2.56	4.59	3.91
(Optimal-BaU)/BaU	12.5%	20.4%	20.0%	21.4%
(2nd subsidy-BaU)/(optimal-BaU)	0.3%	85.6%	80.5%	92.6%
(2nd tax-BaU)/(optimal-BaU)	97.9%	31.3%	50.7%	33.4%

In the row "(Optimal-BaU)/BaU" we measure the relative difference in total discounted wealth between the optimal policy mix and the BaU. As discussed above, in Scenario II to IV the researchers should be shifted swiftly to the clean technology, but without a policy this may never happen or it happens too late. In all these three scenarios it leads to a loss in wealth by approximatly 20%. In Scenario I the welfare difference is smaller (12%), but not insignificant as with much dirty energy production the lack of an extraction tax matters quite a lot.

In the next row "(2nd subsidy-BaU)/(optimal-BaU)", we measure how close a stand alone subsidy to clean R&D can take us from BaU towards the optimal solution. We see that for the three scenarios II to IV, more than 80 percent of the gap is closed. Hence, prioritizing clean R&D seems to be crucial.

Then, in the last row we measure how close a stand alone extraction tax can take us towards the optimal solution. We note that the tax is insufficient for the three scenarios II to IV, but that it takes us nearly all the way in Scenario I (in which case R&D policies are less needed). Hence, an extraction tax seems to be an inefficient instrument for redirecting R&D effort.

6 Discussion and conclusion

In this paper we follow Acemoglu et al (2012) and let patents only last for one period. Hence, a future slump in fossil fuel prices will not redirect private research from dirty to clean energy. Furthermore, since extraction paths are forward biased, the tendency for the market to allocate too many reserachers to fossil fuel technology, is exacerbated. We show that the government should counteract these effects by giving a higher subsidy to clean energy research.

Admittingly, assuming that researchers only look one period ahead may seem unrealistic. The important assumption for our purpose, however, is that reserachers put less weight on future periods than the planner. This is the case in Greaker et. (2018) even though in that paper researchers have complete and perfect foresight. A crucial insight from Greaker et al (2018) is that the results from Acemoglu et al (2012) are robust to changing from myopic researchers to farsighted researchers.

In our paper, R&D in dirty technology only competes with R&D in clean technology. Clearly, "clean technology" could be any new emerging field of technology; the only essential aspect of the alternative technology is that its relative price will with some probability increase *vis-a-vis* the price of fossil fuels. We have chosen to stay within the dirty and clean technology dichtonomy from Acemoglu et al. (2012). One example of a country that fits with this dichtonomy could be Algeria, which currently export oil and gas but could potentiallly produce solar energy both for electricity export and green hydrogen export. Another example could be Norway, which so far has escaped the resource curse, but still uses a lions share of its R&D resources on improving oil and gas extraction techniques, while having great opportunities for off-shore wind development.

Surprisingly, we find that global climate policy may both lead to a resource curse or help the country escaping a potential resource curse. The ripeness of the clean technology is essential for the outcomes: If the clean technology is not too far beyond the dirty technology, a shift to this technology is optimal independent of global climate policy and climate policy can induce this shift. While if the clean technology is underdeveloped, a shift should only happen as a responce to global climate policy, and the government must intervene to make it happen. Our paper could thus have policy implications for fossil fuel exporting countries also employing a major share of their high skilled workesrs in developing new technologies for fossil fuel exploration and extraction.

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Appendix

Appendix A: Theoretical model

A.1 Derivation of First-best

The Lagrangian given by problem 17 is:

$$\begin{aligned}\mathbf{L} \;=\; & \sum_{t=0}^{\infty}\frac{1}{(1+r)^t}\left[\sum\nolimits_j P_{jt}R_{jt}^{\alpha_2}\int_0^1 A_{jit}^{1-\alpha_1}x_{jit}^{\alpha_1}di-\psi\left(\int_0^1 x_{cit}di+\int_0^1 x_{dit}di\right)-c_{dt}R_{dt}-\bar{c}R_{ct}\right]\\ & -\sum_{t=0}^{\infty}\frac{1}{(1+r)^t}\lambda_{ct}(A_{ct}-(1+\gamma(\eta_c\ell_{ct}^{\varpi}+\nu_c))A_{c,t-1})\\ & -\sum_{t=0}^{\infty}\frac{1}{(1+r)^t}\lambda_{dt}(A_{dt}-(1+\gamma(\eta_d(1-\ell_{dt})^{\varpi}+\nu_d))A_{d,t-1})\\ & +\sum_{t=0}^{\infty}\frac{1}{(1+r)^t}\mu_t(Q_{t+1}-Q_t-R_{d,t}),\end{aligned}$$

where λ_{jt} is the shadow value of energy good $j\in\{c,d\}$, ω_{jt} is the shadow value of the average machine quality (the technology stock) in $j\in\{c,d\}$, and μ_t is the shadow cost of the extraction. Note that we set $\ell_{ct}+\ell_{dt}=1$.

The FOC wrt machines x_{jit} is

$$P_{jt}R_{jt}^{\alpha_2}A_{jit}^{1-\alpha_1}\alpha_1 x_{jit}^{\alpha_1-1}-\psi=0,$$

which using the fact $\psi=\alpha_1^2$ can be written

$$x_{jit} = \left(\frac{1}{\alpha_1} P_{jt} R_{jt}^{\alpha_2} L_{jt}^{\alpha_3}\right)^{\frac{1}{1-\alpha_1}} A_{jit},$$

which is the same as the market solution, when $\sigma = 1 - \alpha_1$.

The FOCs wrt resources R_{ct} and R_{ct} are

$$\begin{aligned} \alpha_2 P_{ct} R_{ct}^{\alpha_2 - 1} L_{ct}^{\alpha_3} \textstyle\int_0^1 A_{cit}^{1-\alpha_1} x_{cit}^{\alpha_1} di - \bar{c} &= 0 \\ \alpha_2 P_{dt} R_{dt}^{\alpha_2 - 1} L_{dt}^{\alpha_3} \textstyle\int_0^1 A_{dit}^{1-\alpha_1} x_{dit}^{\alpha_1} di - c_{dt} + \mu_t &= 0, \end{aligned}$$

which using the definition of Y_{jt} can be rewritten

$$\begin{aligned} R_{ct} &= \alpha_2 \frac{P_{ct} Y_{ct}}{\bar{c}} \\ R_{dt} &= \alpha_2 \frac{P_{dt} Y_{dt}}{c_{dt} + \mu_t}. \end{aligned} \tag{21}$$

Comparing he first best resource use (21) with the market solution (5) we see that they are the same as long as the firms face the scarcity rent μ_t. The development of the scarcity rent over time is given by the FOC wrt Q_t which can be written

$$\mu_t = (1 + r)\mu_{t-1} - \frac{\partial c_{dt}}{\partial Q_t} R_t.$$

The relevant FOC for the allocation of scientist is

$$\begin{aligned} \varpi \lambda_{ct} \gamma \eta_c \ell_{ct}^{\varpi - 1} A_{c,t-1} - \varpi \lambda_{dt} \gamma \eta_d (1 - \ell_{ct})^{\varpi - 1} A_{d,t-1} &= 0 \\ &\Leftrightarrow \\ \left(\frac{\lambda_{ct} \eta_c A_{c,t-1}}{\lambda_{dt} \eta_d A_{d,t-1}}\right)^{\frac{1}{1-\varpi}} &= \frac{\ell_{ct}}{1 - \ell_{ct}}. \end{aligned} \tag{22}$$

Now we want to substitute out the $\lambda's$ in (22). To this end, note that the FOC wrt average quality A_{jt} is

$$P_{jt} R_{jt}^{\alpha_2} L_{jt}^{\alpha_3} (1 - \alpha_1) \textstyle\int_0^1 A_{jit}^{-\alpha_1} x_{jit}^{\alpha} di - \lambda_{jt} + \left(\frac{1}{1+r}\right) \lambda_{j,t+1} (1 + \gamma(\eta_c \ell_{c,t+1}^{\varpi} + \nu_c)) = 0. \tag{23}$$

Next we use $A_{jt} \equiv \int_0^1 A_{jit} di$ and the definition of Y_{jt} to rewrite equation (23)

$$P_{jt}(1-\alpha_1)\frac{Y_{jt}}{A_{jt}} - \lambda_{jt} + (\frac{1}{1+r})\lambda_{j,t+1}(1+\gamma(\eta_c \ell_{c,t+1}^{\varpi} + \nu_c)) = 0. \tag{24}$$

Then we use equation (13) to rewrite equation (24):

$$\lambda_{jt} = P_{jt}(1-\alpha_1)\frac{Y_{jt}}{A_{jt}} + (\frac{1}{1+r})\lambda_{jt+1}\frac{A_{j,t+1}}{A_{jt}}. \tag{25}$$

Note that equation (25) can be expanded

$$\begin{aligned}
\lambda_{jt} &= P_{jt}(1-\alpha_1)\frac{Y_{jt}}{A_{jt}} + (\frac{1}{1+r})(P_{jt+1}(1-\alpha_1)\frac{Y_{j,t+1}}{A_{j,t+1}}\frac{A_{j,t+1}}{A_{jt}} \\
&\quad + (\frac{1}{1+r})^2\lambda_{jt+2}\frac{A_{j,t+2}}{A_{j,t+1}}\frac{A_{j,t+1}}{A_{jt}}) \\
&= P_{jt}(1-\alpha_1)\frac{Y_{jt}}{A_{jt}} + (\frac{1}{1+r})(P_{jt+1}(1-\alpha_1)\frac{Y_{j,t+1}}{A_{j,t+1}}\frac{A_{j,t+1}}{A_{jt}} \\
&\quad + (\frac{1}{1+r})^2 P_{jt+2}(1-\alpha_1)\frac{Y_{j,t+2}}{A_{j,t+2}}\frac{A_{j,t+2}}{A_{j,t+1}}\frac{A_{j,t+1}}{A_{j,t}} + (\frac{1}{1+r})^3\lambda_{jt+3}\frac{A_{j,t+3}}{A_{j,t+2}}\frac{A_{j,t+2}}{A_{j,t+1}}\frac{A_{j,t+1}}{A_{jt}},
\end{aligned}$$

and so forth. We use this to obtain

$$\lambda_{jt} = (1-\alpha_1)\frac{1}{A_{jt}}\sum_{v\geq t}(\frac{1}{1+r})^{v-t}P_{jv}Y_{jv}. \tag{26}$$

Then, combining equations (22) and (26), the optimal allocation of scientists is given by (18)

A.2 Partial derivatives of Y_{jt}

In (1) we substitute out R_{jt} using (5) and x_{jit} using (7) to rewrite Y_{jt} as a function of A_{jt}, $\mu_{jt}$and parameters:

$$Y_{jt} = \left(\frac{\alpha_2 P_{jt} Y_{jt}}{c_{jt}+\mu_{jt}}\right)^{\alpha_2} \int_0^1 A_{jit}^{1-\alpha_1} \left[\left(\frac{P_{jt}\left(\frac{\alpha_2 P_{jt} Y_{jt}}{c_{jt}+\mu_{jt}}\right)^{\alpha_2}}{\alpha_1}\right)^{\frac{1}{1-\alpha_1}} A_{jit.}\right]^{\alpha_1} di \tag{27}$$

$$Y_{jt} = \left(\frac{\alpha_2 P_{jt} Y_{jt}}{c_{jt}+\mu_{jt}}\right)^{\alpha_2} A_{jt} \left[\left(\frac{P_{jt}\left(\frac{\alpha_2 P_{jt} Y_{jt}}{c_{jt}+\mu_{jt}}\right)^{\alpha_2}}{\alpha_1}\right)^{\frac{1}{1-\alpha_1}}\right]^{\alpha_1}$$

$$Y_{jt} = A_{jt}^{\frac{1-\alpha_1}{(1-\alpha_2-\alpha_1)}} P_{jt}^{\frac{\alpha_2+\alpha_1}{(1-\alpha_2-\alpha_1)}} \left(\frac{1}{c_{jt}+\mu_{jt}}\right)^{\frac{\alpha_2}{(1-\alpha_2-\alpha_1)}} \alpha_2^{\alpha_2} \left(\frac{\alpha_2^{\alpha_2}}{\alpha_1}\right)^{\frac{\alpha_1}{(1-\alpha_2-\alpha_1)}}.$$

Clearly $\frac{\partial Y_{jt}}{\partial A_{jt}} > 0$ and $\frac{\partial Y_{jt}}{\partial P_{jt}} > 0$. For future reference note that $\frac{\partial Y_{jt}}{\partial c_{jt}} < 0$, $\frac{\partial Y_{jt}}{\partial \mu_{jt}} < 0$, and, since $\frac{1-\alpha_1}{(1-\alpha_2-\alpha_1)} > 1$, we have $\frac{\partial Y_{jt}}{\partial A_{jt}} > 1$. That is Y_j grows by a factor higher than the growth rate of A_j.

A.3 Proof Proposition 2

First we analyze an increase in the allocation to the clan sector keeping $A_{d,t+k}$ unchanged for all k. Note that the optimal subsidy is set so that the right hand side of (20) is equal to one so that $\ell_{c,t+k} = \ell^M_{c,t+k} = \ell^S_{c,t+k}$ for all k. That is the optimal subsidy is implemented and $A_{c,t+k}$ is at the socially optimal level also in the decentralized equilibrium. Then, to show that s^*_{ct} needs to go up at time t it is sufficient to show that

$$\frac{P_{ct} Y_{ct}}{\sum_{k=0}^{\infty} (\frac{1}{1+r})^k P_{c,t+k} Y_{c,t+k}}, \tag{28}$$

goes down. To this end we rewrite the problem slightly, using (27) to get

$$\frac{P_{ct} A_{ct}^{\beta} H_{ct}}{\sum_{k=0}^{\infty} (\frac{1}{1+r})^k P_{c,t+k} A_{c,t+k}^{\beta} H_{c,t+k}},$$

where $\beta = \frac{1-\alpha_1}{1-\alpha_2-\alpha_2}$ and $H_{jt} \equiv P_{jt}^{\frac{\alpha_2+\alpha_1}{1-\alpha_2-\alpha_1}} \left(\frac{1}{c_{jt}+\mu_{jt}}\right)^{\frac{\alpha_2}{1-\alpha_2-\alpha_1}} \alpha_2^{\alpha_2} \left(\frac{\alpha_2^{\alpha_2}}{\alpha_1}\right)^{\frac{\alpha_1}{1-\alpha_2-\alpha_1}}$. Showing that (28) goes down is equivalent to showing that

$$\frac{\sum_{k=0}^{\infty} (\frac{1}{1+r})^k P_{c,t+k} A_{c,t+k}^{\beta} H_{c,t+k}}{P_{ct} A_{ct}^{\beta} H_{ct}} = \frac{\sum_{k=0}^{\infty} (\frac{1}{1+r})^k P_{c,t+k} [\left(1 + (\eta_c \ell_{c,t+k}^{\varpi} + \nu_c)\gamma\right) A_{c,t+k-1}]^{\beta} H_{c,t+k}}{P_{ct}[(1 + (\eta_c \ell_{ct}^{\varpi} + \nu_c)\gamma) A_{ct-1}]^{\beta} H_{ct}}, \tag{29}$$

goes up, where we have used the fact that $A_{c,t+k} = \left(1 + (\eta_c \ell_{c,t+k}^{\varpi} + \nu_c)\gamma\right) A_{c,t+k-1}$. Noticing the compounding feature of innovation on A_c we can cancel out $(1 + (\eta_c \ell_{ct}^{\varpi} + \nu_c)\gamma)\, A_{ct-1}$ as it is present in all elements of the sum in the numerator, and write (29) as

$$\frac{\sum_{k=0}^{\infty} (\frac{1}{1+r})^k P_{c,t+k} [\prod_{v=1}^{k} \left(1 + (\eta_c \ell_{c,t+k}^{\varpi} + \nu_c)\gamma\right)]^{\beta} H_{c,t+k}}{P_{ct} H_{ct}}. \tag{30}$$

Then we have

$$\frac{\partial \left(\frac{\sum_{k=0}^{\infty} (\frac{1}{1+r})^k P_{c,t+k} [\prod_{v=1}^{k} \left(1+(\eta_c \ell_{c,t+k}^{\varpi} + \nu_c)\gamma\right)]^{\beta} H_{c,t+k}}{P_{ct} H_{ct}} \right)}{\partial \ell_{c,t+k}} > 0 \text{ for any } k > 0, \tag{31}$$

and we have thus established our result. Note that a partial change in ℓ_c only at time t, has no impact on s_{ct}.

Next, we also consider that impact of changing $A_{d,t+k}$. When $\ell_{c,t+k}$ increase for some k, $\ell_{d,t+k}$ must decrease with the same amount as $\ell_{c,t+k} + \ell_{d,t+k} = 1$. Due to symmetry in (20) this exacerbates the problem and further enhances the need for s_{ct} to go up at time t when $\ell_{c,t+k}$ increase for some $k > 0$.

Last, for completeness, notice that the derivative given by (31) is equal to zero for $k = 0$, since $\ell_{c,t}$ is canceled out from the fraction.

A.4 Proof of Proposition 3

Similarly to the proof of Proposition 2, showing that the optimal subsidy s_{jt}^* at time t needs to go up (down) to get the right hand side of (20) equal to one amounts to showing that (28) goes down (up). Note that $A_{j,t+k}$ for $j \in c, d$ are given for all k.

First consider the case where prices $P_{j,t+k}$ fall by the same percentage for all k. This does not impact on (28) as the price changes cancel out of the fraction.

Next, having established part i) of the proposition, it is sufficient to show that s_{jt}^* goes down when the prices $P_{j,t+k}$ decreases for any $k > 0$. Recall that in Appendix A.2 we established $\frac{\partial Y_{jt}}{\partial P_{jt}} > 0$.

It follows that $\frac{\partial(P_{j,t+k}Y_{j,t+k})}{\partial P_{j,t+k}} > 0$. Then, for any $k > 0$ we have

$$\frac{\partial\left(\frac{P_{jt}Y_{jt}}{\sum_{k=0}^{\infty}(\frac{1}{1+r})^k P_{j,t+k}Y_{j,t+k}}\right)}{\partial P_{j,t+k}} < 0,$$

and thus (28) goes up when $P_{j,t+k}$ decreases, and consequently s^*_{jt} goes down.

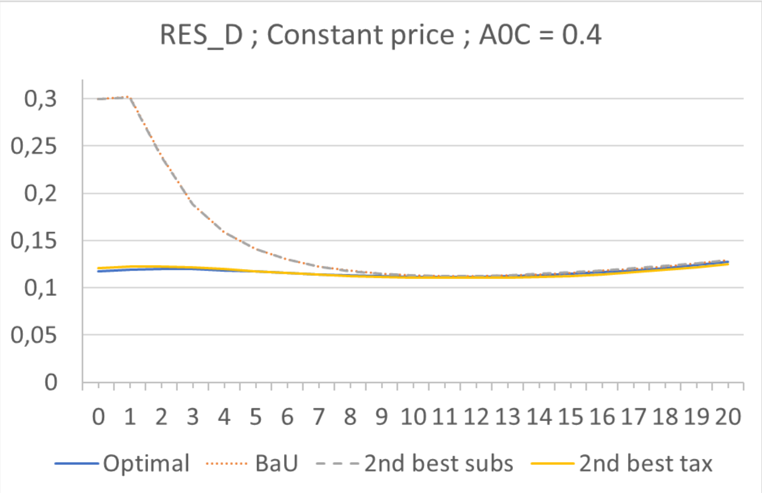
Last, consider the case when prices $P_{j,t+k}$ fall for $k = 0$. For $k = 0$,

$$\frac{\partial\left(\frac{P_{jt}Y_{jt}}{\sum_{k=0}^{\infty}(\frac{1}{1+r})^k P_{j,t+k}Y_{j,t+k}}\right)}{\partial P_{jt}} = \frac{\frac{\partial(P_{jt}Y_{jt})}{\partial P_{jt}}\left(\sum_{k=0}^{\infty}(\frac{1}{1+r})^k P_{j,t+k}Y_{j,t+k} - P_{jt}Y_{jt}\right)}{\left(\sum_{k=0}^{\infty}(\frac{1}{1+r})^k P_{j,t+k}Y_{j,t+k}\right)^2} > 0,$$

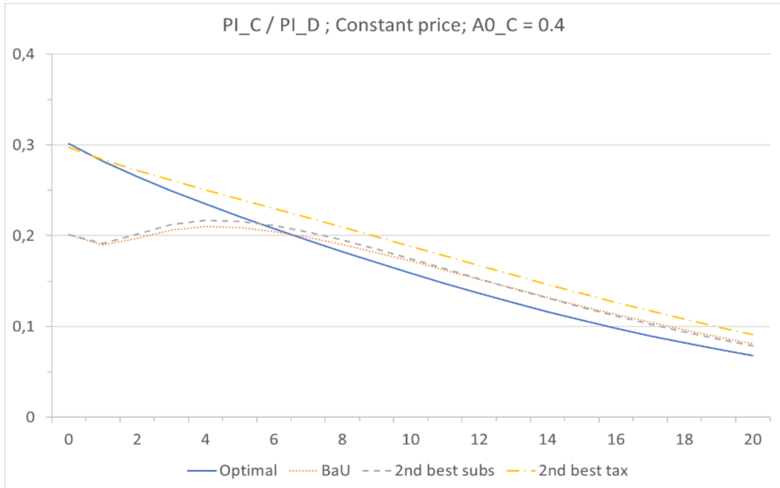
and thus (28) goes down when P_{jt} decreases and consequently s^*_{ct} goes up.

Figure 1 "Constant energy prices and underdeveloped clean sector"

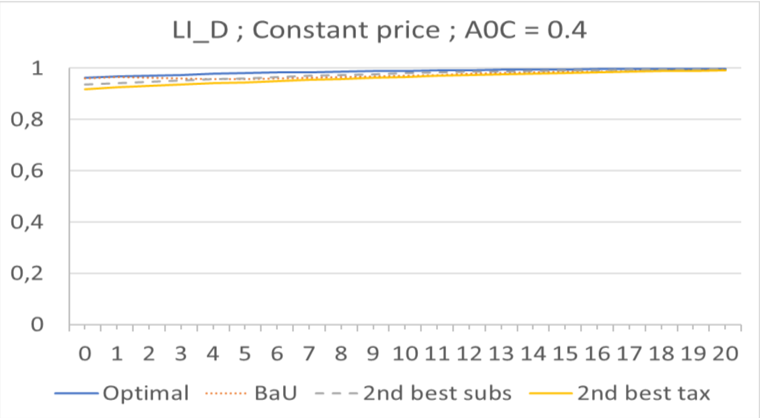
Panel a) Petroleum resource extraction



Panel b) Relative profits in clean versus dirty energy production



Panel c) Allocation of researchers to dirty R&D



Panel d) Tax and subsidy levels (as shares of resource costs and expected R&D profits, respectively)

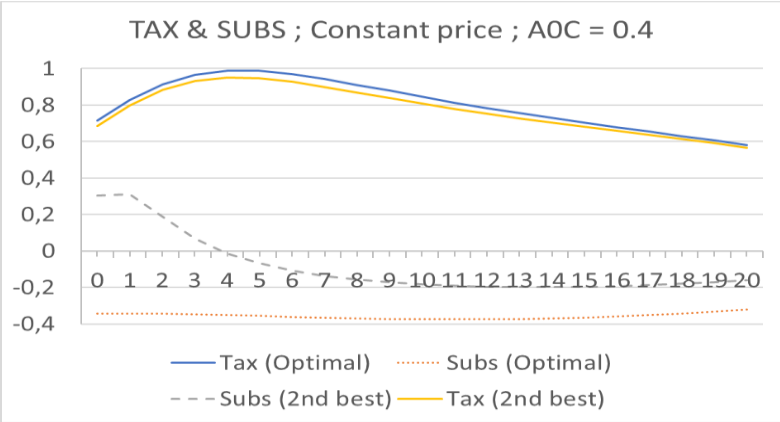
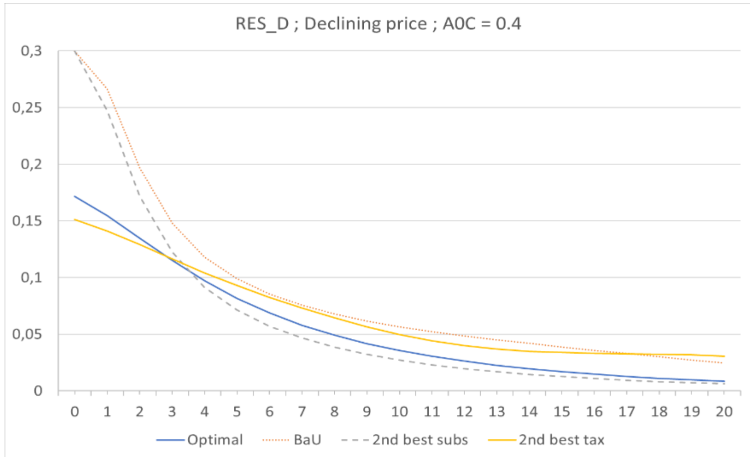
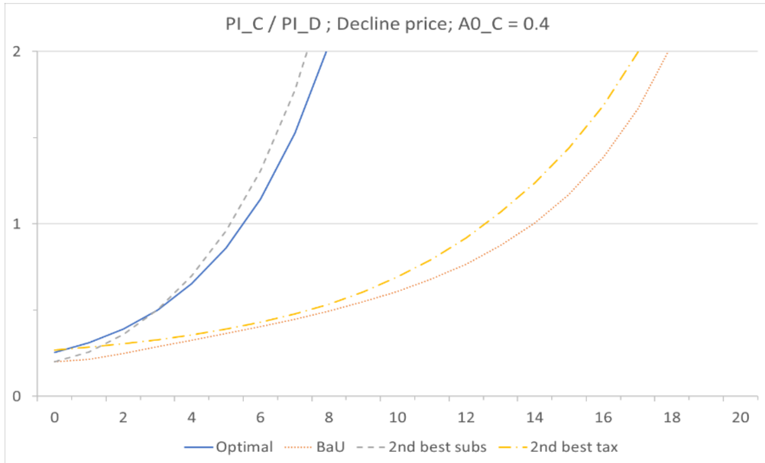


Figure 2 “Declining petroleum prices and underdeveloped clean sector”

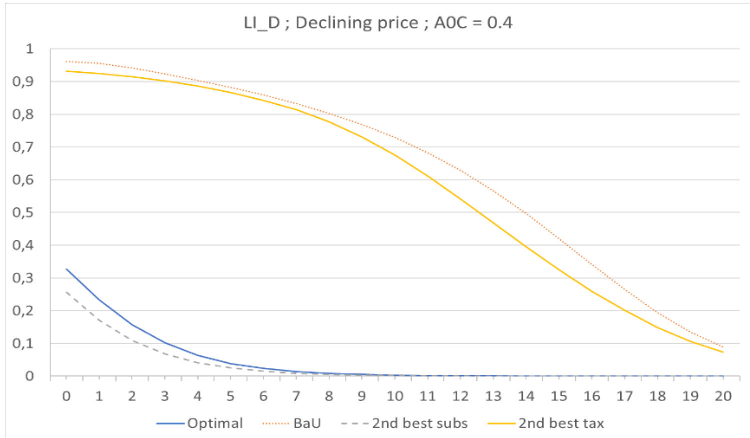
Panel a) Petroleum resource extraction



Panel b) Relative profits in clean versus dirty energy production



Panel c) Allocation of researchers to dirty R&D



Panel d) Tax and subsidy levels (as shares of resource costs and expected R&D profits, respectively)

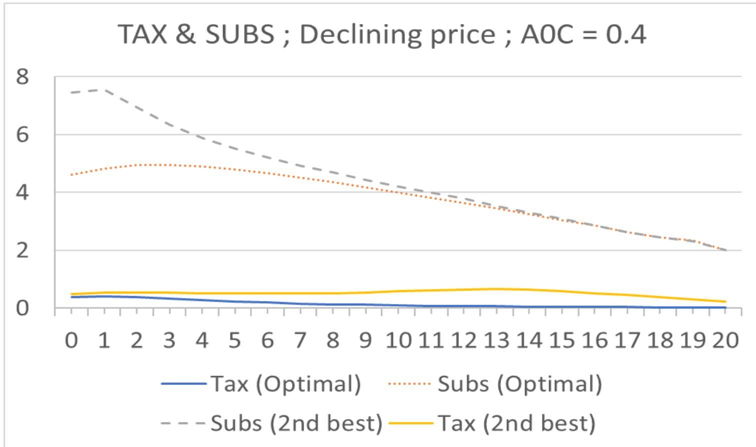
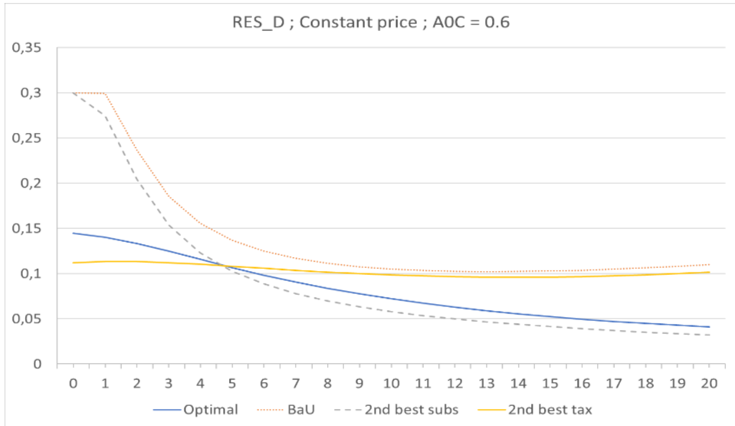
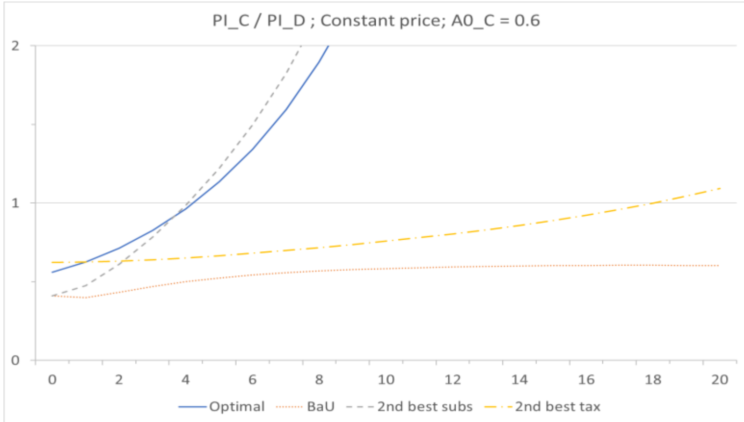


Figure 3 "Constant energy prices and developed clean sector"

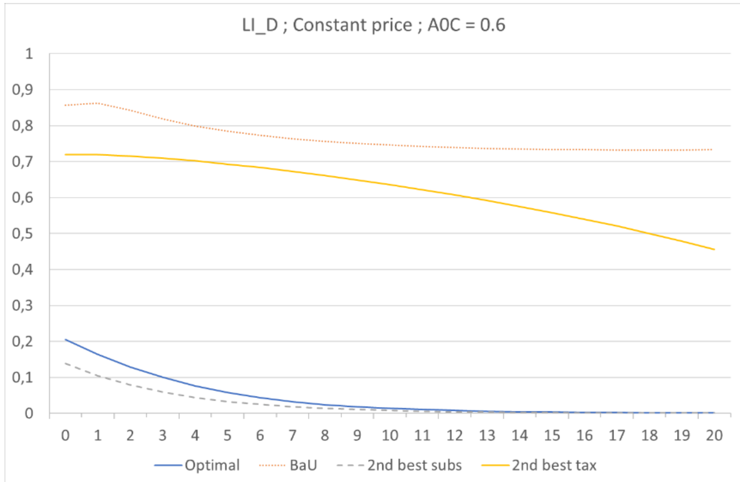
Panel a) Petroleum resource extraction



Panel b) Relative profits in clean versus dirty energy production



Panel c) Allocation of researchers to dirty R&D



Panel d) Tax and subsidy levels (as shares of resource costs and expected R&D profits, respectively)

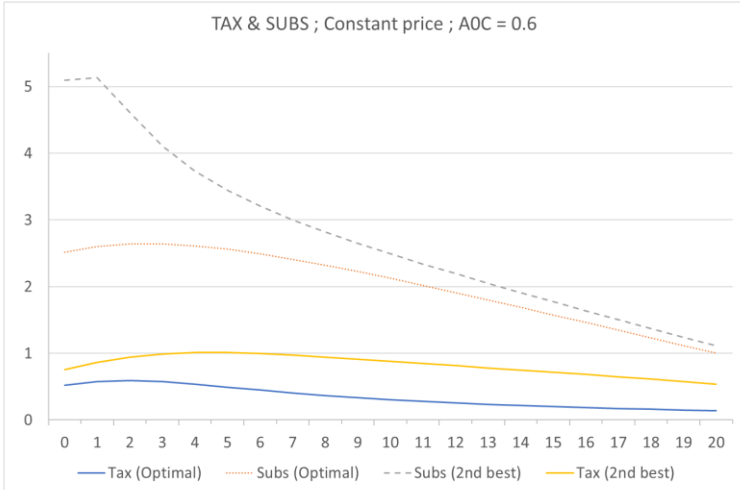
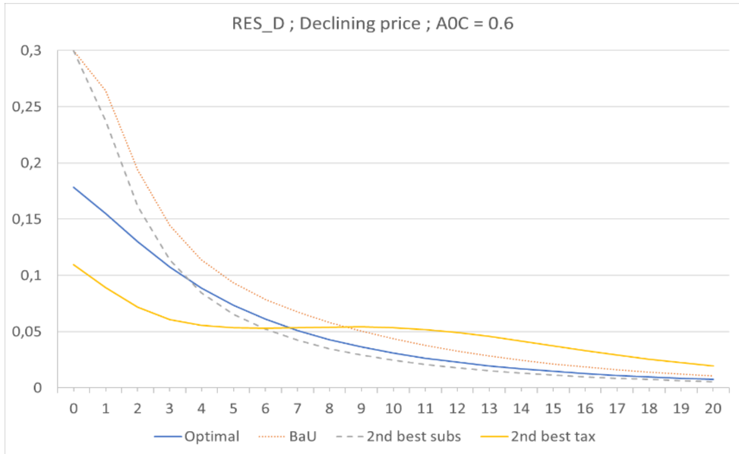
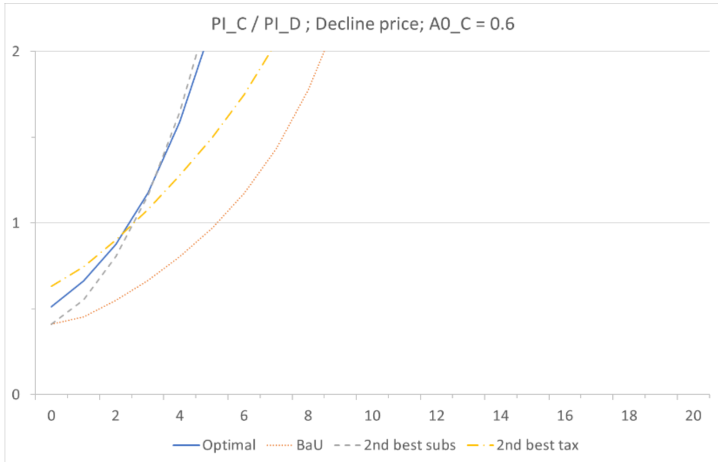


Figure 4 "Declining petroleum prices and developed clean sector"

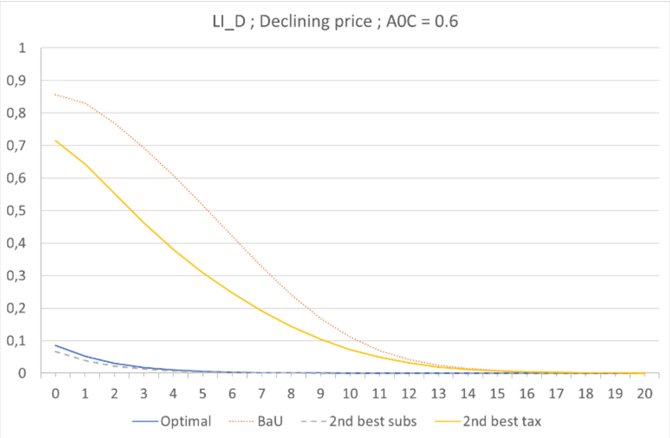
Panel a) Petroleum resource extraction



Panel b) Relative profits in clean versus dirty energy production



Panel c) Allocation of researchers to dirty R&D



Panel d) Tax and subsidy levels (as shares of resource costs and expected R&D profits, respectively)