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Hotelling and Recycling

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Hotelling and Recycling[*]

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We study the exploitation of recyclable exhaustible resources such as metals that are crucial for the energy transition or phosphorus that is crucial for agricultural production. We use a standard Hotelling model of resource exploitation that includes a primary sector and a recycling sector. We show that, when the primary sector is competitive, the price of the recyclable resource increases through time. We then show a new reason why the price of an exhaustible resource may decrease: when the primary sector is monopolistic, the primary producer has incentives to delay its production activities in order to delay recycling. As a consequence, the price path of the recyclable resource may be U-shaped. Numerical simulations show that the date of exhaustion of the virgin resource is further away in time for high and low levels of recoverability than for intermediate levels.

Keywords: non-renewable, recycling, monopoly, competition, market power, optimal control
JEL codes: Q31, Q53.

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1 Introduction

Recyclable exhaustible resources, such as metals (lithium, cobalt, rare earths, nickel, copper, manganese, etc.) and other elements like phosphorus, are increasingly important industrial inputs. Indeed, the aforementioned metals are important inputs for the production of many modern technologies, such as cell phones, light bulbs, automobiles, hybrid car batteries and gearboxes, and wind turbines (Chakhmouradian and Wall, 2012) and phosphorus, derived from phosphate rocks, is essential for soil fertility and has no substitute in agricultural production processes (Cordell et al., 2009).

Historically, the supply of these resources has often been highly concentrated.[1] Moreover, due to economic development and an increasing world population, demand for these resources has been growing rapidly and is expected to grow even more in the future (Alonso et al. 2012, Steen 1998).[2]

One strategy to increase supply and reduce the dependence of other countries on these resources is recycling. In order to assess the effect of recycling, it is necessary to understand how the primary sector may react. Since the main input to the production of recycled materials is the stock of scrap, the emergence of recycling activities may affect the dynamics of both the extraction of the exhaustible resource as well as the price of the final goods.

In this paper, we study the impact of a recycling sector in a stylized economic model of exhaustible resource extraction. We develop a Hotelling model of resource extraction in which the consumption good is produced from virgin or recycled materials. Virgin materials are extracted from a finite stock of a virgin resource and recycled materials are derived from the stock of scrap. The stock of recyclable scrap grows with current consumption of the final good at a given recoverability rate. We assume a competitive recycling sector in which production costs decrease with the stock of recyclable scrap. As a consequence, production in the primary sector generates a positive externality that benefits to the recycling sector. The inverse demand for the consumption good and the cost of recycling are linear. To ensure consistency with the various possible (future and present) market structures in the extraction sectors, we consider two polar cases: competitive and monopolistic extraction.

Our first main result is the following. We show that, if the primary sector is competitive, the optimal level of production for firms in the primary sector is such that the price of the resource grows at the discount rate (this is the so-called Hotelling rule) because these firms assume that their production will not increase the stock of scrap.

Our second main result concerns the case where the recoverability rate is sufficiently high, that is close to 100%. In this case, we are able to derive the following analytical results. The stock of scrap (as well as recycling) increases over time as long as the virgin resource is not exhausted and extraction decreases over time. The scrap stock decreases (slowly) once the

[1] Until 2010, China controlled 95% of the production of rare earths (Chakhmouradian and Wall, 2012), while a handful of countries, including Morocco, China, and the U.S.A, controlled most of the world's Phosphate rock production (IFDC, 2010). However, prospects for the supply of rare earths and Phosphate rocks differ. Since 2010, the supply of rare earths has become less concentrated as China currently possesses less than 40% of rare earth reserves, while the supply of Phosphate rocks has become more concentrated, as 85% of these reserves are currently located in Morocco and Western Sahara.

[2] Alonso *et al.* (2012) predict that the demand for rare earths will increase by 5 to 9 percent per annum until 2025. According to EFMA (2000) and Steen (1998), the demand for phosphorus may increase by as much as 50 to 100% by 2050 with increased global demand for food.

virgin resource is exhausted, until the stock of scrap is depleted. The monopolistic firm has an incentive to postpone extraction compared to a situation without recycling. As a consequence, the price of the resource is U-shaped, that is the price first decreases and then increases.

Our third main result concerns cases where the recoverability rate is not close to 100%. In this case, we are able to solve the model numerically. Our simulations provide some interesting insights. When the recoverability rate is high, our results are qualitatively similar to those obtained in the previous case. The scrap stock first increases and then decreases (in contrast to the previous case, it starts to decrease before the exhaustion date of the virgin resource). Extraction decreases over time and recycling first increases and then decreases and the price path is U-shaped. When the recoverability rate is low, our results are qualitatively affected. The stock of scrap decreases over time and extraction first increases and then decreases. Recycling first decreases (then increases and decreases again), so that the price of the final good can be always increasing. Interestingly, the date of depletion of the virgin resource is the highest for both high and low levels of recoverability.

The main takeaway of the present paper is a new reason why the price of a resource may decrease: a firm with market power in the extraction sector will (strategically) choose to delay extraction in order to reduce the opportunities for recycling.

The remainder of the paper is structured as follows. Section 2 presents the literature review. Section 3 introduces the model in which the monopolist of the exhaustible resource faces a competitive recycling sector. Section 4 studies the price dynamics in the case of a competitive extraction sector. Section 5 focuses on the properties of the optimal path in the case of a monopolistic primary producer. Section 6 concludes.

2 Related Literature

The present paper relates to the literature that deals with the problem of a monopolistic extraction sector facing a competitive recycling sector (Gaskins, 1974; Swan, 1980; Martin, 1982; Suslow, 1986; Hollander and Lasserre, 1988; Grant, 1999) and shows that, despite the presence of a competitive recycling sector, the extraction firm maintains (at least some of) its monopoly rents. Gaskins (1974) shows that recycling leads the monopolist to increase the price of the virgin resource in the short run and to slightly decrease it in the long run. Swan (1980) shows that the monopolist sets a price which approaches its marginal cost of production when there is price discrimination. Baksi and Long (2009) build a model of partial recycling and consider that consumers who participate to the recycling activity are heterogeneous in terms of their recycling cost. They show that the price set by the virgin producer will be close to the competitive price when the rate of recycling is close to one.

Gaudet and Long (2003) consider imperfect competition in the recycling sector and show that, when primary and secondary production decisions are made simultaneously, the presence of the recycling sector may increase the market power of the primary producer. Weikard and Seyhan (2009), motivated by the case of phosphorus, consider a model of competitive resource extraction and the possibility of saturated demand (i.e. taking into account the possibility that soil can become saturated with phosphorus). It is worth stressing that none of the above papers show that the price of the primary good can be U-shaped.

An important aspect of our work, which has not often been considered in the literature, is that we provide insights into the important role of scrap and the feedbacks between the cost of scrap and the market for final goods. An exception is Kaffine (2014), which considers a static model with perfect competition and focuses on very different, policy-driven research questions.

Recently, two papers have investigated the issue of recycling under an energy transition perspective. Pommeret et al. (2022) analyze how the possibility of recycling can affect the timing of the energy transition. They consider the presence of a depreciated green capital that can be fully recycled. They deal with a social planner's problem. They show that recycling influences the steady state in that it increases the stock of green capital and reduces its value. They also show that recycling induces a larger use of minerals (primary resources). Intuitively, this means that the social planner boosts the use of primary resources in order to increase the possibilities of recycling. Fabre et al. (2020) analyze the issue of energy production in the case where minerals and fossil resources are rare by considering that minerals are recyclable. They consider a social planner's problem. They show that the presence of recycling speeds up the investment in renewable capacity and makes the energy mix based on more renewable energy. They also show that a larger recycling rate induces a greater rate of extraction of minerals in the initial period. Our results differ from those two papers in that we show that the extraction rate can be reduced when there is a recycling sector. Another difference is that both papers consider a social planner's problem, while we postulate a competitive/monopolistic framework. It is well acknowledged that the social planner would want an increase in primary resources use to boost future recycling, while we show that the monopolist would strategically choose a reduction in primary resources use to limit the possibilities of recycling. This can explain the differences observed in terms of results. Also, in contrast to Pommeret et al. (2022), our analysis does not only focus on complete recycling.

In this paper, we do not explicitly consider the social or environmental motivations for the development of recycling. The environmental advantages of recycling have long been recognized in the economic literature (Smith, 1972; Weinstein and Zeckhauser, 1974; Hoel, 1978).[3] There is an important literature that includes waste accumulation and environmental damage in their models, making recycling a multiple dividend activity (e.g. Fullerton and Kinnaman, 1995; Palmer et al., 1997; Acuff and Kaffine 2013; Lafforgue and Lorang, 2022). In the present paper, our focus is on the effect of the existence of a recycling sector on the virgin resource extraction sector and not on social welfare.

The present paper is also linked to the literature dealing with durable resources. Durable resources differ from other resources (among which, recyclable resources) in that their demands are for quantities of stock in circulation rather than for flows of production. Producers of durable goods use similar production technologies and consumers typically consume durable goods for a certain period of time. Primary and recycled goods, in contrast, are typically produced using two different production processes. As stated in Levhari and Pindyck (1981), demand is a stock relationship for durable resources while it is a flow relationship for recyclable resources. Our results highlight important differences between recyclable and durable resources. We show that the two assumptions lead to quite different results. Indeed, Levhari and Pindyck (1981) show

[3] André and Cerdá (2006) provide a model that takes into account the interactions of the material composition of output and waste as potentially recyclable products.

that, in the case of a competitive industry that produces a durable good, the price of the resource first decreases and may increase thereafter. In contrast, we find that the price of the resource is always increasing in the context of a competitive extraction sector.

There are other explanations for U-shaped price profiles of exhaustible resources. Pindyck (1978) shows that this may occur when exploration and reserve accumulation are taken into account. In a model with exogenous technical change and endogenous change in grades, Slade (1982) also finds that U-shaped price profiles may occur. These studies do not consider the possibility of recycling.

3 The Model

The economy produces a quantity Q of a consumption good. The consumption good can be produced from a non-renewable resource or from recycled materials. For simplicity we assume that the virgin and recycled materials are perfect substitutes. The primary sector faces a competitive sector of recycling firms.

Non-renewable resource and scrap dynamics

Let $X(t) \geq 0$ be the residual stock of virgin resource at time t, X^0 be the initial stock, with $X(0) \equiv X^0 > 0$, and $x(t) \geq 0$ be the extraction rate at time t, so that:

$$\dot{X}(t) = -x(t). \tag{1}$$

The unit cost of extraction of the virgin resource is assumed to be zero.

Let $S(t) \geq 0$ be the stock of (recyclable) scrap at time t, with an initial stock $S(0) = S_0$. Let $r(t) \geq 0$ be the quantity of recycled materials marketed at time t, so that the total quantity consumed at time t is $Q(t) = x(t) + r(t)$. Let $\alpha \in [0, 1]$ be the proportion of the output that becomes recyclable scrap. It represents the recoverability rate of the final good. Here, $1 - \alpha$ can be interpreted as "dissipated" materials (Gloser et al., 2013) or as a "rate of retirement" (Gaskins, 1974; Grant, 1999). The dynamics of the scrap material thus writes:

$$\dot{S}(t) = \alpha Q(t) - r(t) = \alpha x(t) - (1 - \alpha) r(t). \tag{2}$$

The recycling sector

The recycling sector is assumed to be competitive. The total cost of recycling includes the cost of collecting, processing and transporting waste (included in the price of waste if the recycler buys waste from specialized companies) in addition to the cost of the recycling operation itself. As such, the marginal cost of recycling, denoted $c(S, r)$, is assumed to be a decreasing function of the stock of scrap and an increasing function of the quantity of recycled materials,[4] that is $\frac{\partial c(S,r)}{\partial S} < 0$ and $\frac{\partial c(S,r)}{\partial r} > 0$.

[4] There may be economies of scales, at least for sufficiently low levels of recycling (e.g. see Bohm *et al.* 2010). However, assuming that the marginal cost function of recycling is increasing in recycled materials seems reasonable, and we follow Rosendahl and Rubiano (2019) and Gaudet and Long (2003) who make the same assumption.

In equilibrium in the recycling sector, absent any corner solution, the price of the consumption good must equal the marginal cost of recycling:

$$p\left(Q\left(t\right)\right)=c\left(S\left(t\right),r\left(t\right)\right). \tag{3}$$

The primary sector

The price of primary production is the same as the price of recyled materials. Thus, the discounted profits in the primary sector, with discount rate $\delta \geq 0$, are given by:

$$\int_{0}^{+\infty} e^{-\delta t} pxdt, \tag{4}$$

In the following, we will consider two polar cases: the case of a competitive primary sector and the case of a monopolistic primary sector. In the case of a competitive primary sector, resource owners behave as price takers, and they consider the price of the resource to be a function of time, $p \equiv P\left(t\right)$. In the case of a monopolistic primary sector, the owner of the resource takes into account how extracted quantities affect the total quantity of material supplied (virgin as well as recycled) and the effect of this supply on the price of the resource, that is $p \equiv p\left(Q\left(t\right)\right)$.

4 Competitive primary sector

In this section, we consider the case of a competitive primary sector. In this case, producers take the price, P, as well as the total quantity, Q, as given. They consider the following problem:

$$\underset{\{x(t),t\geq 0\}}{Max} \int_{0}^{+\infty} e^{-\delta t} P\left(t\right)x\left(t\right)dt, \tag{5}$$

$$\begin{aligned} s.t.\ \dot{X}\left(t\right) &= -x\left(t\right), & (6)\\ X\left(t\right) &\geq 0,\ x\left(t\right)\geq 0. & (7)\end{aligned}$$

The Hamiltonian and the Lagrangian for this optimal control problem are as follows:[5]

$$H = Px+\lambda_X\left(-x\right), \tag{8}$$

$$L = H+\mu_X X+\mu_x x, \tag{9}$$

where λ_X is the co-state variable associated with the stock X, and, μ_X, μ_x are the multipliers associated with the non-negativity constraints $X \geq 0$, and $x \geq 0$. The competitive solution is found by solving problem (5) subject to (6) and (7) and then using (3), (2) and $P(t) = p(Q(t))$, $\forall t$, to determine the recycling level and the market clearing price. The Maximum Principle requires that the following conditions hold:

$$\frac{\partial L}{\partial x} = P-\lambda_X+\mu_x = 0, \tag{10}$$

[5]We drop the time index when there is no possible confusion.

$$\dot{\lambda}_X = \delta\lambda_X - \frac{\partial L}{\partial X} = \delta\lambda_X - \mu_X, \tag{11}$$

$$x \geq 0, \mu_x \geq 0, \mu_x x = 0, \tag{12}$$

$$X \geq 0, \mu_X \geq 0, \mu_X X = 0. \tag{13}$$

When both extraction and residual stock levels, $x(t)$ and $X(t)$, are strictly positive, we have $\mu_x = 0$ and $\mu_X = 0$. Substituting these respective values into (10) and (11) yields:

$$P - \lambda_X = 0, \tag{14}$$

$$\dot{\lambda}_X = \delta\lambda_X \tag{15}$$

Differentiating (14) with respect to time gives:

$$\frac{\dot{\lambda}_X}{\lambda_X} = \frac{\dot{P}}{P} \tag{16}$$

From (15), we have:

$$\frac{\dot{\lambda}_X}{\lambda_X} = \delta \tag{17}$$

The combination of (16) and (17) yields:

$$\frac{\dot{P}}{P} = \delta \tag{18}$$

We can thus conclude the following:

Proposition 1: *If the recycling sector is competitive, the optimal extraction path is such that the Hotelling's rule holds: the price of the resource grows over time at a rate equal to the discount rate.*

This proposition shows that the price of the resource increases over time when the recycling sector is competitive. This result reveals a major difference between recyclable goods and durable goods. The price of a durable exhaustible resource decreases with the amount of the durable good in circulation. It is then either always decreasing or U-shaped when the resource extraction sector is competitive (Levhari and Pindyck, 1981).

5 Monopolistic primary sector

In this section, we consider the case of a monopolistic primary sector. We derive several properties regarding the optimal time path of virgin resource extraction, the stock of scrap, the equilibrium recycling quantity, and the price of the consumption good.

For simplicity, we assume in the rest of the paper that the inverse demand for the consumption good and the cost of recycling are linear,

$$p\left(Q\left(t\right)\right) = 1 - Q\left(t\right) \text{ and } c\left(S\left(t\right), r\left(t\right)\right) = 1 - b - \beta\left(S\left(t\right) - r\left(t\right)\right), \tag{19}$$

with $\beta > 0$ and $b \in (0,1)$. Parameter b is a measure of the added value of recycled material compared to scrap.

Solving the recycling sector equilibrium condition (3), we characterize the equilibrium quantity of recycled material at time t as follows:

$$r(t) = \frac{b + \beta S(t) - x(t)}{1+\beta}. \tag{20}$$

This condition holds as long as $S(t) > 0$. When $S(t) = 0$, we must have $r(t) = 0$. Thus, the quantity of recycled material at time t increases with the quantity of scrap and decreases with the quantity of extracted resource. This result is quite intuitive. Since recycling relies on scrap, the higher the stock of scrap, the larger the recycling firms' production. Recycling at time t decreases with the quantity of virgin product sold at time t because recycled and virgin products are strategic substitutes. In the following, we assume that the right hand side of (20) is nonnegative.

Using the equilibrium recycling condition (20) and substituting, we have $p(Q) = \theta(a - x - S)$, where $a = (1 - b + \beta)/\beta$ and $\theta = \frac{\beta}{1+\beta}$. To find the solution to optimal extraction path of the monopolist, we solve the following maximization problem:

$$\underset{\{x\}}{Max} \int_{0}^{+\infty} e^{-\delta t}\theta\left(a - x(t) - S(t)\right) x dt, \tag{21}$$

subject to the dynamic of the resource stock:

$$\dot{X}(t) = -x(t), \tag{22}$$

and to the dynamic of the stock of scrap:

$$\dot{S}(t) = \alpha' x(t) - (1-\alpha)\theta S(t) - b', \tag{23}$$

where $\alpha' = \alpha + \frac{1-\alpha}{1+\beta}$, $b' = \frac{1-\alpha}{1+\beta}b$, $X, S, x \geq 0$, X^0 and $S^0 = 0$ given.

The current value Hamiltonian H and Lagrangian L are defined as follows:

$$H = \theta(a - x - S)x + \lambda_X(-x) + \lambda_S\left(\alpha' x - b' - (1-\alpha)\theta S\right), \tag{24}$$

and,

$$L = H + \mu_X X + \mu_S S + \mu_x x, \tag{25}$$

where λ_X and λ_S are the co-state variables associated with the stocks X and S, and μ_X, μ_S, μ_x are the multipliers associated with the non-negativity constraints $X \geq 0$, $S \geq 0$, and $x \geq 0$.

Thus, the necessary conditions include:

$$\frac{\partial L}{\partial x} = \theta(a - 2x - S) - \lambda_X + \alpha'\lambda_S + \mu_x = 0, \tag{26}$$

$$\dot{\lambda}_X = \delta\lambda_X - \frac{\partial L}{\partial X} = \delta\lambda_X - \mu_X, \tag{27}$$

$$\dot{\lambda}_S = \delta\lambda_S - \frac{\partial L}{\partial S} = \delta'\lambda_S - \mu_S + \theta x, \tag{28}$$

where $\delta' = \delta + (1-\alpha)\theta$,

$$x \geq 0, \mu_x \geq 0, \mu_x x = 0, \tag{29}$$

$$X \geq 0, \mu_X \geq 0, \mu_X X = 0, \tag{30}$$

$$S \geq 0, \mu_S \geq 0, \mu_S S = 0, \tag{31}$$

and $S(0) = 0$ and $X^0 > 0$ are given. We focus on the case where $a > X^0$, which ensures that the virgin resource stock is exhausted in finite time. We also assume that the choke price is sufficiently high (i.e. $a > S_0$) in order to avoid degenerate cases where the price of the resource is zero.

Full resolution of the monopolist's programme yields the extraction and recycling paths as well as the dynamics of the price of the consumption good. Solving the problem involves finding constants that are characterized by nonlinear equations, which limits our ability to study the properties of the solution for any value of the parameters. However, we are able to derive the main properties of the solution when the recoverability rate is high (i.e., when α is arbitrarily close to 100%). We first focus on this case and then provide numerical results for lower levels of the recoverability rate.

5.1 Optimal extraction in high recoverability rate sectors

Near full recoverability seems to be a reasonable assumption for a number of materials, such as copper,[6] vanadium, iron, nickel, palladium, iridium, platinum or gold (see Ciacci *et al.* 2015). In this case, we can show that the optimal path has the following qualitative properties:

Proposition 2: *If the recoverability rate is sufficiently large ($\alpha \to 1$), the optimal extraction path is such that:*
(i) *Extraction $x^*(t)$ is decreasing up to the date of depletion of the virgin resource T^*;*
(ii) *The stock of scrap $S^*(t)$ increases up to T^* and decreases after;*
(iii) *Recycling $r^*(t)$ increases up to T^* and decreases after.*

The optimal level of extraction decreases through time. This result is in line with the standard Hotelling model. Indeed, the extracting firm discounts time, choosing to extract more of the resource today and less tomorrow. The quantity of marketed recycled material, in contrast, increases over time up to the exhaustion of the virgin resource. The intuition of these results is as follows. The recoverability rate is high, thus the stock of scrap increases over time up to the exhaustion of the virgin resource, which reduces the unit cost of recycling. This, in turn, provides incentives for recycling firms to increase their production. At the same time, due to strategic substitutability, the quantity of extracted material decreases, also causing the level of recycling to increase.

Once the virgin resource is exhausted, the stock of scrap decreases over time, which increases the unit cost of recycling. The level of recycling thus decreases.

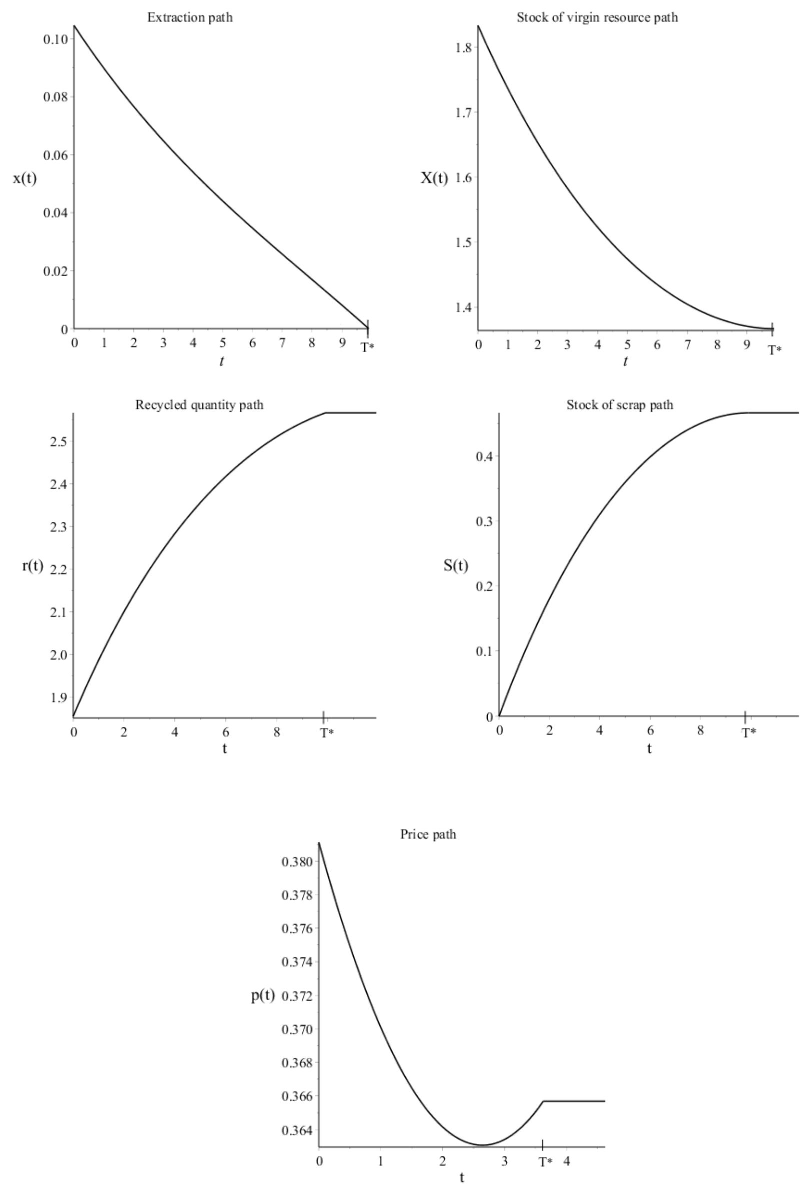
We are now in a position to state our main result. This one concerns the optimal price path:

[6] Copper has a recoverabillity rate around 94-99% for most applications, see Table S5 in Gloser et al. (2013).

Proposition 3: *If the recoverability rate is sufficiently large ($\alpha \to 1$), the optimal price path $p^*(t)$ is U-shaped.*

This result states that the standard result of an increasing resource price does not hold if the recoverabiliity rate is sufficiently high. In the first phase, the price decreases because the amount of recycled material increases over time at a greater rate than the decrease in the amount of extraction ($\dot{p} = -\theta(\dot{x} + \dot{S})$). Intuitively, a low pace of extraction delays accumulation of scrap and then future recycling, which is beneficial to the monopoly. In the second phase, we are getting closer to the date of exhaustion of the virgin resource. Consequently, the marginal cost of extraction becomes increasingly high and then, at some point in time, the price of the resource increases.

Figure 1: Optimal extraction path for high recoverability rate sectors



Notes: The parameter values used to plot these graphs are $X^0 = 0.28$, $\theta = 0.3$,$\delta = 0.05$, $b = 0.9$, $S^0 = 0.5$.

Figure 1 illustrates the results of Propositions 2 and 3. Notice that after the date of exhaustion of the virgin resource, recycling as well as the stock of scrap decrease very slowly and then the price of the resource increases very slowly.

Before going further, it is important to understand why the case where the recoverability rate is high is specific and simpler to solve than the other cases.

The optimal extraction path is given by condition (26). The pace of extraction is thus given by (assuming $x > 0$):

$$\dot{x} = -\frac{1}{2\theta}\dot{\lambda}_X - \dot{S} + (\theta\alpha + 1 - \theta)\,\frac{1}{2\theta}\dot{\lambda}_S. \tag{32}$$

Condition (32) shows that extraction tends to decrease over time when the shadow price of the virgin resource increases over time, when the stock of scrap increases or when the shadow price of scrap decreases over time. The shadow price of the virgin resource always increases over time as the virgin resource becomes scarcer. The shadow price of scrap and the stock of scrap vary over time in opposite directions.

Thus, the evolution of the stock of scrap provides useful information as regards the evolution of extraction. When the stock of scrap increases over time, extraction necessarily decreases over time. When the stock of scrap decreases over time, extraction may increase or decrease over time.

We can now see why the case where the recoverability rate is high is simpler than the other cases. When the recoverability rate is close to 100% ($\alpha \to 1$), the dynamics of the stock of scrap is such that $\dot{S} \to x$ (as long as the stock of virgin resource is not exhausted). Thus, before the date of exhaustion, the stock of scrap grows and then extraction of the virgin resource decreases over time. When the recoverabillity rate is not close to 100%, the stock of scrap may decrease over time over some intervals of time, and thus it is more difficult to conclude as regards the evolution of extraction. It is therefore more difficult to conclude about the qualitative properties of the optimal path and the evolution of the price of the final good.

To provide insight into cases where the recoverability rate is not close to 100%, we perform numerical simulations for lower levels of recoverability in the next section.

5.2 Optimal extraction in various sectors

For some materials, the recoverability rate is quite low. For recyclable elements such as cerium (a rare earth metal), the recoverability rate is as low as 10% (see Ciacci *et al.* 2015).

For cases where the recoverability rate is not close to 100%, we are able to solve the problem numerically for parameter values compatible with our assumptions. Table 1 shows the simulation results for the date of exhaustion of the virgin resource (T^*) and the stock of scrap (T'). We numerically solve for both dates for different values of the recoverability rate and hold the values of all other parameters constant. These simulations suggest that the date of depletion of the stock of scrap increases as the recoverability rate increases, which is intuitive. A less intuitive result is that the date of depletion of the virgin resource is a non monotonic function of the recoverability rate. Indeed, for high levels of recoverability (above 50%), our results suggest that an increase in the recoverability rate leads to an increase in the date of exhaustion of the virgin resource. For low levels of recoverability (below 40%), an increase in the recoverability rate leads to a decrease in the date of exhaustion.

Table 1: Date of exhaustion and recovery rate

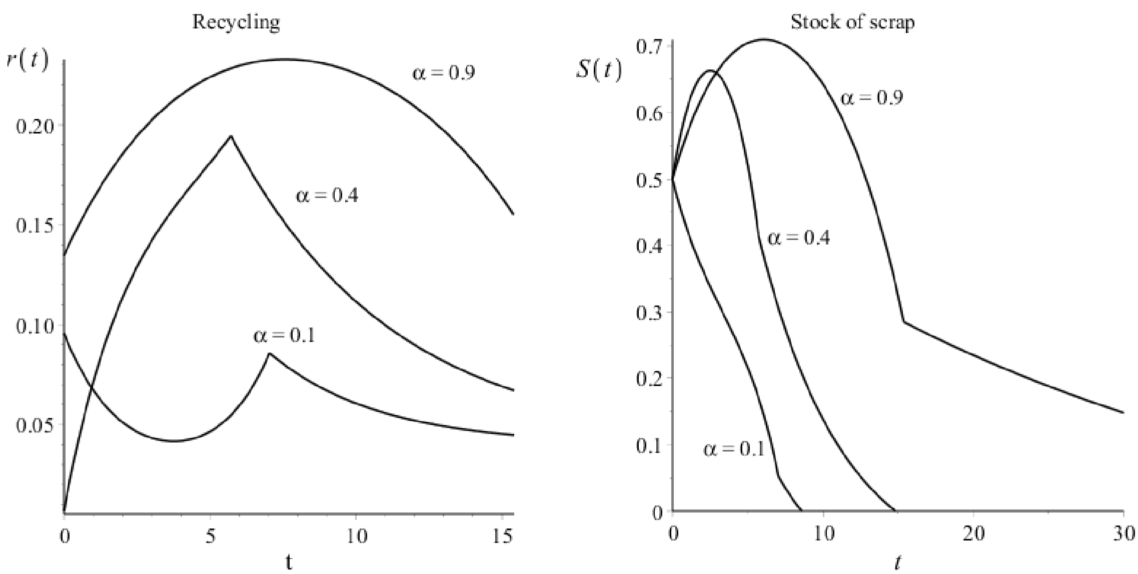
Parameters						Simulations	
α	X^0	S^0	δ	θ	b	T^*	T'
10%	1	0.5	2%	0.3	0.1	7.03	8.60
20%	1	0.5	2%	0.3	0.1	6.14	10.52
30%	1	0.5	2%	0.3	0.1	5.77	12.45
40%	1	0.5	2%	0.3	0.1	5.71	14.82
50%	1	0.5	2%	0.3	0.1	5.91	17.99
60%	1	0.5	2%	0.3	0.1	6.38	22.54
70%	1	0.5	2%	0.3	0.1	7.30	29.70
80%	1	0.5	2%	0.3	0.1	9.24	42.46
90%	1	0.5	2%	0.3	0.1	15.37	60.27

Notes: This Table presents the simulation results of the exhaustion dates T^* (virgin resource) and T' (scrap).

This counterintuitive result can be understood by examining the optimal extraction path for various levels of recoverability. Figure 3 shows the optimal monopoly extraction path of the virgin resource for $\alpha = 90\%$, $\alpha = 40\%$ and $\alpha = 10\%$. As we explain below, the results are consistent with the explanations we provided at the end of Section 5.1.

Figure 2 shows interesting features. When the recoverability rate is as high as 40% or 90%, the stock of scrap is first increasing and then decreasing. Notice that, differently from the case where $\alpha \to 1$, it starts decreasing before the exhaustion date (it is $T^* = 15.37$, 5.71 and 7.03, for $\alpha = 0.9$, 0.4 and 0.1, respectively). The evolution of recycling is similar to the case where the recoverability rate is close to 100% (increasing and then decreasing). When the recoverability rate is as low as 10%, the stock of scrap decreases over time. Recycling has a quite complex dynamics: it is first decreasing, then increasing and then increasing.

Figure 2: Recycling and stock of scrap paths for various sectors

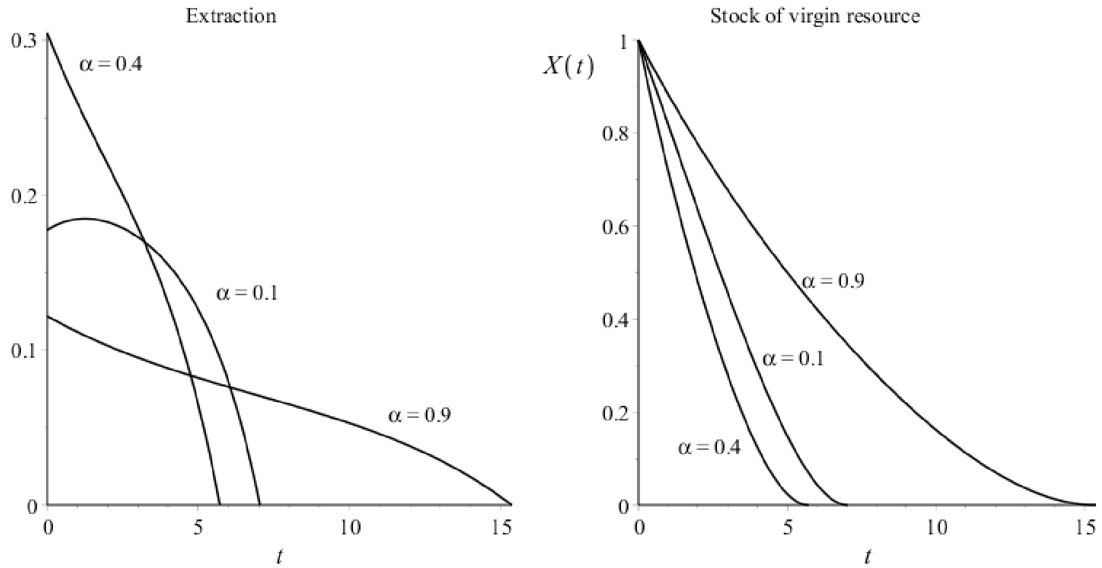


Notes: Parameter values: $X^0 = 1$, $\theta = 0.3$, $\delta = 0.02$, $b = 0.1$, $S^0 = 0.5$.

When looking at the evolution of extraction and the stock of virgin resource (see Figure 3), we can make the following observations. When the recoverability rate is large ($\alpha = 0.4$ or

$\alpha = 0.9$), extraction decreases over time. Moreover, the higher the recoverability rate, the lower the pace of extraction. This highlights the fact that a higher recoverability rate provides the monopoly more incentives to delay extraction. When the recoverability rate is low ($\alpha = 0.1$), the optimal extraction path is not always decreasing over time as in the previous cases, it first increases and then decreases.

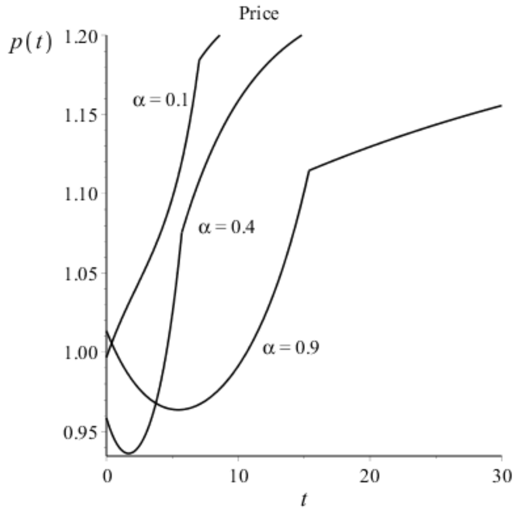
Figure 3: Optimal extraction path for various sectors



Notes: Parameter values: $X^0 = 1$, $\theta = 0.3$ and $\delta = 0.02$, $b = 0.1$, $S^0 = 0.5$.

We can now look at the evolution of the price of the final good (see Figure 4). When the recoverability rate is as large as 40% or 90%, the evolution of the price is similar to the case where the recoverability rate is close to 100%. The price is first decreasing and then increasing through time. When the recoverability rate is only 10%, the price is always increasing over time. This is similar to the situation where there is no recycling (the Hotelling model), but the underlying reason why the price is increasing at the beginning is different. This initial increase in the price of the resource is not due to a decrease in extraction (which is first increasing), but to a decrease in the stock of scrap.

Figure 4: Price path for various sectors



Notes: Parameter values: $X^0 = 1$, $\theta = 0.3$, $\delta = 0.02$, $b = 0.1$, $S^0 = 0.5$.

6 Conclusion

Recycling appears to be a promising strategy to increase the supply of important exhaustible resources.

We have built a model of resource extraction in which the primary sector faces a recycling sector. We have shown that, when the primary sector is competitive, that the price of the recyclable resource increases through time. We have also shown that, when the primary sector is monopolistic, the price of the recyclable resource may be U-shaped when the recoverability rate is sufficiently large. This occurs because the primary producer has incentives to delay the extraction of the resource in order to limit recycling possibilities. We have also shown that virgin resource depletion occurs later when the recoverability rate is large or small than when it is intermediate.

Our results suggest that market power in the primary sector may lead to phases in which the price of the resource decreases.

Appendix A: derivation of the general conditions

Let us assume that the solution is such that $x(t) > 0$ and $X(t) > 0$ over $[0, T^*)$ and $x(t) = X(t) = 0$ for $t \geq T^*$. Also assume that $S(t) > 0$ over $[0, T')$ and $S(t) = 0$ for $t \geq T'$, where $T' \geq T^*$.

First consider the first phase in which $t \in [0, T^*)$. Since $x(t) > 0$, $X(t) > 0$ and $S(t) > 0$, using (29), (30), and (31), we have $\mu_x = \mu_X = \mu_S = 0$. Then (27) writes

$$\dot{\lambda}_X = \delta \lambda_X, \tag{33}$$

and then

$$\lambda_X = c_1 e^{\delta t}, \tag{34}$$

where c_1 is a constant to be determined later.

Conditions (26), and (28) write

$$\theta (a - 2x - S) - c_1 e^{\delta t} + \alpha' \lambda_S = 0, \tag{35}$$

and,

$$\dot{\lambda}_S = \delta' \lambda_S + \theta x, \tag{36}$$

Differentiating (35) with respect to time, we find

$$-2\theta \dot{x} - \theta \dot{S} - \delta c_1 e^{\delta t} + \alpha' \dot{\lambda}_S = 0. \tag{37}$$

Using (35) and (37), we find

$$-2\theta \dot{x} - \theta \dot{S} - \delta c_1 e^{\delta t} - \delta' \left(\theta a - 2\theta x - \theta S - c_1 e^{\delta t} \right) + \alpha' \left(\dot{\lambda}_S - \delta' \lambda_S \right) = 0. \tag{38}$$

Using (36) we obtain

$$-2\dot{x} - \dot{S} + \delta' S + \left(\alpha' + 2\delta' \right) x - \delta' a + (1-\alpha) c_1 e^{\delta t} = 0, \tag{39}$$

Differentiating (23) with respect to time, we obtain

$$\ddot{S} = \alpha' \dot{x} - (1-\alpha)\theta \dot{S}. \tag{40}$$

Substituting (23) and (40) into (39), and rearranging, we have

$$\ddot{S} - \delta \dot{S} - \frac{1}{2} \left[\delta + (1-\alpha)\theta(2+\delta) \right] S = \frac{\alpha'(1-\alpha)}{2} c_1 e^{\delta t} + \left(\frac{\alpha'}{2} + \delta' \right) b' - \frac{\alpha' \delta'}{2} a. \tag{41}$$

Solving for the stock of scrap S, we find

$$S = A + c_2 e^{\gamma^+ t} + c_3 e^{\gamma^- t} - B e^{\delta t}, \tag{42}$$

where c_2 and c_3 are two constants to be determined later, $\gamma^+ = \frac{\delta + \sqrt{\delta^2 + 2\delta + 2(1-\alpha)\theta(2+\delta)}}{2}$, $\gamma^- = \frac{\delta - \sqrt{\delta^2 + 2\delta + 2(1-\alpha)\theta(2+\delta)}}{2}$, $A = \frac{\alpha' \delta' a - (\alpha' + 2\delta') b'}{\delta + (1-\alpha)\theta(2+\delta)}$ and $B = \frac{\alpha'(1-\alpha)}{\delta + (1-\alpha)\theta(2+\delta)} c_1$.

Differentiating (42) with respect to time, we obtain

$$\dot{S} = \gamma^{+} c_2 e^{\gamma^{+} t} + \gamma^{-} c_3 e^{\gamma^{-} t} - B\delta e^{\delta t}. \tag{43}$$

Using (23) and (42), we obtain

$$x = \frac{(1-\alpha)\theta A + b'}{\alpha'} + \left(\frac{\gamma^{+} + (1-\alpha)\theta}{\alpha'}\right) c_2 e^{\gamma^{+} t} + \left(\frac{\gamma^{-} + (1-\alpha)\theta}{\alpha'}\right) c_3 e^{\gamma^{-} t} - \left(\frac{\delta + (1-\alpha)\theta}{\alpha'}\right) B e^{\delta t}. \tag{44}$$

Using $X^0 - X(t) = \int_0^t x dt$ and integrating (44) between 0 and t, we find

$$X^0 - X(t) = \frac{(1-\alpha)\theta A + b'}{\alpha'} t + \left(\frac{\gamma^{+} + (1-\alpha)\theta}{\alpha'}\right) \frac{c_2}{\gamma^{+}} \left(e^{\gamma^{+} t} - 1\right) + \left(\frac{\gamma^{-} + (1-\alpha)\theta}{\alpha'}\right) \frac{c_3}{\gamma^{-}} \left(e^{\gamma^{-} t} - 1\right)$$
$$- \left(\frac{\delta + (1-\alpha)\theta}{\alpha'}\right) \frac{B}{\delta} \left(e^{\delta t} - 1\right). \tag{45}$$

Now consider the second phase in which $t \in [T^*, T')$. We have $x(t) = 0 = X(t)$ and $S(t) > 0$. Using (31), we have $\mu_S = 0$. Condition (28) writes

$$\dot{\lambda}_S = \delta' \lambda_S, \tag{46}$$

and then

$$\lambda_S = c_5 e^{\delta' t}, \tag{47}$$

where c_5 is a constant to be determined later.

Notice that $\dot{S} = -(1-\alpha)\theta S - b'$, and then

$$S = c_4 e^{-(1-\alpha)\theta t} - b'. \tag{48}$$

where c_4 is a constant to be determined. Using (42) and (48) at $t = T^*$, we have:

$$c_4 = \left(A + b' + c_2 e^{\gamma^{+} T^*} + c_3 e^{\gamma^{-} T^*} - B e^{\delta T^*}\right) e^{(1-\alpha)\theta T^*}. \tag{49}$$

Hence, over $t \in [T^*, T')$, we have:

$$S = \left(A + b' + c_2 e^{\gamma^{+} T^*} + c_3 e^{\gamma^{-} T^*} - B e^{\delta T}\right) e^{(1-\alpha)\theta(T^* - t)} - b'. \tag{50}$$

Now consider the third phase in which $t \geq T'$. We have $x = X = S = 0$. The remaining first order conditions are $\mu_x = -a + \lambda_X - \alpha' \lambda_S \geq 0$, $\mu_X = \dot{\lambda}_X - \delta \lambda_X \geq 0$ and $\mu_S = \dot{\lambda}_S - \delta' \lambda_S \geq 0$.

Using (48) at $t = T'$, we obtain

$$S(T') = c_4 e^{-(1-\alpha)\theta T'} - b' = 0, \tag{51}$$

which implies that $T' = \frac{1}{(1-\alpha)\theta} \ln\left(\frac{c_4}{b'}\right)$.

Using $x\left(T^{*}\right)=0$ and (44), we have

$$(1-\alpha)\theta A+b'+\left(\gamma^{+}+(1-\alpha)\theta\right)c_{2}e^{\gamma^{+}T^{*}}+\left(\gamma^{-}+(1-\alpha)\theta\right)c_{3}e^{\gamma^{-}T^{*}}-\left(\delta+(1-\alpha)\theta\right)Be^{\delta T^{*}}=0. \tag{52}$$

Using $X\left(T^{*}\right)=0$ and (45), we have

$$\alpha' X^{0}=\left((1-\alpha)\theta A+b'\right)T^{*}+\left(\gamma^{+}+(1-\alpha)\theta\right)\frac{c_{2}}{\gamma^{+}}\left(e^{\gamma^{+}T^{*}}-1\right)+\left(\gamma^{-}+(1-\alpha)\theta\right)\frac{c_{3}}{\gamma^{-}}\left(e^{\gamma^{-}T^{*}}-1\right)$$
$$-\left(\delta+(1-\alpha)\theta\right)\frac{B}{\delta}\left(e^{\delta T^{*}}-1\right). \tag{53}$$

Using (42) at $t=0$, we have

$$S_{0}=c_{2}+c_{3}+A-B. \tag{54}$$

Using (42) and (48) at $t=T^{*}$, we obtain

$$A+b'+c_{2}e^{\gamma^{+}T^{*}}+c_{3}e^{\gamma^{-}T^{*}}-Be^{\delta T^{*}}=c_{4}e^{-(1-\alpha)\theta T^{*}}. \tag{55}$$

To get an additional condition, we use the following necessary condition:

$$\frac{dL}{dt}=\frac{\partial L}{\partial t}=0 \text{ for all } t \tag{56}$$

Using (23), we have, for $t\in[0,T^{*}]$:

$$\left(-x-\lambda_{S}(1-\alpha)\right)\theta\dot{S}-x\dot{\lambda}_{X}+\dot{S}\dot{\lambda}_{S}=0 \tag{57}$$

Using (36), we obtain:

$$-\lambda_{S}(1-\alpha)\theta\dot{S}-x\dot{\lambda}_{X}+\dot{S}\delta'\lambda_{S}=0, \tag{58}$$

or,

$$\delta\dot{S}\lambda_{S}=x\dot{\lambda}_{X}. \tag{59}$$

Using (35), we have:

$$\delta\dot{S}\left[\theta(a-2x-S)-\lambda_{X}\right]+\alpha' x\dot{\lambda}_{X}=0. \tag{60}$$

At $t=T^{*}$, this condition is equivalent to:

$$\dot{S}(T^{*})\left[\lambda_{X}(T^{*})-\theta a+\theta S(T^{*})\right]=0. \tag{61}$$

Assume that $\dot{S}(T^{*})=0$. Together with (43), (52) and (55), this implies $c_{4}=0$. Hence, using (48), we have $S(t)<0$ when $t\geq T^{*}$, which is impossible. Hence, the last condition is given by:

$$(A-a)\theta+\theta c_{2}e^{\gamma^{+}T^{*}}+\theta c_{3}e^{\gamma^{-}T^{*}}+(c_{1}-\theta B)e^{\delta T^{*}}=0. \tag{62}$$

Appendix B: Proofs of Propositions

Proof of Proposition 2: When $\alpha\to 1$, we have $\alpha'\to 1$, $b'\to 0$ and $\delta'\to\delta$. Thus, we have $\gamma^{+}\to\frac{\delta+\sqrt{\delta^{2}+2\delta}}{2}$, $\gamma^{-}\to\frac{\delta-\sqrt{\delta^{2}+2\delta}}{2}$, $A\to a$ and $B\to 0$. Using condition (52), we have:

$$\gamma^{+}c_2e^{\gamma^{+}T^{*}}+\gamma^{-}c_3e^{\gamma^{-}T^{*}}=0. \tag{63}$$

Using condition (53), we have:

$$X^{0}=c_2\left(e^{\gamma^{+}T^{*}}-1\right)+c_3\left(e^{\gamma^{-}T^{*}}-1\right). \tag{64}$$

Condition (54) becomes:

$$0=c_2+c_3+a. \tag{65}$$

Condition (55) leads to:

$$a+c_2e^{\gamma^{+}T^{*}}+c_3e^{\gamma^{-}T^{*}}=c_4. \tag{66}$$

Solving for c_2,c_3 and c_4 from conditions (63)-(66), we obtain:

$$\begin{aligned}
c_2 &= (a-S_0)\frac{\gamma^{-}e^{\gamma^{-}T^{*}}}{\gamma^{+}e^{\gamma^{+}T^{*}}-\gamma^{-}e^{\gamma^{-}T^{*}}}, && (67)\\
c_3 &= -(a-S_0)\frac{\gamma^{+}e^{\gamma^{+}T^{*}}}{\gamma^{+}e^{\gamma^{+}T^{*}}-\gamma^{-}e^{\gamma^{-}T^{*}}}, && (68)\\
c_4 &= -(a-S_0)\frac{\left(\gamma^{+}-\gamma^{-}\right)e^{\delta T^{*}}}{\gamma^{+}e^{\gamma^{+}T^{*}}-\gamma^{-}e^{\gamma^{-}T^{*}}}+a. && (69)
\end{aligned}$$

and the exhaustion date T^{*} is implicitly characterized by :

$$X^{0}=(a-S_0)\left(1-\frac{\gamma^{+}-\gamma^{-}}{\gamma^{+}e^{\gamma^{+}T^{*}}-\gamma^{-}e^{\gamma^{-}T^{*}}}e^{\delta T^{*}}\right). \tag{70}$$

We conclude that the optimal extraction path is, for $t\in[0,T^{*}]$:

$$x^{*}(t)=\frac{(a-S_0)\delta}{2}\left(\frac{e^{\gamma^{+}T^{*}}e^{\gamma^{-}t}-e^{\gamma^{-}T^{*}}e^{\gamma^{+}t}}{\gamma^{+}e^{\gamma^{+}T^{*}}-\gamma^{-}e^{\gamma^{-}T^{*}}}\right), \tag{71}$$

the stock of scrap is, for $t\in[0,T]$,

$$S^{*}(t)=(a-S_0)\left(1-\frac{\gamma^{+}e^{\gamma^{+}T^{*}}e^{\gamma^{-}t}-\gamma^{-}e^{\gamma^{-}T^{*}}e^{\gamma^{+}t}}{\gamma^{+}e^{\gamma^{+}T^{*}}-\gamma^{-}e^{\gamma^{-}T^{*}}}\right), \tag{72}$$

and the market price, for $t\in[0,T]$,

$$p^{*}(t)=\theta(a-S_0)\left(\frac{\gamma^{+}-\gamma^{-}}{2}\right)\frac{e^{\gamma^{+}T^{*}}e^{\gamma^{-}t}+e^{\gamma^{-}T^{*}}e^{\gamma^{+}t}}{\gamma^{+}e^{\gamma^{+}T^{*}}-\gamma^{-}e^{\gamma^{-}T^{*}}}. \tag{73}$$

Since $\gamma^{+}>0>\gamma^{-}$, the extraction level $x^{*}(t)$ characterized in (71) decreases through time over $[0,T^{*}]$, while the stock of scrap increases through time over this interval, $\dot{S}^{*}(t)=x^{*}(t)\geq 0$. Recycling is given by

$$r^{*}(t)=\frac{b}{\beta}+(a-S_0)\left(1-\frac{\left(\gamma^{+}+\frac{\delta}{2\beta}\right)e^{\gamma^{+}T^{*}}e^{\gamma^{-}t}-\left(\gamma^{-}+\frac{\delta}{2\beta}\right)e^{\gamma^{-}T^{*}}e^{\gamma^{+}t}}{\gamma^{+}e^{\gamma^{+}T^{*}}-\gamma^{-}e^{\gamma^{-}T^{*}}}\right), \tag{74}$$

and increases through time over this interval.

Combining (69) and (70), we have $c_4 = X^0 > 0$. Using (48), we conclude that $T' \to +\infty$ and that S^* is decreasing when $t > T^*$ and so does r^*.□

Proof of Proposition 3:

The right hand side in condition (70) increases when T^* increases. Indeed, its derivative with respect to T^* is:

$$\frac{\theta(a - S_0)}{2} \frac{\sqrt{\delta^2 + 2\delta}}{\gamma^+ e^{\gamma^+ T^*} - \gamma^- e^{\gamma^- T^*}} > 0. \tag{75}$$

Moreover, the right hand side in condition (70) goes to 0 when X^0 goes to 0 and it goes to a when X^0 goes to $+\infty$. Hence, there is a unique solution T^*. Moreover, T^* goes to 0 when X^0 goes to 0 and it goes to $+\infty$ when X^0 goes to a.

From (73), we know that the price of the consumption good is

$$p^*(t) = \theta \frac{(a - S_0)}{2} \sqrt{\delta (2 + \delta)} \frac{e^{\gamma^+ T^*} e^{\gamma^- t} + e^{\gamma^- T^*} e^{\gamma^+ t}}{\gamma^+ e^{\gamma^+ T^*} - \gamma^- e^{\gamma^- T^*}}, \tag{76}$$

The sign of the derivative with respect to time is given by

$$\frac{\partial p^*}{\partial t} \propto \gamma^- e^{\gamma^+ T^*} e^{\gamma^- t} + \gamma^+ e^{\gamma^- T^*} e^{\gamma^+ t}, \tag{77}$$

which is positive if and only if

$$t \geq T^* + \frac{1}{\gamma^+ - \gamma^-} \ln \left(1 - \frac{\delta}{\gamma^+} \right). \tag{78}$$

Hence, $\frac{\partial p^*}{\partial t} \geq 0$ for all $t \in [0, T]$ if and only if

$$T^* \leq \frac{1}{\gamma^+ - \gamma^-} \ln \left(\frac{\gamma^+}{\gamma^+ - \delta} \right). \tag{79}$$

Notice that the right hand side of in condition (70) taken at $T^* = \frac{1}{\gamma^+ - \gamma^-} \ln \left(\frac{\gamma^+}{\gamma^+ - \delta} \right)$ is equal to:

$$(a - S_0) \left(1 - \frac{\gamma^+ - \gamma^-}{\gamma^+ - \gamma^- \frac{\gamma^+ - \delta}{\gamma^+}} \right) \left(\frac{\gamma^+}{\gamma^+ - \delta} \right)^{\frac{\gamma^+}{\gamma^+ - \gamma^-}} < 0 \tag{80}$$

Hence, we must have $T^* > \frac{1}{\gamma^+ - \gamma^-} \ln \left(\frac{\gamma^+}{\gamma^+ - \delta} \right)$.

Hence, p^* is decreasing up to $t = T^* - \frac{1}{\gamma^+ - \gamma^-} \ln \left(\frac{\gamma^+}{\gamma^+ - \delta} \right)$, and increasing after this date.

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