Coordination of sectoral climate policies and life cycle emissions

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Abstract

Drastically reducing greenhouse gas emissions involves numerous specific actions in each sector of the economy. The costs and abatement potential of these measures are interdependent because of sectoral linkages. For instance, the carbon footprint of electric vehicles depends on the electricity mix. This issue has received large attention in the literature on Life Cycle Assessments (LCA). This paper analyzes how life cycle considerations should be integrated into policy design. We model a partial equilibrium with two vertically connected sectors, an upstream (e.g. electricity) and a downstream (e.g. transportation) one. In each sector, a dirty and a clean technology are available. The clean downstream technology consumes the upstream good and may thus shift emissions to the upstream sector. Our main contribution is to detail how optimal subsidies on clean technologies should incorporate life cycle emissions when carbon pricing is limited. The optimal downstream subsidy should be corrected for all external costs generated in the upstream sector, not only unpriced pollution but also the fiscal externality due to the subsidy to the clean upstream technology. We also analyze the joint optimization of upstream and downstream policies. The upstream subsidy should not incorporate features of the downstream sector, whereas the downstream optimal subsidy depends upon the upstream sector characteristics. All results are illustrated using a calibrated example of the electrification of passenger cars.

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1 Introduction

The reduction of Greenhouse Gas (GHG) emissions requires a shift from fossil energy to low-carbon energy. For many energy uses (e.g. transport, industry, heating) such a shift may be achieved through electrification combined with low-carbon power sources (e.g. renewable, nuclear). Other decarbonization options involve a shift from fossil energy sources to hydrogen, which would also require low-carbon production processes (from electrolysis and biomass). In these examples, a downstream sector decarbonizes through technologies that consume an upstream good, the production of which also needs to be decarbonized. As long as the upstream sector is not fully decarbonized, the decarbonization of downstream activities partly shifts emissions upstream. This paper analyzes optimal subsidies for clean technologies in such a configuration and when carbon pricing is imperfect. It aims to clarify the relationship between life cycle emissions and optimal subsidies to low-carbon technologies. Indeed, with an exhaustive Pigovian tax, sectoral interactions, and life cycle emissions do not need to be considered in policy design. But the lack of carbon pricing, and the pervasiveness of subsidies and other instruments, calls for analysis of second-best policies.

Life Cycle Assessments (LCA) quantify life cycle emissions of a product.[1] Some LCA of low-carbon technologies suggest that an increase in upstream emissions can outweigh direct emission reductions, notably for electric vehicles (e.g. Archsmith et al., 2015),[2] raising concerns about the merits of their subsidies. However, the relevance of LCA for policy design is questionable. The original, and most frequent, type of LCA, denominated Attributional LCA (ALCA), accounts for current physical flows and typically utilized upstream average emissions intensity (Earles and Halog, 2011). Such accounting does not describe the impact of adding one unit of the good under scrutiny or increasing the subsidy to this good. To do so, Consequential Life Cycle Assessments (CLCA) integrate economic mechanisms (Earles and Halog, 2011; Rajagopal, 2014). CLCA have been used to evaluate biofuels, taking into account the direct and indirect land use changes that call into question their carbon footprint (e.g Bento and Klotz, 2014).[3] LCA are notably used to set intensity regulations such as fuel standards in the transportation sector (Rajagopal et al., 2017). However, while CLCA quantify the impact of policies on emissions and resources, they do not analyze optimal policies in an economic sense, integrating consumers' surplus and costs.

We develop a partial equilibrium model with two sectors: an upstream and a downstream one. Households consume both goods. In each sector, a dirty and a clean technology are available. The clean downstream technology (e.g. electric vehicles) consumes a part of the upstream production (e.g. electricity) as an input. We analyze the optimal allocation of

[1]The International Organization for Standardization defines LCA as the "compilation and evaluation of the inputs, outputs and the potential environmental impacts of a product system throughout its life cycle" see section 3.2. https://www.iso.org/obp/ui/#iso:std:iso:14040:ed-2:v1:en

[2]See Hajjaji et al. (2013) for a LCA of hydrogen production. The case of hydrogen involves three stages: the fuel market (downstream), hydrogen production (upstream 1), and electricity production (upstream 2) in the case of electrolysis. Another more prospective example is cultured meat which may abate cattle emissions but requires a lot of energy that could be clean electricity (Tuomisto and Teixeira de Mattos, 2011; Mattick et al., 2015).

[3]See Ahlgren and Di Lucia (2014); De Cara et al. (2012) for reviews of the literature on biofuel climate impact which stress the diversity of modeling choices and the differences between economic and CLCA approaches.

production for a given Social Cost of Carbon (SCC), and second-best policies in a flexible setting in which dirty goods may not be taxed at the Pigovian level. This analytical inquiry is completed with a numerical illustration based on the electrification of a transportation sector with an electricity mix composed of gas-fired and renewable electricity.

We investigate how upstream emissions should be integrated into the subsidy of the clean downstream technology. The optimal downstream subsidy should be corrected for all external costs generated in the upstream sector, not only unpriced pollution but also the fiscal externality due to the subsidy to the clean upstream technology. What matters is not upstream average emission intensity, as usually considered in LCA, but the adjustment of both dirty and clean upstream production weighted by their respective external cost.

We relate the formula obtained to the concept of CLCA. From a policy perspective, the relevant metric should be "consequential life-cycle external costs".[4] We further generalize our analysis to alternative policy instruments, among which mandates in either the upstream or downstream sector.

We then analyze the joint optimization of subsidies in both sectors. We show that the optimal upstream subsidy does not directly incorporate features of the downstream sector, whereas the downstream optimal subsidy depends upon the upstream sector characteristics, and notably the difference between the SCC and the upstream carbon tax. This asymmetry is due to the fact that the clean downstream indifferently consumes both clean and dirty upstream production, while the clean upstream production can only be consumed by the clean and not the dirty downstream productions. This result could be extended to multiple downstream sectors. The total welfare loss between first-best and second-best policies only depends on carbon mispricing in both sectors and not on sectoral linkage. The sectoral allocation of this loss is investigated numerically for the electrification of transport. While most emissions and abatement come from the power sector, the transport sector is the main contributor to total welfare. Consequently, total welfare differences are small between first-best and second-best policies, but this hides significant welfare transfers from the downstream sector to the upstream sector. We analyze the cost of a lack of coordination among sectors, comparing the second-best policy to a situation in which the downstream subsidy is set ignoring upstream consequences. Our numerical simulations show that incoordination further magnifies the transfer from the upstream sector to the downstream sector.

From a more descriptive perspective, we describe the influence of each instrument (tax and subsidy in each sector) on total emissions. Such comparative static exercise is similar to a CLCA. We identify mutually exclusive conditions under which total emissions increase with either the downstream subsidy (e.g. to electric vehicles) or the upstream tax (e.g. on gas-fired power plants). While the former case is relatively intuitive, the latter is more surprising. A consequence is that an increase in the SCC, along a transition pathway, may involve an increase in upstream emissions because of the deployment of the clean downstream technology. Our numerical inquiry of electric mobility suggests that such a case is possible for a sufficiently large and carbon-intensive downstream sector (including heavy mobility), because of the likely small elasticity of electricity demand and renewable supply.

Overall, the present work bridges the gap between economic models of the energy transition and LCA discussions in industrial ecology. The latter raises questions for climate policy

[4]We thank an anonymous referee for suggesting a similar wording and putting forward the concept.

design that are not addressed in the former. Some of these questions are due to the observed lack of exhaustive and efficient pricing of emissions (World Bank, 2021).

In their seminal work Lipsey and Lancaster (1956) establish that optimality conditions guiding policy instruments should be modified to integrate pre-existing distortions. Indeed, too small a tax on a polluting good justifies subsidizing clean substitutes. The literature on carbon leakage has analyzed how the regulation of domestic emissions (via tax or tradable permits) should be complemented by subsidies (possibly via output-based rebates) to domestic goods with unregulated foreign substitutes (Fischer and Fox, 2007, 2012; Meunier et al., 2017; Fowlie and Reguant, 2021). Notably, Meunier et al. (2017) and Fowlie and Reguant (2021) analyze how the optimal subsidy depends on the sensitivity of foreign production to home production and foreign emission intensity. The fact that marginal and not average intensities matter for the optimal subsidy is also present in our analysis. Indeed, the domestic-foreign relationship differs from the downstream-upstream one, and these articles do not consider foreign regulations whereas we consider upstream regulation. Galinato and Yoder (2010) consider the optimal combination of tax and subsidy under a net-revenue constrained carbon tax and subsidy program, which explains a departure from the Pigovian rule. They do not model sectoral interconnections, even though they provide numerical illustrations for the electricity and transport sectors because they consider (second generation) biofuels as the clean transport technology and not electric cars. The combination of their analysis with our model is a path for future research.

The relationship between policies on electric mobility and electricity production has been investigated by Holland et al. (2015, 2021) and Gillingham et al. (2021). Holland et al. (2015) analyze optimal second-best subsidies to electric vehicles and how they should integrate emissions from power production. Holland et al. (2021) consider the transition of the transportation sector with an exogenously decarbonizing power sector. In both articles, they do not consider the impact of electricity regulation on the optimal vehicle subsidy, and possibly the joint optimization of policies. Gillingham et al. (2021) analyze how the regulation of the power sector influences the environmental impact of electric vehicles. Their analysis is descriptive and they do not analyze optimal policies.

Finally, our comparative static results are related to a literature that identifies mechanisms through which a subsidy to abatement could increase total emissions, through free entry (Baumol and Oates, 1975) or general equilibrium effects (Kohn, 1992; Mestelman, 1972, 1982). In the present work, a different, and simpler, mechanism is identified, total emissions increase because of upstream polluting production. However, we also identify conditions under which a tax on a polluting (upstream) technology could increase total emissions because of downstream consequences.

The article is organized as follows. In Section 2, we introduce the framework and characterize the optimal (first-best) allocation and the market equilibrium. In Section 3, second-best policies are analyzed, and their relationship with LCA is discussed. The numerical simulation is performed in section 5. Section 6 concludes.

2 The analytical framework

2.1 The model

We consider a partial equilibrium model with two sectors and four technologies (two per sector). There are an upstream sector (e.g. electricity) and a downstream sector (e.g. road transport) labeled $i \in \{U, D\}$. In each sector, a good is consumed by households and can be produced with two technologies: a "dirty" polluting technology and a "clean" emission-free technology labeled $j \in \{d, c\}$. The clean downstream technology uses the upstream good (electricity is both consumed by households and by electric cars). The structure of the model is shown in Figure 1.

For each sector $i = U, D$, the total quantity consumed by households is Q_i and the associated gross consumers surplus is $S_i(Q_i)$, with $S_i' > 0$, and $S_i'' < 0$. On the production side, in sector $i = U, D$, the total quantity produced is $q_{id} + q_{ic}$ the sum of dirty and clean productions, with production costs $C_{ij}(q_{ij})$ with $j = d, c$. Cost functions are positive, increasing and convex, $C_{ij}' > 0$ and $C_{ij}'' \geq 0$.[5] Each clean downstream unit consumes θ units of the upstream good so that the total quantity produced $q_{Uc} + q_{Ud}$ is equal to the quantity consumed by households Q_U and by the clean downstream variety θq_{Dc}: $q_{Ud} + q_{Uc} = Q_U + \theta q_{Dc}$. We will refer to θ as the *linkage intensity*.[6] In sector i, each unit produced by the dirty technology emits α_i tons of CO_2. We denote μ (in €per tCO_2) the Social Cost of Carbon (SCC). Total welfare is then:[7]

$$W(\mathbf{q}, \mu) = \sum_i S_i(Q_i) - \sum_{ij} C_{ij}(q_{ij}) - \mu[\alpha_D q_{Dd} + \alpha_U q_{Ud}] \tag{1}$$

subject to $Q_D = q_{Dd} + q_{Dc}$ and $Q_U + \theta q_{Uc} = q_{Ud} + q_{Uc}$ and $q_{ij} \geq 0$ for $i = U, D$ and $j = d, c$.

[5]In the case of electricity and transportation, the convexity of the clean technology costs is caused by multiple factors. For renewable energy, it comes from a combination of site scarcity, storage, and transportation costs. For electric cars, it mainly comes from the increasing cost associated with density (urban vs rural) and types (weight) of vehicles.

[6]For instance, in the case of electric mobility, linkage intensity would be the energy efficiency of the vehicle: the number of kWh per km.

[7]Other externalities, such as local air pollution or congestion, are not considered even though they constitute important externalities. They could be included in the framework but proper modeling would need to introduce more heterogeneity because the external cost of air pollution varies not only between sectors but also within sectors according to the location of each source of emissions.

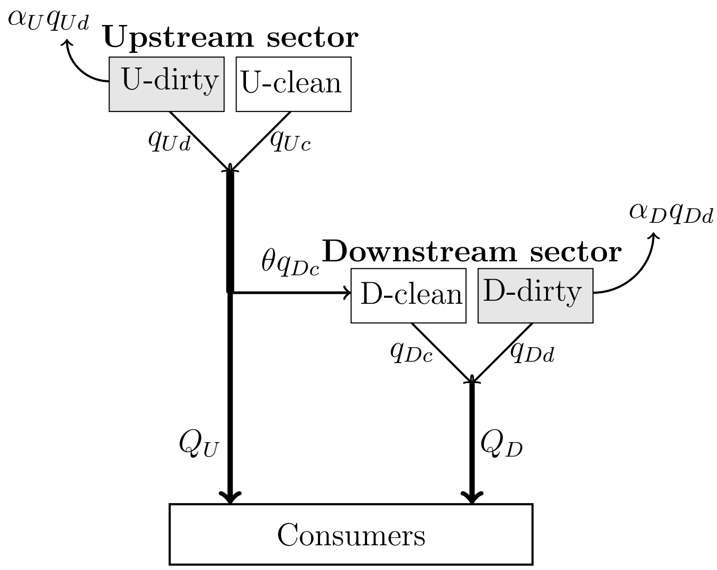

Figure 1: The structure of the model.

In sector $i = U, D$, the selling price of good i is p_i, there is a tax t_i on dirty units and a subsidy s_i on clean units, both can indeed be negative. Net consumer surplus is

$$CS_i(Q_i, p_i) = S_i(Q_i) - p_i Q_i, \tag{2}$$

and the inverse demand function is $P_i(Q_i) = S_i'(Q_i)$.[8] The profit of dirty and clean producers are

$$\pi_{id} = p_i q_{id} - t_i q_{id} - C_{id}(q_{id}) \quad \text{ for } i = U, D \tag{3a}$$
$$\pi_{Uc} = p_U q_{Uc} + s_U q_{Uc} - C_{Uc}(q_{Uc}) \tag{3b}$$
$$\pi_{Dc} = p_D q_{Dc} + s_D q_{Dc} - C_{Dc}(q_{Dc}) - p_U \theta q_{Dc} \tag{3c}$$

Both consumers and producers are assumed to be price takers, and respectively maximize consumers' net surplus and producers' profit, prices clear both markets. Indeed, it is equivalent considering that a single representative firm maximizes the aggregate profit over the two sectors or that a multitude of small producers is doing so for each sector and technology.

Total welfare can be split between the upstream sector and the downstream sector and rewritten as the sum of consumer net surplus, producer profit, and tax revenues:

$$W_i \equiv CS_i + \pi_{id} + \pi_{ic} + t_i q_{id} - s_i q_{ic} - \mu \alpha_i q_{id} \text{ for } i = U, D \tag{4a}$$
$$W = W_U + W_D \text{ from eq. (1), (2) and (3)} \tag{4b}$$

[8]Consumers are assumed to be price takers, and the demand function, $P_i^{-1}(p_i)$, maximizes the net consumer surplus.

To analyze the adjustments of quantities to an increase in the SCC or policy instruments, the following notation helps:

$$\Gamma_{ij} = \begin{cases} \frac{C''_{ij}(q_{ij})}{-S''_i(Q_i)} & \text{if } q_{ij} > 0 \\ +\infty & \text{if } q_{ij} = 0 \end{cases} \tag{5a}$$

$$\Lambda = \frac{S''_U}{S''_D} \tag{5b}$$

The parameter Γ_{ij} is the ratio between the slope of the supply curve of good ij relative and the slope of the price function, which represents the relative adjustment of the supply of good ij compared to the reduction of demand induced by an increase in the price. Parameter Λ compares the responsiveness of upstream and downstream demands to their respective price.

Finally, we assume that with a small SCC only the dirty technology is used, and for a sufficiently large one there are positive quantities consumed in both sectors supplied with the clean technologies.

Assumption 1 *There are $Q^0_D > 0$ and $Q^0_U > 0$ such that*

$$S'_i(Q^0_i) = C'_{id}(Q^0_i) < C'_{ic}(0) \text{ for } i = U, D. \tag{6a}$$

And there are $Q^1_D > 0$ and $Q^1_U > 0$ such that

$$S'_U(Q^1_U) = C'_{Uc}(Q^1_U + \theta Q^1_D) \text{ and } S'_D(Q^1_D) = C'_{Dc}(Q^1_D) + \theta C'_{Uc}(Q^1_U + \theta Q^1_D) \tag{6b}$$

The static framework can be used to analyze a dynamic transition along which the SCC increases and the economy moves from a state with only the dirty technology to a fully clean situation. Along that transition, multiple technology mixes may arise as the clean technologies are progressively phased in and dirty technologies phased out.

The following comments have to be made on the previous modeling choices. First, we consider perfect substitutability on the consumption side between technologies in each sector. It simplifies the analysis and help focuses on the impacts of sectoral linkage, it also allows to have a well-defined MAC of substituting a dirty unit for a clean one. In the power sector technologies are not perfect substitutes because of storage cost and the variability of demand and renewable supply (e.g. Baranes et al., 2017). We consider that the convexity of the upstream costs includes storage costs (as in Coulomb et al., 2019). In the mobility sector, electric and gasoline vehicles are more or less substitutes depending on the use (distance traveled per trip and per year, population density, weather conditions...). Second, the clean downstream technology cannot discriminate among upstream technologies and consumes the same mix as other consumers.[9]

Third, our framework is static and does not include dynamic aspects such as adjustment cost, learning-by-doing, or sectoral inertia. Indeed, linear investment costs, stable over time,

[9]For instance, we do not consider the possibility to charge electric cars at night, so that the content of the electricity used to charge is not the same as the total mix of the grid. Also, a fuel-cell powered technology could choose its source of hydrogen (from electrolysis or gas).

could be considered included in the production cost of clean technologies.[10] Fourth, a peculiar type of sectoral linkage is considered here. The clean downstream technology creates a vertical sectoral linkage with the ”upstream” sector. A more general and realistic setting would introduce sectoral relationships in an input/output framework, all sectors would be already linked before the introduction of the clean downstream technologies which would be associated with other technical coefficients.[11]

To get explicit formula and make simulations, we will use the following quadratic specification (see Appendix A for the expressions of Q_i^0 and Q_i^1, $i = U, D$):

Specification 1 *For $i \in \{U, D\}, j \in \{e, d\}$*

$$S_i(Q_i) = a_i Q_i - \frac{b_i}{2} Q_i^2 \tag{7a}$$

$$C_{ij}(q_{ij}) = c_{ij} q_{ij} + \frac{b_i \Gamma_{ij}}{2} q_{ij}^2 \tag{7b}$$

with $a_i, b_i, c_{ij}, \Gamma_{ij}$ all non-negative real numbers.

2.2 Optimal allocation

We first consider the optimal allocation and clarify the relationship between marginal abatement costs (MAC) and life cycle emissions. Indeed, MACs, obtained by substituting a dirty unit with a clean unit, in both sectors should be equalized with the SCC. For a given SCC, the cost of the clean downstream technology depends upon the upstream sector consumers surplus and production costs. Upstream emissions should be encompassed in the computation of the downstream MAC consistently with the cost considered (Lemma 1).

The optimal allocation $\mathbf{q}^{FB}(\mu) = (q_{ij}^{FB}(\mu))_{i,j}$ maximizes welfare (1). Denoting ϕ_{ij} the Lagrange multiplier of the positivity constraint $q_{ij} \geq 0$, the first order conditions are:

$$P_U(Q_U) = C'_{Ud}(q_{Ud}) + \alpha_U \mu - \phi_{Ud} \tag{8a}$$

$$= C'_{Uc}(q_{Uc}) - \phi_{Uc} \tag{8b}$$

$$P_D(Q_D) = C'_{Dd}(q_{Dd}) + \alpha_D \mu - \phi_{Dd} \tag{8c}$$

$$= C'_{Dc}(q_{Dc}) + \theta P_U(Q_U) - \phi_{Uc} \tag{8d}$$

$$Q_U + \theta q_{Dc} = q_{Ud} + q_{Uc} \tag{8e}$$

$$Q_D = q_{Dc} + q_{Dd} \tag{8f}$$

[10] For dirty technologies, the situation is less simple since they are phased out and some capacity may remain idle.

[11] In a full input-output setting, each of our four “technologies” would be a sector. Indeed, the upstream-downstream relationship would be less clear, but what matters is that the clean downstream technology i) consumes much more electricity than the dirty downstream technology and ii) cannot discriminate between the two upstream technologies. The overall message is that subsidy to clean technologies should be adjusted for all life cycle externalities: not only unpriced emissions but also ”upstream” subsidies would remain even though the precise formula would likely be much more intricate, notably if demands are elastic and all costs convex.

At the optimal allocation in each sector, a positive quantity is produced and consumed thanks to Assumption 1, and the marginal consumer surplus is equal to the marginal costs of each technology used. Note that the marginal cost of the clean downstream technology encompasses the marginal benefit from the upstream good consumption P_U.

Lemma 1 *At the optimal allocation, if all technologies are used, the SCC is equal to the MACs of substituting a dirty by a clean unit in both sectors. In the downstream sector, a relevant MAC should weight similarly upstream marginal costs and upstream emissions:*

$$\forall \omega \in [0,1],\ \mu = \frac{C'_{Dc} + \theta[(1-\omega)C'_{Uc} + \omega C'_{Ud}] - C'_{Dd}}{\alpha_D - \theta\omega\alpha_U} \tag{9}$$

In each sector, there are two ways to reduce emissions: reducing demand or substituting a dirty unit by a clean one. If all quantities are positive, at the optimal allocation the MAC associated with each option should be equal to the SCC. These MAC should be computed with direct emissions:

$$\mu = \frac{C'_{Uc} - C'_{Ud}}{\alpha_U} = \frac{C'_{Dc} + \theta P_U - C'_{Dd}}{\alpha_D}$$

At first glance, indirect emissions of the clean downstream technology do not intervene in those formula. However, the marginal cost of the clean downstream technology encompasses the marginal value of the upstream good P_U which depends upon the SCC and upstream emissions intensity. Replacing the expression of P_U with either equations (8a) or (8b) gives the two relations:

$$\mu = \frac{C'_{Dc} + \theta C'_{Uc} - C'_{Dd}}{\alpha_D} = \frac{C'_{Dc} + \theta C'_{Ud} - C'_{Dd}}{\alpha_D - \theta\alpha_U}$$

These equations tell us that the upstream emissions taken into account at the denominator of the MAC should be consistent with the upstream cost at the numerator. Indeed, it works with any weighting of the two technologies as long as marginal costs and emissions rates are similarly weighted. The marginal clean downstream unit could be produced using any mix of upstream technologies, and the MAC should be computed accordingly taking into account upstream emissions.

The implicit assumption that other quantities are held constant is only valid at the first-best optimum, an issue not specific to our setting, the analysis of second-best policies will highlight the limited relevance of MACs in second-best contexts.

It is possible that not all technologies are used. Indeed, for small (resp. large) SCC only dirty (resp. clean) technologies are used. In between, all configurations can arise depending on parameter values. Indeed, if a clean technology is not used then the MAC associated to it is below the SCC. Furthermore, the clean downstream quantity is not used if indirect upstream emissions are larger than downstream ones.

Lemma 2 *At the social optimum, if $\alpha_D < \theta\alpha_U$, then the clean downstream quantity is null if the dirty upstream quantity is positive.*

The proof is straightforward and relies on Assumption 1:

$$C'_{Dc}(0) + \theta(C'_{Ud} + \alpha_U\mu) \geq C'_{Dd}(Q_D^{FB}) + \alpha_D\mu$$

Together with equations (8c) and (8b) implies that q_{Ud}^{FB} is positive only if q_{Dc}^{FB} is null.

Intuitively, if the clean downstream is not used without pollution ($\mu = 0$) because of its larger costs, then it is not used if its indirect emissions are larger than the emission of the dirty technology it replaces. The relevant upstream emission intensity is the dirty technology intensity and not the average across both clean and dirty technologies. In that case, in a dynamic perspective, as the SCC increases the clean downstream technology is used only once the upstream sector is fully decarbonized. We assume that it is not the case for the rest of the article.

Assumption 2 *Emission intensity α_D is larger than indirect emission intensity $\theta\alpha_U$.*

2.3 Market equilibrium and comparative static

Market equilibrium prices and quantities satisfy the equations, denoting ψ_{ij} the Lagrange multiplier of the positivity constraint $q_{ij} \geq 0$:

$$S'_i(Q_i) = p_i = C'_{id}(q_{id}) + t_i - \psi_{id} \text{ for } i = U, D \tag{10a}$$

$$P_D(Q_D) = C'_{Dc}(q_{Dc}) + \theta p_U - s_D + \psi_{Dc} \tag{10b}$$

$$P_U(Q_U) = C'_{Uc}(q_{Uc}) - s_U + \psi_{Uc}. \tag{10c}$$

Lemma 3 *The first-best can be decentralized with Pigovian taxes $t_i = \alpha_i\mu$ and $s_i = 0$*

This textbook results helps clarify two important points: if all emissions are taxed when emitted, then life cycle considerations are not required to design the optimal policy. Furthermore, there is no need to coordinate policies, each local regulator sets the same tax level. However, both of these points only hold when taxes are optimally set at the Pigovian level, a case rarely met in the real world, so it is worth investigating consequences of departure from this situation.

Before analyzing optimal couple of subsidies, let us look at the impact of each instruments on total emissions. Indeed, one would expect that taxes on dirty goods and subsidies on clean good both reduce total pollution.

Proposition 1 *At the market equilibrium:*

- *Total emissions always decrease with upstream subsidies or downstream taxes.*
- *Total emissions increase with respect to the subsidy on the clean downstream quantity if and only if $q_{Dc} > 0$ and*

$$\frac{\alpha_D}{1 + \Gamma_{Dd}} < \frac{\theta\alpha_U}{\Gamma_{Ud}\left(1 + \frac{1}{\Gamma_{Ud}} + \frac{1}{\Gamma_{Uc}}\right)} \tag{11}$$

- *Total emissions increase with respect to the tax on the dirty upstream quantity if and only if* $q_{Ud} > 0$ *and*

$$\alpha_U < \frac{\alpha_D}{1+\Gamma_{Dd}} \left[\theta + \frac{1}{\theta\Lambda}\left(1+\frac{1}{\Gamma_{Uc}}\right)\left(\Gamma_{Dc}+\frac{\Gamma_{Dd}}{1+\Gamma_{Dd}}\right)\right]^{-1} \tag{12}$$

Proof in Appendix B.1. To understand the mechanisms behind Proposition 1, it is useful to consider the impact of a change of a quantity on the three others, rather than to look at the impact of the associated instrument (it is equivalent to use a price instrument or a quota to set a given quantity). In each case, emissions avoided in one sector are compared to emissions added in the other. In equation (11), the left-hand side corresponds to emissions avoided in the downstream sector by an increase in the clean downstream quantity and the right-hand side corresponds to emissions generated in the upstream sector to satisfy the associated demand. In equation (12), the left-hand side corresponds to emissions avoided in the upstream sector by a reduction of the dirty upstream quantity and the right-hand side corresponds to the increase in downstream emissions triggered by the increase in the upstream price and the substitution between the clean and dirty downstream productions.

The detailed expressions of the adjustments of quantities are obtained from the market equilibrium equations (10). Parameters Γ_{ij} and Λ, defined in equations (5), intervene in those changes as they characterize the slopes of demand and supply functions.

An increase in the subsidy s_D on the clean downstream, the second point of the Proposition, has a direct effect on the clean downstream quantity and an indirect effect on the three others.[12] The change dq_{Dc} of the clean downstream quantity, generated by a change of s_D, is associated with a reduction of the dirty downstream quantity, from equation (10a) if $q_{Dd} > 0$:[13]

$$P'_D.[dq_{Dc} + dq_{Dd}] = C''_{Dd}.dq_{Dd} \text{ so } dq_{Dd} = -\frac{dq_{Dc}}{1+\Gamma_{Dd}}.$$

And in the upstream sector from equation (10c) and (10a):

$$P'_U.[dq_{Ud} + dq_{Uc} - \theta dq_{Dc}] = C''_{Ud} dq_{Ud} = C''_{Uc} dq_{Uc}$$

and, dividing by $-P'_U$, the two upstream quantities are adjusted as follow:

$$dq_{Uj} = \frac{\theta dq_{Dc}}{\Gamma_{Uj}\left(1+\frac{1}{\Gamma_{Ud}}+\frac{1}{\Gamma_{Uc}}\right)} \text{ for } j = d, c \tag{13}$$

These adjustments will play a crucial role in the analysis of the optimal policies.[14]

And concerning the third point of Proposition 1, the subtraction of a dirty upstream unit induces a adjustment of both markets: both prices rise and in the downstream sector clean

[12] In full rigor the equilibrium quantities should be defined as functions of the four instruments $q^E_{ij}(t_U, t_D, s_U, s_D)$, and the marginal variations dq_{ij} considered in the main text would be more rigorously written as $\partial q^E_{ij}/\partial s_D$ (see Appendix B.1). The analysis of the influence of t_U follows the same logic with $\partial q^E_{ij}/\partial t_U$.

[13] Actually, the formula for dq_{Dd} also holds for $q_{Dd} = 0$ by definition of Γ_{Dd} (cf equation 5a).

[14] Concerning corners situations, if $q_{Dd} = 0$ the inequality is satisfied since the left-hand side is null ($\Gamma_{Dd} = +\infty$), and if $q_{Ud} = 0$ it is not, since the right-hand side is null ($\Gamma_{Ud} = +\infty$).

units are partly replaced by dirty ones. In condition (12), the right-hand side describes the emissions generated by that replacement, it is the product of the substitution between the dirty and clean downstream quantities (first factor) and the increase in the clean downstream quantity (second factor). An increase in the tax on the dirty upstream variety is more likely to increase emissions if the downstream emission intensity α_D is large, the dirty downstream quantity is highly responsive to the clean downstream quantity (Γ_{Dd} small), and the upstream demand and clean upstream supply are price inelastic ($-P_U'$ and Γ_{Uc} large). The influence of the linkage intensity is not monotonic, for extreme values, an additional upstream unit has little influence on the clean downstream quantity either because it plays a negligible role in the total cost (small θ) or because a lot is required per unit (large θ). In between, for intermediary values of θ, the reduction in downstream pollution can compensate the rise in upstream pollution. We discuss the empirical relevance of these conditions in the simulation section 5.5.

From these comparative statics, one can deduce the impact of the SCC on the optimal allocation as summarized in the following corollary.

Corollary 1 *A marginal increase in the SCC induces the following changes in the optimal allocation:*

- *Total and downstream emissions decrease. Quantities consumed by households decrease in both sectors.*
- *The clean upstream quantity increases, and, the clean downstream quantity decreases if and only if $q^*_{Dc} > 0$ and condition (11) holds.*
- *The dirty upstream quantity increase if and only if $q^*_{Ud} > 0$ and condition (12) holds.*

The proof is in Appendix B.1. Most consequences of an increase in the SCC are intuitive except for dirty upstream and clean downstream quantities. Dirty upstream production increases if condition (11) is satisfied, and clean downstream production decreases if condition (11) is satisfied. When the SCC increases both the demand for and the cost of the clean downstream technology increases and if condition (12) is satisfied the latter dominates and the clean downstream quantity decreases. In the upstream sector, the cost of the dirty technology increases with the SCC but the demand for the upstream good, emanating from the downstream sector, increases and, if condition (12) holds, can compensate for the cost increase and requires an expansion of the dirty technology. The two conditions (11) and (12) are mutually exclusive,[15] the increase in dirty upstream production occurs only if the clean downstream technology expands.

If dirty technologies have linear costs, $\Gamma_{id} = 0$ for $i = U, D$, then, if they are used, they set the price in both sectors ($p_i = c_{id} + \alpha_i \mu$), and the two conditions (11) and (12) are simplified. Notably, condition (11), under which the clean downstream decreases with respect to the SCC, is $\alpha_D < \theta\alpha_U$, which contradicts Assumption 2.

[15]Multiplying both sides of (11) by θ gives $\alpha_D/(1+\Gamma_{Dd}) > \alpha_U$, while condition (12) implies $\alpha_D/(1+\Gamma_{Dd}) < \alpha_U$.

3 Optimal policies

In this section, we analyze the implications of sectoral linkages on the optimal downstream policy. First, we derive the optimal downstream subsidy and analyze how upstream features intervene. We generalize those results to other policy instruments. Second, we use these results to discuss the relevance and limitations of ALCA and CLCA for policy design.

3.1 Optimal downstream policies

3.1.1 Optimal subsidy

Let us start with a discussion of the optimal downstream subsidy for a given tax on the dirty downstream technology and regulation (both tax and subsidy) in the upstream sector. For instance, in the case of electric mobility, the question is whether emissions associated with electricity production should influence the optimal subsidy on electric vehicles. Even though the subsidy is initially justified by the unpriced negative externality from the dirty downstream technology, it should also be adjusted to the sub-optimal upstream regulation.

For an instrument τ (a tax or a subsidy), the maximization of the welfare function given by equation (1) gives the first-order condition:

$$0 = \sum_{i,j} \frac{\partial W}{\partial q_{i,j}} \frac{\partial q_{i,j}}{\partial \tau} \tag{14}$$

Injecting equations (10a), (10b), (10c), satisfied at the market equilibrium gives:

$$s_U \frac{\partial q_{Uc}}{\partial \tau} + (\alpha_U \mu - t_U) \frac{\partial q_{Ud}}{\partial \tau} + s_D \frac{\partial q_{Dc}}{\partial \tau} + (\alpha_D \mu - t_D) \frac{\partial q_{Dd}}{\partial \tau} = 0 \tag{15}$$

for all instruments, each derivative $\frac{\partial q_{ij}}{\partial \tau}$ only depends on θ, Γ_{ij} and Λ (defined in equations (5)), which enables to obtain an explicit formula for the optimal downstream subsidy, as given by the following proposition.

Proposition 2 *For given downstream tax t_D and upstream tax and subsidy t_U and s_U, the optimal downstream subsidy, is:*

$$s_D = \frac{1}{1 + \Gamma_{Dd}} (\alpha_D \mu - t_D) - \theta \sigma \tag{16}$$

with σ defined as:

$$\sigma \equiv \frac{1}{1 + \frac{1}{\Gamma_{Ud}} + \frac{1}{\Gamma_{Uc}}} \left[(\alpha_U \mu - t_U) \frac{1}{\Gamma_{Ud}} + s_U \frac{1}{\Gamma_{Uc}} \right] \tag{17}$$

The proof is in Appendix B.3. The term σ accounts for the influence of the upstream sector on the optimal subsidy. It can be interpreted as the (marginal) indirect subsidy to the downstream technology due to upstream inefficient regulations, we come back to it below.

The optimal downstream subsidy is justified by externalities, indeed, if emissions are taxed at the Pigovian level, so $t_i = \alpha_i \mu$ and $s_U = 0$, the optimal subsidy is null. Otherwise, the subsidy is influenced by external benefits and costs generated in both the downstream and upstream sectors. In the downstream sector, an increase in clean production reduces dirty production by an amount determined by the slopes of consumer demand and dirty marginal cost. If either the demand is inelastic or dirty cost is linear the rate of substitution is equal to minus one.[16]

Concerning the influence of the upstream sector regulation $\theta\sigma$: First, if emissions are efficiently priced ($t_U = \alpha_U \mu$, $s_U = 0$) the upstream sector characteristics do not intervene in the formula ($\sigma = 0$). It is so because the environmental cost is already encompassed in the upstream price. Second, the optimal downstream subsidy does not depend on the average mix in the upstream sector but on the emission intensity of the marginal unit which is a weighted sum of dirty and clean production, the weights depending on the slope of the respective marginal costs. With a linear dirty upstream cost ($\Gamma_{Ud} = 0$) that marginal unit is dirty as long as there is some dirty production, and, in such a case, the optimal downstream subsidy is:

$$s_D = \frac{1}{1+\Gamma_{Dd}}(\alpha_D\mu - t_D) - \theta(\alpha_U\mu - t_U). \tag{18}$$

The term σ is the sum of external costs generated by additional upstream demand, it consists of the increase in the two upstream quantities times the associated external cost due to the mispricing of pollution ($\alpha\mu - t_U$) and the subsidy (s_U). The increases in upstream quantities are determined by the slope of upstream costs and demand. Indeed, to supply the additional unit there is a reduction of consumption and an increase in production from the two technologies.

3.1.2 Alternative instruments

In practice, in many countries, multiple regulations are in place in candidate upstream and downstream sectors, notably in the electricity, transportation, industry, and building sectors. For instance, in the EU, the power sector and several industrial sectors are covered by the EU Emission Trading Scheme, and low-carbon technologies (renewable and nuclear where it is still developed) are subsidized with targeted shares of renewables in total electricity production.[17] Furthermore, in both the US and the EU, transportation sectors are subject to several regulations: an EU standard on fossil cars emission intensity and subsidies on electric cars.

Here, we do not aim at an exhaustive analysis of second-best instruments coordination, and only explore optimal downstream regulation in two contexts: First, we analyze the optimal downstream subsidy when there is an alternative (non-price) upstream instrument (Proposition 3). Second, we analyze an optimal downstream alternative instrument with the upstream sector regulated with a tax and a subsidy (Proposition 4). In both cases, the general result obtained is declined to the specific case of a mandate in the Upstream sector (Corollary 2), and in the downstream sector (Corollary 2).

[16] The formula could be generalized to take into consideration an imperfect substitution between dirty and clean downstream goods on the consumer side as empirically investigated by Xing et al. (2021).

[17] Concerning the EU-ETS, it is a cap and trade system with several additional features, most notably the market stability reserve that makes the cap flexible, total emissions are therefore more or less fixed.

Proposition 3 *The optimal downstream subsidy is the difference between the marginal external benefit from reduced downstream emissions and the sum of the adjustments of upstream quantities to face the additional demand weighted by their respective implicit subsidies. These implicit subsidies relate to the mispricing of pollution and subsidy of the clean upstream technology and depend on the upstream regulation.*

Let us formalize, and prove Proposition 3. Consider that the upstream regulation is fixed and upstream quantities adjust according to the supply curves $q^r_{Uj}(p_U)$ for $j \in \{d, c\}$. The upstream market equilibrium is described by the market clearing equation:

$$q^r_{Ud}(p_U) + q^r_{Uc}(p_U) = P_U^{-1}(p_U) + \theta q_{Dc}.$$

Any increase in the clean downstream quantity is associated with an adjustment on the upstream market described by the two equations:

$$dq_{Uj} = q^{r\prime}_{Uj} dp_U = q^{r\prime}_{Uj} \frac{\theta dq_{Dc}}{q^{r\prime}_{Uc} + q^{r\prime}_{Ud} - 1/P'_U} \text{ for } j = d, c.$$

By analogy with Γ_{Uj}, we note $\Gamma^r_{Uj} = -1/(P'_U q^{r\prime}_{Uj})$, the ratio of the slopes of the supply of technology $j = d, c$ and upstream demand, and obtain a generalization of σ and formula (16):

$$\begin{aligned} s_D &= \frac{\alpha_D \mu - t_D}{1 + \Gamma_{Dd}} - \theta \sigma^r \\ \text{with } \sigma^r &= \frac{1}{1 + \frac{1}{\Gamma^r_{Ud}} + \frac{1}{\Gamma^r_{Uc}}} \left[(\alpha_U \mu - (p_U - C'_{Ud})) \frac{1}{\Gamma^r_{Ud}} + (C'_{Uc} - p_U) \frac{1}{\Gamma^r_{Uc}} \right]. \end{aligned} \tag{19}$$

The downstream subsidy needs to be reduced by the indirect subsidy encompassed in the upstream price. The second term above is the sum of the implicit subsidies, in parenthesis, times the adjustment of the associated quantity. The parameters $\Gamma^{r\prime}_{Uj}$ depend on the regulation in the upstream sector. The following Corollary illustrates this result with an upstream mandate.

Corollary 2 *If there is a mandate of a (binding) share r_U of clean production in the upstream sector ($q_{Uc} = r_U(q_{Ud} + q_{Uc})$), the optimal downstream subsidy is:*

$$s_D = \frac{\alpha_D \mu - t_D}{1 + \Gamma_{Dd}} - \theta \frac{(1 - r_U)\alpha_U \mu}{1 + r_U^2 \Gamma_{Uc} + (1 - r_U)^2 \Gamma_{Ud}} \tag{20}$$

The proof is in Appendix B.4. With an upstream mandate, the share of the dirty upstream production is fixed at $(1 - r_U)$, and $\alpha_U(1 - r_U)$ is then the average emission intensity of upstream production. The optimal downstream subsidy should encompass these upstream emissions weighted by the reduction of upstream demand. Indeed, if upstream demand is inelastic, or upstream costs are linear, then $\Gamma_{Uj} = 0$ for $i = d, c$ and formula (20) simplifies to

$$s_D = \frac{\alpha_D \mu - t_D}{1 + \Gamma_{Dd}} - \theta(1 - r_U)\alpha_U \mu. \tag{21}$$

In that specific situation (a mandate and inelastic demand), the optimal downstream subsidy should be corrected by the average upstream emissions.

Let us now consider an optimal downstream general regulation, as above we first enunciate the general principle obtained before formalizing and applying it to the case of mandate.

Proposition 4 *An optimal regulation of the downstream sector is such that the marginal downstream benefit is equal to $\theta\sigma$, that is, the sum of the upstream implicit subsidies times the adjustment of corresponding quantities.*

The marginal downstream benefit is the difference between the benefits from emissions avoided and the implicit subsidy to the clean downstream technology. The adjustment of upstream quantities is independent of the downstream regulation.

Let us consider a generic downstream regulation r which, together with the upstream price, determines the two downstream quantities. The upstream market clears according to equations (10a) and (10c) so that the adjustment of upstream quantities is still described by equation (35) and does not directly depends on r. The welfare impact of a change of r is, injecting (10a) and (10c) into (15):

$$(P_D - C'_{Dc} - \theta P_U)\frac{dq_{Dc}}{dr} + (P_D - C'_{Dd} - \alpha_D\mu)\frac{dq_{Dd}}{dr} - s_U\frac{dq_{Uc}}{dr} - (\alpha_U\mu - t_U)\frac{dq_{Ud}}{dr}.$$

And the optimal r satisfies the following equation (making use of (35)):

$$\Big[\alpha_D\mu - (P_D - C'_{Dd})\Big]\frac{-dq_{Dd}/dr}{dq_{Dc}/dr} - \Big[C'_{Dc} + \theta P_U - P_D\Big] = \theta\sigma. \tag{22}$$

The left-hand side is the marginal net benefit of an increase in the clean downstream quantity due to a change of the regulation. It is the difference between the benefit from reduced pollution and the implicit downstream subsidy. The right-hand side is the marginal net cost in the upstream sector of an increase in the clean downstream quantity. It corresponds to the upstream corrective term found for the optimal downstream subsidy in equation (19).

If the downstream instrument is a mandate, it is well known that a mandate can be described as a combination of an implicit subsidy on the clean mandated technology and an implicit tax on the dirty one (Holland, 2012). The optimal mandate formula should be adjusted to account for downstream demand adjustment and upstream external costs.

Corollary 3 *If the downstream sector is regulated with a mandate, the optimal share of the clean downstream (r_D) is such that:*

$$C'_{Dc} + \theta P_U - C'_{Dd} = \Big[\alpha_D\mu - \theta\sigma\Big] + \Big[(1-r_D)\alpha_D\mu + r_D\theta\sigma\Big]\frac{-1}{Q_D}\frac{dQ_D}{dr_D}. \tag{23}$$

with $\theta\sigma$ given by equation (17).

The proof is in Appendix B.5. There are several ways to characterize the optimal mandate. The formula (23) allows isolating the influence of the upstream demand elasticity.[18]

[18] Holland et al. (2009) and Lemoine (2017) analyze the design of mandate with an incomplete carbon pricing, motivated by the design of low-carbon fuel standard in California. Holland et al. (2009) consider the choice of the average carbon intensity with two fuels (gasoline and ethanol), which corresponds to the choice of r in our framework. With more than two fuels, Lemoine (2017) shows that the regulator should also fix emissions rating different from the true emission rates. In both articles, vertical interactions with an upstream market are not modeled.

The optimal mandate should be set such that the difference between the clean and dirty marginal costs (left-hand side) is equal to the marginal external benefit of substituting a dirty by clean downstream unit (first bracketed term) plus the marginal external benefit from reducing the total downstream demand (second bracketed term). The benefit of substituting a dirty by a clean downstream unit is the difference between the marginal environmental damage and the indirect subsidy $\theta\sigma$. It is as if the clean downstream were assigned an external cost $\theta\sigma$.

3.2 Attributional and Consequential LCA

ALCA, the most common type of LCA, focuses on the physical flows that compose the life cycle of a product (from material extraction to end of life), and aims at attributing all those flows. CLCA aims at assessing the changes in quantities as a consequence of a decision (Rajagopal and Zilberman, 2013; Rajagopal, 2014). The choice of methodology should be consistent with the intended use of the metric. Here, we clarify how both approaches fit with our economic analysis by first assessing the relationship of these metrics to the optimal allocation, and, second their relation to second-best policies.

3.2.1 LCA and the optimal allocation

In our setting, an ALCA of emissions from the clean downstream sector is

$$E_{ALCA} = q_{Dc} \times \theta \frac{\alpha_U q_{Ud}}{q_{Uc} + q_{Ud}}. \tag{24}$$

It attributes upstream emissions to end-uses proportionally, so that all upstream emissions are attributed. According to Lemma 1, ALCA can be used to compute a policy-relevant MAC as long as upstream marginal costs at the numerator are weighted consistently with upstream emissions at the denominator, that is, $\omega = \alpha_U q_{Ud}/(q_{Uc} + q_{Ud})$. Indeed, Lemma 1 establishes that any weighting would work. Furthermore, according to Lemma 2 it is sufficient that indirect emissions $\theta\alpha_U$ be larger than direct emissions α_D, the maximum abated, for the clean downstream to be unused at the optimum. The market shares of the two upstream technologies do not matter for that decision.

ALCA is not conceived to assess the impact on emissions of adding a unit of the clean downstream good. This is the purpose of CLCA. Proposition 1 can be interpreted as a stylized CLCA of each of the four goods, and four instruments, present in our model. According to Corollary 1, a technology with positive consequential emissions should be less used, at the optimal allocation, as the SCC increases. Therefore, ALCA are linked to static considerations: the attribution of current emissions to end-uses and the computation of MAC. CLCA are linked to the evolution of quantities along the (optimal) transition.

The result that a good (whether an input or a consumption good) with positive consequential emissions should be less produced as the external cost of emissions increases, is general and not specific to our model. First, the optimal allocation corresponds to the market equilibrium with Pigovian taxes. Second, the consequential emissions of a good can be interpreted as an assessment of the substitutability of a good with total emissions, a negative (resp. positive) consequential emissions means that the two are substitutes (resp.

complements). And, as the external cost of emissions increase, goods that are substitutes to emissions should be produced more and complements produced less (cf the proof of Corollary 1 in Appendix B.1).

3.2.2 CLCA and second-best subsidies

The optimal subsidy of the clean downstream sector described by formula (16) in Proposition 2 can be linked to life cycle considerations. If all three other instruments are null the optimal subsidy is:

$$s_D = \mu \left[\alpha_D \frac{1}{1+\Gamma_{Dd}} - \theta \alpha_U \frac{1}{\Gamma_{Ud}} \frac{1}{1 + \frac{1}{\Gamma_{Ud}} + \frac{1}{\Gamma_{Uc}}} \right]$$

which corresponds to the SCC times the consequential emissions from an additional unit of the clean downstream. The first term corresponds to the emissions avoided in the downstream sector and the second one to the emissions generated in the upstream sector (cf discussion below Proposition 1). In that case, the indirect subsidy σ is the SCC times the increase in upstream emissions.

With non-null taxes and subsidies, the formula should be corrected to only account for externalities. From a welfare perspective, all mispriced goods generate externalities, not only pollutants but also subsidized quantities which generate fiscal externalities. The term $\theta\sigma$ is the policy-relevant consequential assessment and may be named "consequential life cycle external costs". Furthermore, the adjustment of the two upstream quantities depends on the policy in place. Proposition 3, and the optimal subsidy described by equation (19), illustrate those two points. Consequential and Attributional LCA coincide in the specific case of an upstream mandate and an inelastic demand as illustrated by equation (21).

4 Policy coordination with imperfect carbon pricing

This section investigates the second-best policies in the two sectors when carbon pricing is unavailable. We start by computing the second-best policies in the two sectors when taxes are bounded in both sectors (Proposition 5). Then, with Specification 1, we compare quantities and welfare between first-best and second-best policies. Finally, we investigate the case of ad-hoc uncoordinated policies and the associated welfare loss.

4.1 Optimal downstream and upstream subsidies

With Specification 1 we can compare quantities and welfare between first-best and second-best policies.

The following Proposition provides the formula of optimal subsidies that are jointly optimized.

Proposition 5 *For given taxes t_D and t_U, the optimal second-best subsidies s_D^{SB} and s_U^{SB} satisfy the following equations, if $q_{id} > 0$ in both sectors $i = U, D$,*

$$s_D^{SB} = \frac{1}{1+\Gamma_{Dd}}\big(\alpha_D\mu - t_D\big) - \theta\frac{1}{1+\Gamma_{Ud}}\big(\alpha_U\mu - t_U\big) \tag{25a}$$

$$s_U^{SB} = \frac{1}{1+\Gamma_{Ud}}\big(\alpha_U\mu - t_U\big) \tag{25b}$$

While the optimal downstream subsidy still encompasses elements from the upstream sector, it is not so for the upstream subsidy.[19] The optimal upstream subsidy is only determined by substitution between clean and dirty production in the upstream sector but not in the downstream sector. The ratio Γ_{Ud} only encompasses local sector substitution and not the adjustment of demand emanating from the upstream sector. It is so because the downstream subsidy optimally adjusts and absorbs change in the upstream price. There is an asymmetry between the two sectors because a subsidy on the clean downstream good rises the demand for the upstream good whether clean or dirty, whereas a subsidy on the clean upstream good has only an impact on the supply of the clean downstream technology and not the dirty one. Proposition 5 could easily be extended to several downstream sectors.

The above optimal couple of subsidies is obtained for an interior situation in which all technologies are used. In that case, the substitution between clean and dirty production plays a crucial role, the motivation for the subsidies precisely being to reduce dirty production. However, for sufficiently large SCC the two sectors are eventually decarbonized, in that case, the subsidies are used to ensure that dirty production is not profitable. Of particular interest is the case in which the upstream sector is decarbonized but the downstream is not. This case will arise in our numerical illustration. The following lemma characterizes the optimal couple of subsidies in that case.

Lemma 4 *The optimal couple of subsidies satisfies, if $q_{Ud} = 0$ and $q_{Dd} > 0$ at the second-best:*

$$s_D^{SB} = \frac{1}{1+\Gamma_{Dd}}\big(\alpha_D\mu - t_D\big) - \theta s_U^{SB} \tag{26}$$

$$\begin{aligned} s_U^{SB} &= C'_{Uc}(q_{Uc}) - [C'_{Ud}(0) + t_U] \\ &= C'_{Uc}(q_{Uc}) - P_U(q_{Uc} - \theta q_{Dc}) \end{aligned} \tag{27}$$

The Proof is in Appendix B.6. The downstream subsidy formula is familiar, it is the difference between avoided emissions in the downstream sector and the upstream subsidy. Even though the upstream sector is fully decarbonized the downstream subsidy needs to be reduced by the indirect subsidy in the upstream price. The upstream subsidy does not directly depends on the SCC, it is set to keep dirty upstream production unprofitable. The upstream price is equal to the marginal cost of the dirty technology at zero: $p_U = C'_{Ud}(0) + t_U$.

[19] It is not exactly true in full rigor since the characteristics of the downstream sector indirectly influence Γ_{Ud} in the general model, but not in a quadratic specification.

Therefore, as the SCC keeps increasing the clean downstream technology expands, and to face that additional demand the upstream subsidy also increases. Contrary to the situation in which both upstream technologies produce (Proposition 5), the optimal upstream subsidy depends on downstream characteristics.

4.2 Welfare loss from carbon mispricing

With the quadratic specification 1, one can get explicit expressions of equilibrium and optimal (first-best and second-best) quantities, and also of the welfare loss of the second-best policy compared to the first-best.

Corollary 4 *Given specification 1, and two taxes t_U and t_D, at the second-best policy, if all quantities are positive:*

- *The clean quantities in the upstream and downstream sectors are equal to their first-best values; the dirty quantities are larger than their first-best values.*
- *The welfare loss between the first-best and the second-best policy does not depend on sectoral linkage, it is:*

$$W^{FB} - W^{SB} = \frac{1}{2}\frac{1}{1+\Gamma_{Ud}}\frac{(\alpha_U\mu - t_U)^2}{b_U} + \frac{1}{2}\frac{1}{1+\Gamma_{Dd}}\frac{(\alpha_D\mu - t_D)^2}{b_D}. \tag{28}$$

- *The welfare of the upstream sector is always higher in the first-best than in the second-best. In contrast, the welfare of the downstream sector may be higher in the second-best than in the first-best:*

$$W_U^{FB} - W_U^{SB} = \frac{1}{2}\frac{1}{1+\Gamma_{Ud}}\frac{(\alpha_U\mu - t_U)^2}{b_U} + \theta s_U^{SB} q_{Dc}^{SB} \tag{29a}$$

$$W_D^{FB} - W_D^{SB} = \frac{1}{2}\frac{1}{1+\Gamma_{Dd}}\frac{(\alpha_D\mu - t_D)^2}{b_D} - \theta s_U^{SB} q_{Dc}^{SB} \tag{29b}$$

The proof is provided in Appendix B.7. It is not *a priori* straightforward to compare second-best and first-best clean quantities. Remarkably, they coincide with the quadratic specification. There are two opposite effects: a satiation effect and a substitution effect. With above optimal dirty quantities, clean production is less necessary to satisfy consumers (satiation effect) but is used to substitute for the dirty technology (substitution effect). In the quadratic specification, these two effects compensate exactly (cf Appendix B.7).

The welfare loss given by equation (28) is a quadratic function of the absolute mispricing of carbon emissions in each sector $\alpha_i\mu - t_i$. The slopes of the demand function and the dirty production function intervene in the welfare loss. The larger these slopes the lower the loss. These slopes can be interpreted as a measure of the elasticity of demand and dirty supply. At the extreme, with inelastic demands or inelastic dirty supply functions the second-best subsidies can mimic the first-best. The welfare loss is independent of θ, it is a peculiarity of the quadratic specification, linked to the first point of the Lemma.

The last point of the corollary compares welfare at the sectoral level between first-best and second-best policies. Differences (29) in sectoral welfare are the sum of a term from carbon mispricing and a transfer $\pm\theta s_U^{SB} q_{Dc}$ from the upstream to the downstream sector related to the clean upstream subsidy. The latter may be interpreted in two ways. First, it translates the implicit subsidy from the upstream sector to the clean downstream technology. Second, it is equal to the cost of mispriced emissions indirectly emitted by the clean downstream technology. Surprisingly, the downstream sector is better off with the second-best policy than with the first-best one if the clean quantity is sufficiently large:

$$q_{Dc} > \frac{1}{2\theta b_D} \frac{1+\Gamma_{Ud}}{1+\Gamma_{Dd}} \frac{(\alpha_D \mu - t_D)^2}{\alpha_U \mu - t_U}.$$

The numerical illustration will exhibit such a situation.

4.3 Welfare loss from incoordination

In what follows, we investigate the benefits of coordinating upstream and downstream subsidies. We will simulate numerically such policies in the next section. The causes of incoordination are multiple. Typically, policies could be developed by distinct regulators, each one with her own agenda. With regulators from distinct jurisdictions, each regulator might ignore pollution arising in the other jurisdiction.[20] Also, a regulator could show a limited understanding on the functioning of the other sector. At the extreme, one may consider regulators that do not consider linkage when designing their policies.

It is outside the scope of the present article to develop a fully-fletched model of strategically interacting regulators and we will content ourselves with the following ad-hoc set of uncoordinated policies:

$$s_D^{Inc} = \frac{1}{1+\Gamma_{Dd}} \big(\alpha_D \mu - t_D\big) \tag{30a}$$

$$s_U^{Inc} = \frac{1}{1+\Gamma_{Ud}} \big(\alpha_U \mu - t_U\big) \tag{30b}$$

The key point of uncoordinated policies is that the downstream regulator does not integrate the indirect emissions caused by the downstream clean technology in the upstream sector. Consequently, the downstream subsidy is too large and so is the quantity of the clean downstream technology. Indeed, the upstream regulator also ignores the influence of its policy on the downstream sector, but still sets the second-best subsidy.[21] The following lemma exhibits the welfare losses associated with uncoordinated policies.

Lemma 5 *Given specification 1, the welfare losses from lack of coordination are*

$$W^{SB} - W^{Inc} = \frac{1}{2} \frac{\theta^2}{\kappa} \frac{(\alpha_U \mu - t_U)^2}{(1+\Gamma_{Ud})^2} \tag{31}$$

[20]For instance, when subsidizing electric vehicles the Californian regulator might ignore the rise of emissions from electricity production in other US states

[21]Note that given the subsidy s_D^{Inc} the upstream regulator should not set the subsidy s_U^{SB} but a subsidy that would take into account downstream effect. The second-best subsidy s_U^{SB} does not have downstream components only when s_D is optimized and set at s_D^{SB}.

in which

$$\kappa = b_D \frac{\Gamma_{Dd} + \Gamma_{Dc} + \Gamma_{Dd}\Gamma_{Dc}}{1 + \Gamma_{Dd}} + b_U \frac{\theta^2}{1 + \Gamma_{Ud}^{-1} + \Gamma_{Uc}^{-1}}$$

The proof is in Appendix B.8. It also discusses welfare losses at the sectoral level. Welfare losses increase with the linkage intensity and with upstream carbon mispricing. The factor κ is the inverse of the sensitivity of the clean downstream quantity to its subsidy, which relates to both upstream and downstream market characteristics. It is interesting to note that for large sectoral linkage, the welfare losses from the lack of coordination can be larger than the ones due to the lack of carbon mispricing (given in Corollary 4). Indeed, reducing carbon mispricing would solve both issues.

5 Numerical illustration

5.1 Motivation, calibration, and set-up

This section completes the formal results with a calibrated numerical example, inspired by the electrification of passenger cars. While Proposition 5 and Corollary 4 provide explicit and concise formulas, we lack simple, and interpretable, formulas for quantities and sectoral welfare. The numerical illustration fills this gap. Furthermore, the calibration itself enables to discuss the real-world relevance of the conditions identified in Proposition 1 and Corollary 4.

It considers the French situation for sector sizes and technology costs but with a hypothetical electricity mix composed of gas-fired plants and renewables.[22] The relevant data are shown in Table 1. We detail the calibration of the demand and costs functions in Appendix C.[23] We consider gas-fired plants ($\alpha_U = 0.35$ tCO_2/MWh) and gasoline engines ($\alpha_D = 0.12$ tCO_2/km) as dirty technologies, and renewable power and electric vehicles as clean technologies. We assume that both technologies do not directly emit CO_2. The linkage intensity is given by the typical energy efficiency of the engine of electric vehicles ($\theta = 0.2kWh/km$).[24] We assume that both dirty technology costs are linear ($\Gamma_{id} = 0$).

Our numerical illustration compares the market equilibrium of the following four policy scenarios with a social cost of carbon of 150€/tCO_2:

- BAU: Business-As-Usual scenario, used as a reference, without any tax or subsidy;
- FB: First-Best scenario with Pigovian taxes and no subsidies;
- SB: Second-Best scenario with carbon pricing with taxes set a 20% of the Pigovian level and second-best subsidies given by equations (25) with $\Gamma_{id} = 0$ for $i = U, D$;

[22] The aim is to illustrate the theoretical results obtained through a stylized simulation, we abstract from many relevant issues, most notably: the regulated price of electricity, the partial interconnection of European power systems, the uncertain future of nuclear power, the variability of electricity demand and car charging, and the actual policies in both sectors among which the EU-Emission Trading Scheme.

[23] The model is solved using complementarity methods with disjunctive constraints as described in Gabriel et al. (2012). The python code is available upon request. Data sources are mainly from French Ministries and official agencies and from the academic literature.

[24] We disregard energy and emissions from the manufacturing of cars in this example.

- Inc: a scenario in which policies are uncoordinated as discussed in section 4.3. The regulator only considers direct emissions when setting subsidies, in which there is partial carbon pricing with taxes set at 20% of the Pigovian level and subsidies given by:

$$s_U^{Inc} = \alpha_U \mu - t_U \tag{32a}$$
$$s_D^{Inc} = \alpha_D \mu - t_D \tag{32b}$$

	Upstream (U)	Downstream (D)
a_i	395 €/MWh	1.215 €/km
b_i	0.46 10^{-6}€/MWh^2	1.63 10^{-9}€/km^2
c_{id}	177 €/MWh	0.5 €/km
α_i	0.35 tCO_2/MWh	0.12 $kgCO_2$/km
θ		0.2 kWh/km
c_{ic}	180 €/MWh	0.467 €/km
Γ_{ic}	0.75	0.035
Γ_{id}	0	0
μ	150 €/ tCO_2	

Table 1: Parameter values

There are three main takeaways from this numerical illustration. First, due to differences in $\alpha_i\mu/c_{id}$, the upstream sector achieves most of the overall abatement while it accounts for a moderate share of overall welfare. Second, sub-optimal carbon pricing reduces abatement compared to the first-best and induces an important welfare transfer from the upstream to the downstream sector. Third, the lack of coordination between sectoral policies increases the latter transfer and implies larger downstream abatement. Nevertheless, the associated welfare losses are limited compared to the second-best case. Fourth, we describe relevant changes of parameter values compatible with the situations described in Proposition 1. Total emissions would increase with electric mobility deployment if coal-fired plants are used, instead of gas-fired plants. And, total emissions would increase with the upstream tax if the transportation sector is larger (adding heavy mobility) with more emitting vehicles and smaller power demand (*e.g.* without electric heating) than in our calibration.

We detail these findings in the following subsections that deal with (1) equilibrium prices and quantities (2) surpluses and (3) abatement and (4) marginal effects of instruments on emissions.

5.2 Quantities and prices

Policy	BAU		FB		SB		Inc	
Sector	U	D	U	D	U	D	U	D
Q_i (TWh, 10^6 km)	473	436	361	425	451	434	451	434
q_{ic}	0	0	279	87	279	87	279	234
q_{id}	473	436	99	338	189	347	218	200
P_i (€/MWh, 10^{-3}€/km)	177	500	229	518	187	504	188	504
t_i	0	0	52	18	10	4	10	4
s_i	0	0	0	0	42	6	42	14

Table 2: Quantities and prices

Table 2 gives the quantities and prices in each sector for the four scenarios considered. Indeed, there is no clean production in BAU.

Carbon pricing and policy instruments in general have a much bigger impact in the upstream sector than in the downstream sector. For instance, first-best taxes increase the upstream price by 30% and the downstream price by only 4%. Consequently, consumption is reduced by 23% in the upstream sector and only by 2% in the downstream sector. In scenarios SB and Inc, consumption is almost unaffected because of the much lower tax levels. Clean good quantities are equal in FB and SB (Corollary 4), but larger in Inc since downstream subsidies are larger than in SB. The corresponding difference in the upstream consumption is entirely satisfied by the dirty upstream technology (because of its linear cost).

5.3 Surpluses and emissions

	FB (%)	SB (%)	SB/FB	Inc (%)	Inc/FB
$(W - W^{BAU})/W^{BAU}$	5.89	4.80	0.81	4.46	0.76
$(W_U - W_U^{BAU})/W_U^{BAU}$	36.74	27.43	0.75	23.13	0.63
$(W_D - W_D^{BAU})/W_D^{BAU}$	0.20	0.63	3.08	1.02	5.01
$\theta s_U q_{Dc}$	0.00	0.69	-	1.90	-
$(E - E^{BAU})/E^{BAU}$	-65.63	-50.64	0.77	-55.11	0.84
$(E_U - E_U^{BAU})/E_U^{BAU}$	-79.97	-60.74	0.76	-55.96	0.70
$(E_D - E_D^{BAU})/E_D^{BAU}$	-21.79	-19.77	0.91	-52.49	2.41
E_{CLCA} [a]	5.88	5.88	1.00	15.87	2.70

[a]: Consequential LCA emission from the clean downstream technology: $E_{CLCA} = \theta \alpha_U q_{Dc}$

Table 3: Total and sectoral welfare gains and emissions reduction relative to the BAU case (in %) for the scenario FB, SB, and Inc. SB and Inc scenarios are also compared with the FB scenario in columns SB/FB and Inc/FB. The table also indicates the welfare transfer from the upstream to the downstream sector $\theta s_U q_{Dc}$ (in M€) along with Consequential LCA emissions E_{CLCA} (in MtCO_2).

Table 3 shows surpluses and emissions at the global and sectoral levels, in the three scenarios. In addition, Table 7 shows the absolute values of total and sectoral surpluses (in M€) and emissions (in $MtCO_2$).

First, the upstream sector represents almost 75% of total emissions but only 15% of total welfare in the BAU scenario. For this reason, overall welfare gains from environmental policies are relatively small: 5.9% in FB, 4.8% in SB, and 4.5% in Inc. However, welfare gains in the upstream sector are large with increases of 36% in FB and 27% in SB. In comparison, welfare gains in the downstream sector are much smaller with increases of 0.2% in FB and 0.6% in SB.

Second, the welfare of the downstream sector is larger in SB than in FB and even larger in Inc than in SB. In the downstream sector, the losses from imperfect carbon pricing are outweighed by the transfers from the upstream sector ($\theta s_U q_{Dc}$) as formalized in Corollary 4. Indeed the clean downstream technology, and hence the downstream sector, benefits from the lack of upstream taxation. Given the relatively small contribution of consumption reduction in the downstream sector, this transfer explains nearly all the gain between SB and FB.

Third, the Inc scenario differs from the SB scenario as follows. Indeed, overall welfare is lower in Inc than in SB, but given that most welfare gains happen in the upstream sector, the difference is limited (below 1%). However, results on sectoral welfare show important differences, Inc induces a larger welfare transfer from the upstream sector to the downstream sector along with achieving a larger abatement.

5.4 Decomposition of abatement

Sector	Abatement source	FB	SB	Inc
Upstream	Consumption	27.6	7.1	6.7
	Clean	68.5	88.7	83.1
	Total	96.0	95.8	89.8
Downstream	Consumption	0.9	0.2	0.2
	Clean	7.3	9.5	23.9
	Consequential	-4.3	-5.5	-13.9
	Total	4.0	4.2	10.2
Total	Total	100.0	100.0	100.0

Table 4: Allocation of Abatement (in %). Total abatement in each scenario is decomposed as the sum of demand reduction and clean technology production: $E^0 - E = \alpha_U(Q_U^0 - Q_U) + \alpha_U q_{Uc} + \alpha_D(Q_D^0 - Q_D) + \alpha_D q_{Dc} - \alpha_U \theta q_{Dc}$, the last term being consequential life cycle emissions.

Table 4 decomposes the effort among sectors and the two channels: consumption reduction and clean technology deployment. In FB, 65% of emissions are abated (see Table 7). Most of the abatement is achieved in the electricity sector, which cuts 96% of its emissions, an effort that is nearly evenly allocated between a reduction of consumption and a deployment of clean electricity. In SB, only 50% of emissions are cut, since the reduction of demand is lower, the effort is slightly reallocated to mobility, the demand of which being the less elastic.

In Inc, clean technologies are further mobilized and the share of effort of the downstream sector more than doubles.

5.5 Effect of instruments on emissions

	Parameters				Effect on emissions
Calibration	α_D	α_U	b_U	b_D	$\frac{\partial E}{\partial s_D} < 0,\ \frac{\partial E}{\partial t_U} < 0$
Variations	Modification of parameters				
• Coal-fired electricity	–	$\times 2$	–	–	$\frac{\partial E}{\partial s_D} > 0,\ \frac{\partial E}{\partial t_U} < 0$
• High-emission cars • Higher car-mileage • Smaller power sector	$\times 2.5$	–	$\times 0.7$	$\times 2.5$	$\frac{\partial E}{\partial s_D} < 0,\ \frac{\partial E}{\partial t_U} > 0$

Table 5: Effects of instruments on emissions based on two alternative sets of parameters

Before closing this section, let us discuss the empirical relevance of the comparative static results of Proposition 1 on the influence of policy instruments on total emissions. The ambiguity relates to the effect of the subsidy on the clean downstream variety and the tax on the dirty upstream variety. With our calibration, emissions decrease with respect to both instruments but we identify realistic modifications of the parameters that reverse these effects. Table 5 summarizes these findings.

First, concerning the influence of the subsidy to electric cars, since $\Gamma_{id} = 0$ (dirty variety cost are linear) condition (11) amounts to a comparison of the emission rate of gasoline cars $\alpha_D = 0.12$ with indirect emissions of electric cars $\theta\alpha_U$. With gas-fired power plants, the latter $(0.2 \times 0.35 = 0.7)$ is lower than the former and emissions decrease with the deployment of electric cars. However, with coal-fired electricity, the typical emission rate of which approaches $1tCO_2/MWh$, the comparison is reversed, and subsidizing electric cars would increase total emissions. It is important to remind that in such a case electric cars should not be subsidized and deployed as long as coal-fired electricity is not completely phased out, and once it is the subsidy to electric cars would be set according to Lemma 4.

Second, concerning the upstream tax, for total emissions to increase with respect to t_U, condition (12) needs to be satisfied and with Γ_{id} being null it amounts to:

$$\alpha_U \left[\theta + \frac{b_D}{\theta b_U}(1 + \frac{1}{\Gamma_{Uc}})\Gamma_{Dc}\right] < \alpha_D.$$

With our calibration total emissions decrease with the tax on polluting power since the left-hand side equates 0.85 which is larger than $\alpha_D = 0.12$.

The condition above is more likely to hold if: (1) the transportation sector is large (low b_D) so a change in the electricity price has a large impact on the quantity of electric cars supplied (which is proportional to the market size); (2) the electric sector is small, or electricity demand is inelastic (high b_U), so the electricity price increases substantially with the tax ; (3) the emission rate of gasoline cars is large. In our numerical setting, a division by 2 of b_D a reduction by 30% of b_U, and a multiplication by 2.5 of α_D ensure that condition (12) is satisfied. Such adjustment of parameters would hold if: The transportation sector is expanded to include light-duty and heavy-duty vehicles. Electric heating is less frequent than in France, where it is much more common than in other comparable countries. And the emission rate of gasoline cars is equal to the US average, which is around 250gCO_2/km.

6 Conclusion

We analyzed the coordination of sectoral decarbonization policies in an economy with interconnected sectors. Such issues are particularly important in the debate on the carbon footprint of electric vehicles, for other electrification options (e.g. heating, cultured meat), and for hydrogen and biogas deployments. We focused on the influence of carbon mispricing on the design of downstream subsidies and the coordination between downstream and upstream policies. We related this analysis to LCA, and notably the concept of Consequential LCA.

The analysis of the second-best subsidy in the downstream sector stressed two main points. First, externalities and not emissions influence the optimal subsidy, these externalities consist of not only unpriced emissions but also subsidized (clean) quantities. This first point highlights the shortcomings of CLCA in designing subsidies. Second, it is the adjustment of these quantities that matters, notably the marginal upstream emission intensity and not the average one that influences the downstream subsidy or mandate. We generalized these results for alternative instruments and notably mandate in either the upstream or downstream sector. Subsidies on clean technologies in both sectors should be coordinated. In the second-best policy, the upstream subsidy does not incorporate features of the downstream sector whereas the optimal downstream subsidy should be reduced to account for the indirect subsidy due to upstream pollution mispricing.

There are several avenues for further research. First, the analysis could be adapted for bio-energies (biofuels and wood energy) the carbon footprint of which is controversial because of life cycle considerations.[25] And the design of support for bio-energies taking into account life cycle emissions is a topical policy question for the energetic transition. Second, even though our model could be used to describe a dynamic transition with an increasing SCC, it is fundamentally static and should be extended into a dynamic framework taking into account inertia and technical change. Third, our policy framework assumes that sectoral regulations are designed by a single entity. However, this might not be the case if the two sectors are in different jurisdictions or in a federal system in which the upstream sector is regulated at the federal level and multiple downstream sectors at the state level. In these cases, both pollution externalities and fiscal externalities will likely play a key role to explain

[25] Emissions arising from their consumption are partly compensated by carbon off-takes at the production stage but several economic analyses have stressed that direct and indirect land use changes (mostly deforestation) can severely reduce their net climate footprints (e.g. Searchinger et al., 2008; Keeney and Hertel, 2009).

inefficient decentralized regulations. Our framework could be easily used to assess those inefficiencies. Finally, our analysis of second-best policies could be improved by introducing explicit constraints on carbon pricing in the spirit of the work of Galinato and Yoder (2010) to better understand how such constraints transfer efforts and wealth among interconnected sectors and jurisdictions.

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A Quadratic specification

We provide here the expressions of quantities in the two situations considered in Assumption 1 with Specification 1.

With only dirty technologies:

$$Q_i^0 = \frac{1}{b_i(1+\Gamma_{id})}(a_i - c_{id}) \text{ and } p_i^0 = \frac{c_{id} + \Gamma_{id} a_i}{1+\Gamma_{id}}.$$

With only clean technologies, the two quantities Q_U^1 and Q_D^1 if positive satisfy the couple of equations

$$\begin{aligned}
a_U - b_U Q_U &= c_{Uc} + b_U \Gamma_{Uc}(Q_U + \theta Q_D) \\
a_D - b_D Q_D &= c_{Dc} + b_D \Gamma_{Dc} Q_D + \theta\big(c_{Uc} + b_U \Gamma_{Uc}(Q_U + \theta Q_D)\big);
\end{aligned}$$

the unique solution of which is:

$$\begin{aligned}
Q_U^1 &= \frac{1}{\Delta}\left[(b_D(1+\Gamma_{Dc}) + b_D\Gamma_{Dc} + \theta^2 b_U \Gamma_{Uc})(a_U - c_{Uc}) - \theta b_U \Gamma_{Uc}(a_D - c_{Dc} - \theta c_{Uc})\right] \\
Q_D^1 &= \frac{1}{\Delta}\left[b_U(1+\Gamma_{Uc})(a_D - c_{Dc} - \theta c_{Uc}) - \theta b_U \Gamma_{Uc}(a_U - c_{Uc})\right] \\
\text{with } \Delta &= b_U(1+\Gamma_{Uc})((b_D(1+\Gamma_{Dc}) + \theta^2 b_U \Gamma_{Uc}) - (\theta b_U \Gamma_{Uc})^2
\end{aligned}$$

Assumptions 1 are satisfied under the following conditions on parameters: First, there is no clean production initially if $p_U^0 < c_{Uc}$ and $p_D^0 < c_{Dc} + \theta p_U^0$. Second, a fully clean situation with positive production and consumption of both goods exists if and only if

$$\left[\theta + \frac{b_D(1+\Gamma_{Dc})}{\theta b_U \Gamma_{Uc}}\right](a_U - c_{Uc}) > a_D - c_{Dc} - \theta c_{Uc} > \theta \frac{\Gamma_{Uc}}{1+\Gamma_{Uc}}(a_U - c_{Uc}).$$

The first inequality ensures a non-negative consumption upstream and the second a non-negative consumption downstream. The clean upstream production should be sufficiently abundant, Γ_{Uc} small, to serve both markets (both extreme expressions are equal for $\Gamma_{Uc} = +\infty$). And the relative size of the upstream sector (b_D/b_U) should be sufficiently large to ensure that the downstream sector does not completely absorb the clean upstream production.

B Proofs

B.1 Proof of Proposition 1

We provide the proof for the case in which all technologies are used ($q_{ij}^* > 0$), other cases in which a technology is not used can be obtained as a specification of that case.

Equilibrium quantities are functions of the four instruments $q_{ij}^E(t_D, s_D, t_U, s_U)$ for $i, j \in \{U, D\} \times \{c, d\}$, the equilibrium upstream price is $p_U^E(t_D, s_D, t_U, s_U)$. These functions satisfy the four equations (10a), (10b), (10c).

It is useful to derive some general expressions of the change of quantities on each market with respect to an instrument $\tau \in \{t_D, s_D, t_U, s_U\}$. From the equilibrium on the downstream market, taking the full derivative of the couple of equations $P_D(q_{Dd}^E + q_{Dc}^E) = C'_{Dd}(q_{Dd}) + t_D = C'_{Dc}(q_{Dc}) - s_D + \theta p_U^E$ with respect to τ, using the definition of Γ_{Uj} (equation (5a)), and the Kronecker delta ($\delta_{ij} = 1$ if $i = j$ and 0 otherwise), gives

$$\begin{bmatrix} \frac{\partial q_{Dd}^E}{\partial \tau} \\ \frac{\partial q_{Dc}^E}{\partial \tau} \end{bmatrix} = \frac{1}{(-P'_D)(\Gamma_{Dd}\Gamma_{Dc} + \Gamma_{Dd} + \Gamma_{Dc})} \begin{bmatrix} 1+\Gamma_{Dc} & -1 \\ -1 & 1+\Gamma_{Dd} \end{bmatrix} \begin{bmatrix} -\delta_{t_D,\tau} \\ \delta_{s_D,\tau} - \theta \frac{\partial p_U^E}{\partial \tau} \end{bmatrix} \tag{33}$$

Similarly, on the upstream market, derivation of the couple of equations $P_U(q_{Ud}^E + q_{Uc}^E - \theta q_{Dc}) = C'_{Ud}(q_{Ud}) + t_U = C'_{Uc}(q_{Uc}) - s_U$ gives:

$$\begin{bmatrix} \frac{\partial q_{Ud}^E}{\partial \tau} \\ \frac{\partial q_{Uc}}{\partial \tau} \end{bmatrix} = \frac{1}{\Gamma_{Ud}\Gamma_{Uc} + \Gamma_{Ud} + \Gamma_{Uc}} \begin{bmatrix} 1+\Gamma_{Uc} & -1 \\ -1 & 1+\Gamma_{Ud} \end{bmatrix} \begin{bmatrix} -\frac{\delta_{t_U,\tau}}{-P'_U} + \theta \frac{\partial q_{Dc}^E}{\partial \tau} \\ \frac{\delta_{s_U,\tau}}{-P'_U} + \theta \frac{\partial q_{Dc}^E}{\partial \tau} \end{bmatrix} \tag{34}$$

Downstream instruments The consequences of downstream regulations on the upstream quantities are mediated through the quantity of the clean downstream, from equations (34):

$$\frac{\partial q_{Uj}^E}{\partial \tau} = \frac{\theta/\Gamma_{Uj}}{1 + \frac{1}{\Gamma_{Ud}} + \frac{1}{\Gamma_{Uc}}} \frac{\partial q_{Dc}^E}{\partial \tau} \tag{35}$$

and

$$\frac{\partial p_U^E}{\partial \tau} = \frac{-\theta P_U'}{1 + \frac{1}{\Gamma_{Ud}} + \frac{1}{\Gamma_{Uc}}} \frac{\partial q_{Dc}^E}{\partial \tau} \tag{36}$$

Let us now look at each downstream instrument in turn.

- Downstream tax t_D: the principle of the proof is to write the changes of other quantities as functions of the change in q_{Dd}^E.

  From the first order equation (10b), the impact of the downstream tax on the quantity of clean downstream is such that (from equation (33)):

$$\frac{\partial q_{Dc}^E}{\partial t_D} = \frac{-1}{1 + \Gamma_{Dc}} \left[\frac{\partial q_{Dd}^E}{\partial t_D} + \frac{\theta}{-P_D'} \frac{\partial p_U^E}{\partial t_D} \right];$$

  injecting into equation (36), the change in the upstream price as a function of the change in the dirty downstream is:

$$\frac{\partial p_U^E}{\partial t_D} = -\frac{\partial q_{Dd}^E}{\partial t_D} \left[\frac{-1}{\theta P_U'} \left(1 + \frac{1}{\Gamma_{Ud}} + \frac{1}{\Gamma_{Uc}} \right) (1 + \Gamma_{Dc}) + \frac{\theta}{-P_D'} \right]^{-1}.$$

  From equation (33), we get that $\partial q_{Dd}^E / \partial t_D$ is negative because $\partial p_U^E / \partial \tau$ is negatively related to it.

  So, a small increase in t_D leads to a change of emissions equal to

$$\begin{aligned} \frac{\partial q_{Dd}^E}{\partial t_D} \times & \left\{ -\alpha_D + \alpha_U \frac{1}{-P_U' \Gamma_{Ud}} \left[\frac{-1}{\theta P_U'} \left(1 + \frac{1}{\Gamma_{Ud}} + \frac{1}{\Gamma_{Uc}} \right) (1 + \Gamma_{Dc}) + \frac{\theta}{-P_D'} \right]^{-1} \right\} \\ &= -\alpha_D + \theta \alpha_U \left[\left(1 + \Gamma_{Ud} + \frac{\Gamma_{Ud}}{\Gamma_{Uc}} \right) (1 + \Gamma_{Dc}) + \Gamma_{Ud} \frac{\theta^2 P_U'}{P_D'} \right]^{-1} \\ &\leq -\alpha_D + \theta \alpha_U \leq 0 \end{aligned}$$

- Downstream subsidy:

  From equation (10a)

$$\frac{\partial q_{Dd}^E}{\partial s_D} = -\frac{1}{1 + \Gamma_{Dd}} \frac{\partial q_{Dc}^E}{\partial s_D} \tag{37}$$

  and, $\partial q_{Dc}^E / \partial s_D$ is positive from equations (33) and (36). Then, from equation (35) the effect of s_D on total emissions is:

$$\frac{\partial q_{Dc}^E}{\partial s_D} \left\{ -\frac{\alpha_D}{1 + \Gamma_{Dd}} + \alpha_U \frac{\theta / \Gamma_{Uj}}{1 + \frac{1}{\Gamma_{Ud}} + \frac{1}{\Gamma_{Uc}}} \right\},$$

  condition (11) follows.

Upstream instruments Upstream instruments will influence downstream quantities through the upstream price as described by equations (33).

- Upstream tax: we write all changes as a function of the dirty upstream quantity change.

  Let us denote ϕ the slope of the clean downstream with respect to the upstream price:

$$\phi = -\frac{\partial q_{Dc}^E/\partial t_D}{\partial p_U^E/\partial t_D} = \frac{\theta(1+\Gamma_{Dd})}{(-P_D')(\Gamma_{Dd}\Gamma_{Dc}+\Gamma_{Dd}+\Gamma_{Dc})}.$$

  The change in the upstream price solves:

$$\begin{aligned}\frac{\partial p_U^E}{\partial t_D} &= P_U' \times \left(\frac{\partial q_{Ud}^E}{\partial t_U} + \frac{\partial q_{Uc}^E}{\partial t_U} - \theta\frac{\partial q_{Dc}^E}{\partial t_U}\right) = P_U'\frac{\partial q_{Ud}^E}{\partial t_U} - \frac{1}{\Gamma_{Uc}}\frac{\partial p_U^E}{\partial t_U} + \theta P_U'\phi\frac{\partial p_U^E}{\partial t_U} \\ &= P_U'\frac{\partial q_{Ud}^E}{\partial t_U} \times \left[1 + \frac{1}{\Gamma_{Uc}} - \theta P_U'\phi\right]^{-1}\end{aligned}$$

  then, indeed $\partial q_{Ud}^E/\partial t_U \leq 0$ and, using $\partial q_{Dd}^E/\partial t_U = \phi(1+\Gamma_{Dd})(\partial p_U^E/\partial t_U)$, the change of emissions is

$$\frac{\partial q_{Ud}^E}{\partial t_D}\left\{\alpha_U + \alpha_D\frac{\phi}{1+\Gamma_{Dd}}\frac{P_U'}{1+\Gamma_{Uc}-\theta P_U'\phi}\right\} = \frac{\partial q_{Ud}^E}{\partial t_D}\left\{\alpha_U - \frac{\alpha_D}{1+\Gamma_{Dd}}\frac{1}{(1+\frac{1}{\Gamma_{Uc}})/(-\phi P_U')+\theta}\right\}$$

  and injecting the expression of ϕ gives condition (12).

- Upstream subsidy:

  The upstream price derivative is:

$$\begin{aligned}\frac{\partial p_U^E}{\partial S_U} &= P_U' \times \left(\frac{\partial q_{Ud}^E}{\partial s_U} + \frac{\partial q_{Uc}^E}{\partial s_U} - \theta\frac{\partial q_{Dc}^E}{\partial s_U}\right) = P_U'\frac{\partial q_{Uc}^E}{\partial s_U} - \frac{1}{\Gamma_{Ud}}\frac{\partial p_U^E}{\partial s_U} + \theta P_U'\phi\frac{\partial p_U^E}{\partial s_U} \\ &= P_U'\frac{\partial q_{Uc}^E}{\partial s_U} \times \left[1 + \frac{1}{\Gamma_{Ud}} - \theta P_U'\phi\right]^{-1}\end{aligned}$$

  then, $\partial q_{Uc}/\partial s_U \geq 0$ and $\partial p_U^E/\partial s_U \leq 0$, so the change in emissions:

$$\alpha_D\phi(1+\Gamma_{Dd})\frac{\partial p_U^E}{\partial S_U} + \alpha_U\frac{1}{C_{Dd}''}\frac{\partial p_U^E}{\partial S_U} \leq 0$$

B.2 Proof of Corollary 1

The optimal allocation corresponds to the market equilibrium with $t_i = \alpha_i\mu$ and $s_i = 0$, in sector $i = U, D$. The corollary can then be proved by using the above calculations with:

$$\frac{dq_{ij}^*}{d\mu} = \alpha_D\frac{\partial q_{ij}^E}{dt_D} + \alpha_U\frac{\partial q_{ij}^E}{dt_U}.$$

A more elegant way to proceed is to isolate one good $(i,j) \in \{U,D\} \times \{d,c\}$, and define the economic benefit as a function of E and q_{ij}:

$$B(E, q_{ij}) = \max_{q_{kl},(k,l)\neq(i,j)} \{W(\mathbf{q}) + \mu E | \alpha_D q_{Dd} + \alpha_U q_{Ud} \leq E\}.$$

Indeed $E = \alpha_D q_{Dd} + \alpha_U q_{Ud}$ must be larger than $\alpha_i q_{ij}$ if $j = d$. The optimal allocation E^* and q^*_{ij} then solves

$$\frac{\partial B}{\partial E} = \mu \text{ and } \frac{\partial B}{\partial q_{ij}} = 0$$

and an increase in the SCC μ leads to an increase in q^*_{ij} if and only if

$$\frac{\partial^2 B}{\partial E \partial q_{ij}} \leq 0,$$

which means that emissions and the good ij are substitutes.

The optimal allocation is decentralized with $t_U = \alpha_U \mu$, $t_D = \alpha_D \mu$, and $s_D = s_U = 0$. With an additional tax t on good ij (on top of $\alpha_i t_i$ if $j = d$) equilibrium emissions E^E, which is equal to $\sum_i \alpha_i q^E_{id}$, and quantity q^E_{ij} solve

$$\frac{\partial B}{\partial E} = \mu \text{ and } \frac{\partial B}{\partial q_{ij}} = t;$$

an increase in t (keeping μ constant and thus the three other instruments) increases emissions if and only if $\partial^2 B/\partial E \partial q_{ij} \leq 0$. Therefore, we can state that

Result The quantity q^*_{ij} decreases with respect to the SCC, if and only if the emissions decrease with respect to τ_i in the Pigovian regulation $(t_D, t_U, s_D, s_U) = (\alpha_D \mu, \alpha_U \mu, 0, 0)$, with $\tau_i = t_i$ if $j = d$ and $t_i = -s_j$ if $j = c$.

The corollary then follows from Proposition 1.

B.3 Proof of propositions 2 and 5

Proposition 2 The optimal downstream subsidy solves equation (15), with $\tau = s_D$. The derivatives of each quantity with respect to s_D are given by equation (37) for the downstream dirty quantity and by equation (35) for upstream quantities. The formula (16) follows.

Proposition 5 The couple of optimal subsidies solves two equations (15), with $\tau = s_D$ and $\tau = s_U$. From the market equilibrium conditions (10a), dirty quantities change are given by, for $i = U, D$:

$$\frac{\partial q^E_{Dd}}{\partial s_i} = -\frac{1}{1+\Gamma_{Dd}} \frac{\partial q^E_{Dc}}{\partial s_i} \text{ and } \frac{\partial q^E_{Ud}}{\partial s_i} = \frac{1}{1+\Gamma_{Ud}} \left[\frac{\partial q^E_{Uc}}{\partial s_i} - \theta \frac{\partial q^E_{Dc}}{\partial s_i}\right].$$

Injecting these two equations into equation (15), gives:

$$\left[s_U - \frac{\alpha_U \mu - t_U}{1+\Gamma_{Ud}}\right] \frac{\partial q^E_{Uc}}{\partial s_i} + \left[s_D - \frac{\alpha_D \mu - t_D}{1+\Gamma_{Dd}} + \theta \frac{\alpha_U \mu - t_U}{1+\Gamma_{Ud}}\right] \frac{\partial q^E_{Dc}}{\partial s_i} = 0$$

for $i = U, D$, the two expressions (25) solve these first order conditions.

B.4 Proof of Corollary 2

With a mandate r_U, denoting q the total upstream production, $q^r_{Uc}(p_U) = r_U q(p_U)$, $q^r_{Ud}(p_U) = (1 - r_U)q(p_U)$ and $q(p_U)$ solves

$$p_U = r_U C'_{Uc}(r_U q) + (1 - r_U)C'_{Ud}((1 - r_U)q).$$

Therefore, $q^{r\prime}_{Uc}(p_U) = r_U q'(p_U)$, $q^{r\prime}_{Ud}(p_U) = (1 - r_U)q'(p_U)$, and the Γ^r_{Uj} are then:

$$\frac{1}{\Gamma^r_{Uc}} = r_u(-P'_U)q', \text{ and } \frac{1}{\Gamma^r_{Ud}} = (1 - r_u)(-P'_U)q'.$$

Injecting the above expressions into formula (19):

$$\begin{aligned} s_D &= \frac{\alpha_D\mu - t_D}{1 + \Gamma_{Dd}} - \frac{\theta(-P'_U)q'}{1 - q'P'_U}\Big[\left(\alpha_U\mu - r_u\left(C'_{Uc} - C'_{Ud}\right)\right)(1 - r_u) + (1 - r_U)\big(C'_{Uc} - C'_{Ud}\big)r_U\Big] \\ &= \frac{\alpha_D\mu - t_D}{1 + \Gamma_{Dd}} - \frac{\theta\alpha_U\mu(1 - r_u)}{1 + \frac{1}{(-P'_U)q'}} \end{aligned}$$

And replacing $q' = [r_U^2 C'_{Uc} + (1 - r_U)^2 C'_{Ud}]^{-1}$ (from the upstream suppliers first-order conditions) gives formula (20).

B.5 Proof of Corollary 3

With a downstream mandate r_D, $q_{Dc} = r_D Q_D$ and $q_{Dd} = (1 - r_D)Q_D$ and the downstream market equilibrium is described by:

$$p_D = r_D[C'_{Dc} + \theta P_U] + (1 - r_D)C'_{Ud}.$$

Let us denote δ the difference between the clean marginal cost and the dirty marginal cost:

$$\delta = C'_{Dc} + \theta P_U - C_{Dd}$$

Formula (22) writes

$$[\alpha_D\mu - r_D\delta]\frac{\partial q_{Dd}}{\partial r_D} - [(1 - r_D)\delta + \theta\sigma]\frac{\partial q_{Dc}}{\partial r_D} = 0$$

Then

$$[\alpha_D\mu - r_D\delta]\left[Q_D - (1 - r_D)\frac{\partial Q_D}{\partial r_D}\right] - [(1 - r_D)\delta + \theta\sigma]\left[Q_D + r_D\frac{\partial Q_D}{\partial r_D}\right] = 0$$

regrouping terms

$$[\alpha_D\mu - \delta - \theta\sigma]\,Q_D - [(1 - r_D)\alpha_D\mu + r_D\theta\sigma]\frac{\partial Q_D}{\partial r_D} = 0$$

and equation (23) follows.

B.6 Proof of Lemma 4

In Proposition 5, dirty quantities were supposed to be positive. However, for large μ they are null, and subsidies are used to keep them null. Formally, the welfare function is not continuously differentiable everywhere with respect to instruments because of corners situations in which one of the q_{ij} is null. Regarding q_{Ud}, it is null if $p_U \leq t_U + C'_{Ud}(0)$, that is, if q^E_{Uc} and q^E_{Dc} are such that

$$P_U(q^E_{Uc} - \theta q^E_{Dc}) \leq t_U + C'_{Ud}(0).$$

For s_U, s_D such that $p_U < t_U + C'_{Ud}(0)$ the derivative of welfare with respect to s_i is

$$-s_U \frac{\partial q^E_{Uc}}{\partial s_i} - \left[s_D - \frac{\alpha_D \mu - t_D}{1 + \Gamma_{Dd}} \right] \frac{\partial q^E_{Dc}}{\partial s_i}$$

and the couple (s_U, s_D) which cancels this equation implies $p_U > t_U + C'_{Ud}(0)$, so that if $q^E_{Ud} = 0$ at the second-best, then welfare is maximized along the boundary $P_U(q^E_{Uc} - \theta q^E_{Dc}) = t_U + C'_{Ud}(0)$. So s_U, s_D maximize

$$W + \lambda[q^E_{Uc} - \theta q^E_{Dc} - P_U^{-1}(t_U + C'_{Ud}(0))]$$

for some $\lambda > 0$. The optimality conditions are then

$$(\lambda - s_U) \frac{\partial q^E_{Uc}}{\partial s_i} - \left[s_D - \frac{\alpha_D \mu - t_D}{1 + \Gamma_{Dd}} - \theta\lambda \right] \frac{\partial q^E_{Dc}}{\partial s_i} = 0 \text{ for } i = U, D$$
$$q^E_{Uc} - \theta q^E_{Dc} = P_U^{-1}(t_U + C'_{Ud}(0)).$$

Therefore, $\lambda = s_U$ and s_D is given by equation (26). The two subsidies are such that $p^E_U = t_U + C'_{Ud}(0)$ and since, at the market equilibrium, $s_U = C'_{Uc}(q^E_{Uc}) - p^E_U$ equation (27) follows.

B.7 Proof of Corollary 4

The exposition of the proof is lighter by rewriting welfare as a function of the difference between actual and optimal quantities. Let us define:

$$z_1 = q_{Dc} - q^{FB}_{Dc},\ z_2 = q_{Uc} - q^{FB}_{Uc},\ z_3 = q_{Dd} - q^{FB}_{Dd},\ z_4 = q_{Ud} - q^{FB}_{Ud},$$

Rewrite welfare as:

$$W(z_1, z_2, z_3, z_4) = W^{FB} - \sum_i \frac{\gamma_i}{2} z_i^2 - \sum_{j \neq i} \frac{\gamma_{ij}}{2} z_i z_j$$

with $\gamma_{ij} = \gamma_{ji}$ (it is straightforward to write the γs as functions of the parameters of Specification 1 but not necessary for the proof, the result holds more generally).

Quantities The optimal first-best quantities are $z_i^{FB} = 0$. Our second-best case corresponds to a situation in which there are two subsidies on quantities 3 and 4 to be denoted ρ_3 and ρ_4 ($\rho_3 = \alpha_D\mu - t_D$ and $\rho_4 = \alpha_U\mu - t_U$), and welfare is optimized with respect to z_1 and z_2.[26] We show that the optimal z_1 and z_2 are null.

First, the two quantities z_3 and z_4 depends on z_1 and z_2, they solve:

$$\frac{\partial W}{\partial z_i}(z_1, z_2, z_3, z_4) = -\rho_i \text{ for } i = 3, 4$$

which gives:

$$\begin{bmatrix} \gamma_3 & \gamma_{34} \\ \gamma_{34} & \gamma_4 \end{bmatrix} \begin{bmatrix} z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} \rho_3 - \gamma_{13}z_1 - \gamma_{23}z_2 \\ \rho_4 - \gamma_{14}z_1 - \gamma_{24}z_2 \end{bmatrix}$$

inverting the matrix, denoting $\delta = \gamma_3\gamma_4 - \gamma_{34}^2$:

$$\begin{bmatrix} z_3 \\ z_4 \end{bmatrix} = \frac{1}{\delta} \begin{bmatrix} \gamma_4 (\rho_3 - \gamma_{13}z_1 - \gamma_{23}z_2) - \gamma_{34} (\rho_4 - \gamma_{14}z_1 - \gamma_{24}z_2) \\ \gamma_3 (\rho_4 - \gamma_{14}z_1 - \gamma_{24}z_2) - \gamma_{34} (\rho_3 - \gamma_{13}z_1 - \gamma_{23}z_2) \end{bmatrix}.$$

Second, the second-best z_1 solves:

$$0 = \frac{\partial W}{\partial z_1} + \frac{\partial W}{\partial z_3}\frac{\partial z_3}{\partial z_1} + \frac{\partial W}{\partial z_4}\frac{\partial z_4}{\partial z_1} = -\left[\gamma_1 z_1 + \gamma_{12}z_2 + \gamma_{13}z_3 + \gamma_{14}z_4\right] - \sum_{i=3,4} \rho_i \frac{\partial z_i}{\partial z_1}.$$

The last term is the substitution effect mentioned in the main text and is equal to:

$$-\rho_3 \frac{(-\gamma_4\gamma_{13} + \gamma_{34}\gamma_{14})}{\delta} - \rho_4 \frac{(-\gamma_3\gamma_{14} + \gamma_{34}\gamma_{13})}{\delta}.$$

The first bracketed term is the satiation effect mentioned in the main text, it depends on z_3 and z_4 which are linear in ρ_3 and ρ_4 with:

$$\frac{\partial}{\partial \rho_3}(\gamma_{13}z_3 + \gamma_{14}z_4) = \frac{1}{\delta}\left[\gamma_{13}\gamma_4 - \gamma_{14}\gamma_{34}\right]$$

Therefore, the terms in ρ_3 and ρ_4 cancel out, and the first-order condition above simplifies to a linear equation with a null fixed term:

$$\begin{aligned} 0 = -\gamma_1 z_1 - \gamma_{12}z_2 &- \frac{\gamma_{13}}{\delta}\left[(\gamma_{34}\gamma_{14} - \gamma_4\gamma_{13})z_1 + (\gamma_{34}\gamma_{24} - \gamma_4\gamma_{23})z_2\right] \\ &- \frac{\gamma_{14}}{\delta}\left[(\gamma_{34}\gamma_{13} - \gamma_3\gamma_{14})z_1 + (\gamma_{34}\gamma_{23} - \gamma_4\gamma_{24})z_2\right]. \end{aligned}$$

Following the same steps for z_2 gives another linear equation with a null fixed term.

The optimal second-best solution is then $z_1 = z_2 = 0 = z_1^{FB} = z_2^{FB}$.

[26] Any subsidy couple (ρ_1, ρ_2) is associated with a unique quantity couple (x_1, x_2), and vice-versa; the two other quantities z_3 and z_4 can indifferently be written as a function of the former or the latter.

Welfare comparison At the second-best welfare is:

$$W = W^{FB} - \frac{\gamma_3}{2} z_3^2 - \frac{\gamma_4}{2} z_4^2 - \gamma_{34} z_3 z_4.$$

And, from Specification 1, the coefficients are $\gamma_{34} = \partial^2 W / \partial q_{Ud} q_{Dd} = 0$, $\gamma_3 = b_D(1 + \Gamma_{Dd})$ and $\gamma_4 = b_U(1 + \Gamma_{Ud})$. Therefore, $z_3 = \gamma_4 \rho_3 / \delta = \rho_3/\gamma_3$, and $z_4 = \rho_4/\gamma_4$. Welfare is:

$$W = W^{FB} - \frac{\rho_3^2}{2\gamma_3} - \frac{\rho_4^2}{2\gamma_4} = W^{FB} - \frac{(\alpha_D \mu - t_D)^2}{2b_D(1 + \Gamma_{Dd})} - \frac{(\alpha_U \mu - t_U)^2}{2b_U(1 + \Gamma_{Ud})}.$$

Sectoral welfares We use our specification 1. Since $q_{ic}^{FB} = q_{ic}^{SB}$ we have $P_U^{FB} - P_U^{SB} = s_U^{SB}$ (from market equilibrium condition (10c)) and $q_{id}^{FB} - q_{id}^{SB} = Q_i^{FB} - Q_i^{SB}$. The difference of gross consumer surplus can be written as:

$$S_i^{FB} - S_i^{SB} = (Q_i^{FB} - Q_i^{SB})[a_i - \frac{b_i}{2}(Q_i^{FB} + Q_i^{SB})] = \frac{1}{2}(Q_i^{FB} - Q_i^{SB})(p_i^{FB} - p_i^{SB})$$

and the difference of dirty production costs (with a similar manipulation):

$$C_{id}^{FB} - C_{id}^{SB} = \frac{1}{2}(q_{id}^{FB} - q_{id}^{SB}) \left[C'_{id}(q_{id}^{FB}) + C'_{id}(q_{id}^{SB})\right] = \frac{1}{2}(Q_i^{FB} - Q_i^{SB}) \left[(p_i^{FB} - \alpha_i \mu) - (p_i^{SB} - t_i)\right]$$

The difference in welfare between FB and SB is then (clean production costs cancel out):

$$W_U^{FB} - W_U^{SB} = S_U^{FB} - S_U^{SB} - \left[C_{Ud}^{FB} - C_{Ud}^{SB}\right] + \theta(p_U^{FB} - p_U^{SB}) q_{Dc} \tag{38}$$
$$= \frac{1}{2}\big(Q_U^{SB} - Q_U^{FB}\big)\big(\alpha_U \mu - t_U\big) + \theta s_U^{SB} q_{Dc} \tag{39}$$

and

$$W_D^{FB} - W_D^{SB} = -\theta s_U^{SB} q_{Dc} + \frac{1}{2}\big(Q_D^{SB} - Q_D^{SB}\big)\big(\alpha_D \mu - t_D\big).$$

And $b_i(Q_i^{SB} - Q_i^{FB}) = p_i^{FB} - p_i^{SB} = b_i \Gamma_{id}(q_{id}^{FB} - q_{id}^{SB}) + \alpha_i \mu - t_i$ Hence $Q_i^{SB} - Q_i^{FB} = \frac{1}{b_i(1+\Gamma_{id})}\big(\alpha_i \mu - t_i\big)$, expression (29) follows.

B.8 Proof of Lemma 5

Welfare could be written as a function of the subsidies. With the quadratic specification, quantities are linear functions of the subsidies, and welfare is then a quadratic function of these.

It is then remarkably simple to compare welfare with our uncoordinated subsidies to the second-best situation. We have $s_U^{Inc} = s_U^{SB}$ so that welfare is:

$$W\big(s_U^{Inc}, s_D^{Inc}\big) = W^{SB} + \frac{1}{2}(s_D^{Inc} - s_D^{SB})^2 \frac{\partial^2 W}{\partial s_D^2} \tag{40}$$

And the derivative of welfare is:

$$\frac{\partial W}{\partial s_D} = s_U \frac{\partial q_{Uc}^E}{\partial s_D} + (\alpha_U \mu - t_U)\frac{\partial q_{Ud}^E}{\partial s_D} + s_D \frac{\partial q_{Dc}^E}{\partial s_D} + (\alpha_D \mu - t_D)\frac{\partial q_{Dd}^E}{\partial s_D}$$

so that $\partial^2 W/\partial s_D^2 = \partial q_{Dc}^E/\partial s_D$. Then, from equations (33) and (36):

$$\frac{\partial q_{Dc}^E}{\partial s_D} = \frac{1}{b_D}\frac{1+\Gamma_{Dd}}{\Gamma_{Dd}+\Gamma_{Dc}+\Gamma_{Dd}\Gamma_{Dc}}(1-\theta\frac{\partial p_U}{\partial s_D}), \text{ and } \frac{\partial p_U}{\partial s_D} = \frac{\theta b_U}{1+\frac{1}{\Gamma_{Ud}}+\frac{1}{\Gamma_{Uc}}}\frac{\partial q_{Dc}^E}{\partial s_D}.$$

Equation (31) follows.

It is also possible to consider sectoral welfare differences using the same approach:

$$W_i\big(s_U^{Inc}, s_D^{Inc}\big) = W_i^{SB} + (s_D^{Inc} - s_D^{SB})\frac{\partial W_i}{\partial s_D} + \frac{1}{2}(s_D^{Inc} - s_D^{SB})^2 \frac{\partial^2 W_i}{\partial s_D^2} \tag{41}$$

This leads to:

$$W_D^{Inc} - W_D^{SB} = \frac{1}{2}(\theta s_U^{SB})^2 \frac{\partial q_{Dc}}{\partial s_D} - \theta s_U^{SB}\frac{\partial p_U}{\partial s_D}\big(q_{Dc}^{SB} + \frac{\theta s_U^{SB}}{2}\frac{\partial q_{Dc}}{\partial s_D}\big) \tag{42a}$$

$$W_U^{Inc} - W_U^{SB} = -(\theta s_U^{SB})^2 \frac{\partial q_{Dc}}{\partial s_D} + \theta s_U^{SB}\frac{\partial p_U}{\partial s_D}\big(q_{Dc}^{SB} + \frac{\theta s_U^{SB}}{2}\frac{\partial q_{Dc}}{\partial s_D}\big) \tag{42b}$$

We do not further develop formulas but note that $\frac{\partial p_U}{\partial s_D} > 0$ (see above).

These formulas show that uncoordinated policies have two effects on sectoral welfare. For the downstream (resp. upstream) sector, a gain (resp. loss) from the subsidy paid by the upstream sector and a loss (resp. a gain) from the increase in the upstream price induced by the additional downstream clean quantity. Note that the former effect is null as soon as the dirty upstream cost is linear (as in the simulation). Hence, incoordination may either increase or decrease sectoral welfare depending on the relative magnitude of both effects. However, the case where incoordination reduces downstream welfare may seem contradictory. As defined, uncoordinated policies do not maximize sectoral welfare. Subsidies that unilaterally maximize sectoral welfare would constitute a Nash equilibrium of a game to be specified, and it is left to future work.

C Calibration

	Upstream: power sector		Downstream: passenger road transport	
	Value	Source	Value	Source
Q_i^0	473 TWh	RTE[a]	436 10^6 km	French Ministry of Ecology[b]
ϵ_i	0.8	INSEE[c]	0.7	Graham and Glaister (2004)
c_{id}	177 €/MWh	RTE[a]	0.5 €/km	French Ministry of the Economy[d]
α_i	0.350 tCO_2/MWh	RTE[a]	0.120 kCO_2/km	ADEME[e]
θ			0.2 kWh/km	ADEME[e]

a: https://www.rte-france.com/en/eco2mix

b: https://www.statistiques.developpement-durable.gouv.fr/chiffres-cles-du-transport-edition-2019

c: https://www.insee.fr/fr/statistiques/4467133?sommaire=4467460

d: https://www.economie.gouv.fr/particuliers/bareme-kilometrique

e: https://carlabelling.ademe.fr/

Table 6: Parameter values and sources

Our numerical illustration is based on the electrification of passenger cars in France. The upstream sector is then the power sector and the downstream sector is the transport sector, restricted to passenger cars. We use several data sources from different official agencies and from the academic literature. In each sector, demand and costs are calibrated using values from 2019. The parameter values and sources are given in Table 6. To calibrate the demand function, we proceed as follows: from a BAU price equal to the dirty marginal cost c_{id}, a BAU quantity Q_i^0 and a price elasticity of demand ϵ_i in sector $i = U, D$, we infer a_i and b_i from

$$
\begin{aligned}
a_i &= c_{id}\big(1 + \frac{1}{\epsilon_i}\big) \\
b_i &= \frac{c_{id}}{Q_i^0 \epsilon_i}
\end{aligned}
$$

We did not find any available data source to easily calibrate c_{ic} et Γ_{ic}. Therefore, we choose two SCCs μ_i^0 in which clean technology starts to be competitive.

$$
\begin{aligned}
c_{Uc} &= c_{Ud} + \alpha_U \mu_U^0 \\
c_{Dc} &= c_{Dd} - \theta c_{Ud} + (\alpha_U - \theta \alpha_D)\mu_D^0
\end{aligned}
$$

We choose $\mu_U^0 = 10$€ / tCO_2 and $\mu_D^0 = 50$€ / tCO_2.

Then, we choose two μ_i^1 together with a share z_i of the clean technology in sector i such that $q_{ic}(\mu_i^1) = z_i Q_i^0$ (μ_i^1 is not too large to ensure that the q_{jd} are positive). We compute Γ_{ic}

	BAU	FB	SB	Inc
W	175.40	185.50	183.50	182.90
CS	208.00	178.40	201.60	201.60
$W + \mu E$	208.00	196.80	199.70	198.00
E	217.90	75.30	107.80	100.50
W_U	27.50	37.30	34.70	33.40
CS_U	52.30	30.40	47.50	47.50
$W_U + \mu E_U$	52.30	42.50	44.60	44.90
E_U	165.60	34.80	66.20	76.50
W_D	147.90	148.20	148.90	149.50
CS_D	155.70	148.00	154.10	154.10
$W_D + \mu E_D$	155.70	154.30	155.10	153.10
E_D	52.30	40.50	41.60	24.00
E_{Dc}	0.00	1.60	2.50	7.20
E_{CLCA}	0.00	6.10	6.10	16.40
$\theta s_U q_{Dc}$	0.00	0.00	0.70	2.00

Table 7: Total and sectoral surpluses (in M€) and emissions (in $MtCO_2$) in absolute values, for the four scenarios.

from the formula:

$$
\begin{aligned}
c_{Uc} &= c_{Ud} + \alpha_U \mu_U^0 \\
c_{Dc} &= c_{Dd} - \theta c_{Ud} + (\alpha_U - \theta \alpha_D)\mu_D^0 \\
\Gamma_{Uc} &= \frac{1}{b_U} \frac{\alpha_U(\mu_U^1 - \mu_U^0)}{z_U^1 Q_U^0} = \frac{\epsilon_U \alpha_U (\mu_U^1 - \mu_U^0)}{c_{Ud} z_U^1} \\
\Gamma_{Dc} &= \frac{1}{b_D} \frac{(\alpha_D - \theta\alpha_U)(\mu_D^1 - \mu_D^0)}{z_D^1 Q_D^0} = \frac{\epsilon_D (\alpha_D - \theta\alpha_U)(\mu_D^1 - \mu_D^0)}{c_{Dc} z_D^1}
\end{aligned}
$$

We choose $\mu_D^1 = 300$ tCO_2 and $z_D^1 = 0.5$ for the downstream sector and $\mu_U^1 = 200 tCO_2$ and $z_U = 0.8$ for the upstream sector.

D Simulations results