Subsidies and rebound effect with incomplete carbon pricing: an application to biogas[1]

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Preliminary draft

Abstract

Bioenergies from dedicated crops or wood have faced substantial criticisms due to their significant land requirements. Certain bioenergy pathways, such as biogas generated from crop residues, manure, or food waste, appear to be exempt from this criticism. However, these feedstocks are byproducts of agricultural activities that generate emissions not covered by current climate policies in most countries. We analyze the optimal subsidy to biogas production in a second-best setting where emissions from food production and fossil gas are under-taxed. We show analytically how the indirect effect of the biogas subsidy on food production should be taken into account, as well as the welfare implications. We provide numerical simulations calibrated on the French dairy market and methanization of livestock manure. We compare a second-best situation in which dairy emissions are not taxed with a first-best situation in which they are taxed. This illustration indicates that for a small Social Cost of Carbon, the rebound effect on milk production is moderate, and the optimal subsidy departs from the Pigouvian one accordingly. The second-best quantity of biogas is slightly larger than the first-best one. For a large Social Cost of Carbon, the rebound effect is important, and the gap between the first-best and second-best widens considerably.

Keywords

Biogas, Life cycle emissions, Climate policy, Second-best policy, Rebound effect

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[1] Guy Meunier acknowledges the support of the chair Energy and Prosperity, under the aegis of the Risk Foundation. This work has been carried out at the Energy4Climate Interdisciplinary Center (E4C) of IP Paris and Ecole des Ponts ParisTech, which is in part supported by the 3rd Programme d'Investissements d'Avenir [ANR-18-EUR-0006-02] and by the Foundation of Ecole Polytechnique (ENGIE Foundation's Funding "Resilience of Energy Systems to Climate Change and Negative Emissions").

1 Introduction

Bioenergy will likely play an essential role in a carbon-neutral economy, alongside a decarbonized electricity system (Creutzig et al. 2015). There are multiple bioenergy pathways that vary according to the form of bioenergy (solid, liquid, gaseous) and the type of feedstock used (wood, crops, residues, waste). Bioenergies from dedicated crops or wood have faced substantial criticisms due to their significant land requirements. Certain bioenergy pathways appear to be exempt from this criticism, such as biogas generated from crop residues, manure, or food waste. However, these feedstocks are byproducts of agricultural activities that produce emissions not covered by current climate policies in most countries. The purpose of this article is to analyze the design of subsidies to bioenergies given the suboptimal regulation of agricultural activities and energy markets. This article is part of a research agenda that aims to consider realistic situations where GHG emissions are lightly taxed, and low-carbon energies are subsidized. The rebound effect of green subsidies on polluting activities is a recurring criticism of these policies,[2] it is essential to clarify whether it can undermine the merit of such subsidies and how it should be taken into account in their design.

For instance, the French national low carbon strategy foresees the consumption of 460TWh per year of bioenergy to achieve carbon neutrality, which is close to the yearly electricity consumption (470TWh in 2021), and approximately half of which would be biogas.[3] Biogas can be produced through methanization of agricultural residues, biowaste, dedicated crops or intermediate energy crops. Agricultural residues include harvest residues, processing residues and livestock manure. In order to achieve the objectives set, French farmers benefit from a feed-in tariff for biogas. Given the importance of cattle emissions particular caution should be put for biogas produced with manure. Indeed, agriculture accounts for 18.5% of the French total emissions, of which CH4 emissions account for almost half, and these emissions are still unregulated.

We start with a partial equilibrium model of two interacting markets for food and for gas. Biogas is jointly produced with food and is a perfect natural gas substitute. Natural gas combustion and food production are polluting and taxed but below the optimal Pigouvian level. We analyze the impact of a subsidy on biogas and its optimal design given suboptimal taxes on food and biogas. We then specify the framework by introducing farmers' heterogeneity, with the dairy market in mind. We provide a range of numerical simulations based on the French context to assess the effects' magnitude.

On the analytical front, we first describe the first-best regulation in which natural gas and food are priced at their Pigouvian level. At the first-best biogas should be subsidized if it reduces emissions in the farm, which is the case for biogas produced with livestock manure or agricultural residues. If taxes on food and natural gas are below their Pigouvian level, the optimal (second-best) subsidy to biogas encompasses two additional terms related to the adjustment of food and natural gas quantities to the supply of biogas. These adjustments should be weighted by their external costs. If food and biogas are complement in the production process, food production increases with the quantity of biogas produced, and the optimal biogas subsidy should be lowered to take into account the increase of emissions due to the increase of food production. With a quadratic specification, the second-best quantity of biogas is identical to the first-best one obtained with Pigouvian taxes and subsidies.

Then, we provide a numerical illustration based on the French dairy market. The simulation reveals a rebound effect on milk production due to biogas subsidization, and we show how total emissions first decrease and then increase with the biogas subsidy. We then compare a second-best situation in which dairy is not taxed with the first-best situation in which dairy is efficiently taxed. For a Social Cost of Carbon (SCC) of 100€/tCO_2 the rebound effect is moderate and the effect on livestock emissions is offset by the reduction of the milk carbon footprint due to the adoption of methanisation. The rebound effect calls for a relatively minor reduction of the biogas subsidy, and the optimal quantity of biogas produced is slightly larger when livestock emissions are underpriced compared to the first-best. We further investigate how this gap depends on the SCC. Indeed, at the first-best, the quantity of biogas first increases until all manure available is methanised, and then it decreases together with the total amount of milk produced. In our

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[2] The rebound effect is usually defined as the increase of energy consumption following an improvement in energy efficiency (e.g. Berkhout et al. 2000). Here, we define the "rebound effect" as the increase in the polluting good production (e.g. dairy) per additional unit of the complementary clean good (e.g. biogas). Indeed, one can interpret an increase of the use of methanisation as an efficiency improvement since it induces less emissions per unit produced.

[3] The last version available of the strategy is here https://www.ecologie.gouv.fr/sites/default/files/Projet%20SNBC%20EN.pdf. French electricity production and consumption numbers are available here: https://www.rte-france.com/actualites/bilan-electrique-2021

second-best scenario, with underpriced farming emissions, the amount of abatement feasible is limited, and once all manure is methanised, the biogas quantity plateaus and the difference with the first-best widen.

Numerous articles, whether in the grey or academic literature, have attempted to evaluate emissions reduction from various bioenergy pathways using Life Cycle Assessment (LCA) or numerical simulations. Bio-energies, specially biofuels, have been controversial for years mostly because of their land requirement, and the associated competition with food production or carbon sinks. Taking into account indirect emissions due to land use change considerably reduces the environmental benefits of these bioenergies (Searchinger et al. 2008; Beckman et al. 2011; Khanna et al. 2021).

LCAs of biogas production pathways show that manure cattle and other agricultural residues are the most environmentally-friendly feedstocks to produce biogas (Poeschl et al. 2012a, 2012b). Notably, biogas produced from manure avoids some methane emissions from manure in the farm (Esnouf et al. 2021; Poeschl et al. 2012a; Amon et al. 2006; Battini et al. 2014; Fusi and Pirlo 2023). These standard LCA ignore market adjustments notably on the agricultural and energy markets.[4] Numerical simulations have been used to assess the impact of biogas production policies in various contexts, and the use of dedicated crops like maize has been questioned because of the effect on land uses and food prices (e.g. Britz and Delzeit 2013).

Concerning pathways using crop residues or manure, some rebound effects have been quantified. In two different contexts, Bartoli et al. (2016) and Appel et al. 2016 find complementarities between biogas production and milk or meat production and a possible increase in the cattle herd due to biogas support. Still, Bartoli et al. (2019) show that supporting manure use while reducing the subsidy to biomethane was better for the environment than using maize to produce biogas. Moreover, Gérard and Jayet (2023) find that an increase in the price of crop residues could lead to an increase in crop areas, yields, and associated emissions. To our knowledge, there is no attempt to draw normative implications of these rebound effects other than calls for caution. We show that these rebound effects do not undermine the rationale for supporting these pathways, even though subsidies should be adjusted.

Finally, on the theoretical front, the literature on the design of climate policy and bioenergy has focused on biofuels and wood energy. The design of fuel blend mandates has received some attention (De Gorter and Just 2009; Holland et al. 2009; Lapan and Moschini 2012) when biofuel is produced by a dedicated crop and directly competes with food production. Similarly, several theoretical articles have investigated the trade-offs between carbon removal from forests and bioenergy production (Hoel 2020; Hoel et al. 2014). For instance, Hoel (2020) consider second best subsidy to biofuel from wood in a dynamic setting, the subsidy is justified by the substitution with the untaxed fossil fuel, and eventually is negative when biofuels need be phased out. None of these theoretical articles consider a situation in which agricultural activity and bioenergy are complements. Our work shares similarities with that of Hoarau and Meunier (2023), although their article focuses on electric vehicles rather than bioenergy. They examine the design of subsidies for electric vehicles in contexts where both the electricity and mobility sectors are imperfectly regulated. They show how the optimal subsidy to electric mobility should encompass life cycle emissions from electricity production (see also Fell et al. 2023, on hydrogen subsidies). A similar logic applies to our study despite the differences in the interactions between clean and polluting activities.

The rest of the article is organized as follows. In Section 2, the general model is introduced. Optimal policies are described in Section 3. In Section 4, we introduce farmers' heterogeneity. Numerical simulations are presented and discussed in Section 5. Section 6 concludes.

2 Model

We start with a simple and general model before opening the black box of the cost function by introducing heterogeneous farmers.

We consider a partial equilibrium model of a food product market (e.g. dairy) and its interaction with the gas market. The total quantity of food produced is X kg, and the total quantity of gas is Y kWh. Consumer prices are p_x and p_y for food and gas, respectively. The gross consumer surplus from food is $U(X)$

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[4] "Consequential" LCA integrate economic mechanisms (Earles and Halog 2011; Rajagopal 2014), and have been used to evaluate biofuels, taking into account food and energy market adjustment and land use changes. (Bento and Klotz 2014) further propose policy-based LCA, a methodology that further closes the gap with economics policy evaluation.

and from gas is $V(Y)$, two positive increasing and strictly concave functions. Net consumer surpluses are the difference between gross surplus and expanses p_xX and p_yY.

The total quantity of gas Y is the sum of Y_b kWh of biogas, from methanization, and Y_f kWh of natural gas, they are perfect substitutes. The production of Y_f cost $C_f(Y_f)$, with $C_f(.)$ a positive, increasing and convex function.

Farmers are aggregated into a representative farmer with a cost function $C(X, Y_b)$, which is assumed to be positive, increasing in both arguments and convex, that is:

$$\frac{\partial^2 C}{\partial X^2} > 0,\ \frac{\partial^2 C}{\partial Y^2} > 0 \ \text{ and } \frac{\partial^2 C}{\partial X^2}\frac{\partial^2 C}{\partial Y^2} - \left[\frac{\partial^2 C}{\partial X \partial Y}\right]^2 > 0 \tag{1}$$

The cross derivative of the cost function can be positive or negative; it captures the interaction between food production and bioenergy. If it is positive the two are substitutes, increasing the quantity of bioenergy increases the food production cost, as is the case for dedicated crops that use agricultural land. If it is negative, the two are complements, as is the case when crop residues or manure are used for methanization, which is the case we have in mind. We will further specify the model for the latter situation in Section 4, micro-founding the aggregated cost function and its cross derivatives.

Emissions arise from the combustion of natural gas (CO_2) and the joint production of food and bioenergy. These emissions are all converted into CO_2 equivalent and denoted: e_f tCO_2 per kWh of natural gas, and $e(X, Y_b)$ for emissions from the joint production of X kg of food and Y_b kWh of biogas. These emissions are assumed to be increasing with respect to X, decreasing with respect to Y_b, and convex.[5] For specific cases, one needs to assess farm emissions carefully. We further discuss and detail the origin of emissions for the dairy sector in Section 4. The combustion of biogas is assumed carbon neutral because it is compensated by CO_2 removal during plant growth, whether these plants are used directly for food and bioenergy production or used to feed livestock, the manure of which is used for biogas production. Farm emissions will depend on the type of activity and will mostly consist of CH_4 from cattle, and N_20 from fertilisers.

We consider the following policy setting: a tax τ on food, a tax t on natural gas, and a subsidy s on biogas. The profit of the representative farmer is

$$\Pi_x(p_x, p_y, X, Y_b) = (p_x - \tau)X + (p_y + s)Y_b - C(X, Y_b). \tag{2}$$

The profit of the representative natural gas producer is

$$\Pi_f = (p_y - t)Y_f - C_f(Y_f) \tag{3}$$

The SCC is δ €/ tCO_2, and total Welfare is the difference between consumers' surplus and production costs, including environmental damages:

$$W(X, Y_f, Y_b) = U(X) + V(Y_f + Y_b) - C(X, Y_b) - C_f(Y_f) - \delta[e_fY_f + e(X, Y_b)]. \tag{4}$$

It can also be written as the sum of consumers' net surplus, producers' profit, and tax revenues minus environmental costs.

3 Optimal first-best and second-best policies.

We first describe the optimal allocation before considering the second-best subsidy to biogas. The production of biogas connects the two markets. The optimal allocation can be decentralized with a Pigouvian tax on all emissions or alternatively appropriate taxes on food and natural gas and a subsidy to biogas. If taxes are given and lower than their optimal level, the optimal biogas subsidy should be adjusted to account for its indirect effects on the two polluting quantities.

[5] We restrict ourselves to cases in which farm emissions are decreasing with bioenergy production, which is the case for complements (crop residues, manure, food waste, forestry co-products) but not for dedicated crops.

The optimal allocation is denoted X^*, Y_b^*, Y_f^*, and consumers' marginal surplus is denoted p_x^* and p_y^* since they coincide with consumer prices at the market equilibrium with Pigouvian taxes. If all quantities are positive, they satisfy the first order conditions:

$$U'(X^*) = p_x^* = \frac{\partial C}{\partial X}(X^*, Y_b^*) + \delta \frac{\partial e}{\partial X}(X^*, Y_b^*) \tag{5a}$$

$$V'(Y_f^* + Y_b^*) = p_y^* = \frac{\partial C}{\partial Y}(X^*, Y_b^*) + \delta \frac{\partial e}{\partial Y}(X^*, Y_b^*) \tag{5b}$$

$$= C_f'(Y_f^*) + \delta e_f \tag{5c}$$

The two sectors are linked via the production cost of the farming sector. For instance, increasing the demand for food would induce an increase of biogas production, and a decrease of gas price and natural gas consumption.

3.1 Market equilibrium

All agents are assumed price takers, consumers' decisions maximize consumers' net surpluses, and producers' decisions maximize the representative profits given by equations (2) and (3). The three following equations then describe the market equilibrium:

$$U'(X) = p_x = \frac{\partial C}{\partial X}(X, Y_b) + \tau \tag{6a}$$

$$V'(Y_f + Y_b) = p_y = \frac{\partial C}{\partial Y}(X, Y_b) - s \tag{6b}$$

$$= C_f'(Y_f) + t. \tag{6c}$$

Lemma 1 *The first best allocation is obtained with Pigouvian pricing which, in our setting, consists in*

$$\tau = \delta \frac{\partial e}{\partial X}(X^*, Y_b^*),\ t = \delta e_f \text{ and } s = -\delta \frac{\partial e}{\partial Y_b}(X^*, Y_b^*)$$

The policy described involves taxing food production and subsidizing biogas at their marginal external costs and benefits. In our setting, it is equivalent to a Pigouvian tax δ on farm emissions $e(X, Y_b)$. Indeed, farm emissions are difficult to measure, and taxes and subsidies for final products limit the so-called monitoring, reporting, and verification costs. It should be emphasized that the biogas subsidy is justified by the reduction of farm emissions and not the substitution with natural gas.

To set the optimal subsidy, the impact of this subsidy on equilibrium quantities is required. From equation (6a), an increase of the subsidy s modifies the quantity of food produced as follows:

$$\frac{\partial X}{\partial s} = -\frac{\partial^2 C}{\partial X \partial Y} \frac{1}{\partial^2 C/\partial X^2 - U''} \frac{\partial Y_b}{\partial s} \tag{7}$$

If the two goods are complements, a subsidy for biogas increases food production by reducing its marginal production costs. Conversely, if the goods are substitutes, the subsidy decreases food production. The impact is larger when demand and supply are more elastic.

Concerning the effect on natural gas, equation (6c) tells us that

$$\frac{\partial Y_f}{\partial s} = \frac{V''}{C_f'' - V''} \frac{\partial Y_b}{\partial s}. \tag{8}$$

The rate of substitution between biogas and natural gas is given by the fraction on the right side. If the production cost of natural gas is linear, the price of gas does not change with an increased supply of biogas, and the substitution rate is -1. Each kWh of biogas replaces one kWh of natural gas. It is also the case if gas demand is inelastic (V'' infinite). Otherwise, supply and demand adjustment induces a substitution rate lower than 1, and possibly null if supply is inelastic (c_f'' infinite).

Let us denote by ρ_X and ρ_f the adjustment in quantities of food X and natural gas Y_f, respectively, for each additional unit of biogas:

$$\rho_X = -\frac{\partial^2 C}{\partial X \partial Y}\frac{1}{\partial^2 C/\partial X^2 - U''} \text{ and } \rho_f = \frac{V''}{C_f'' - V''}. \tag{9}$$

The quantity of natural gas always decreases with the quantity of biogas, ρ_f is negative. The parameter ρ_X will be called the *rebound effect* in the following. It is positive (resp. negative) if food and biogas are complement (resp. substitute). The effect of an increase of biogas on total emissions is therefore:

$$\frac{\partial e}{\partial Y_b} + \rho_X \frac{\partial e}{\partial X} + \rho_f e_f \tag{10}$$

It could be interpreted as a consequential life cycle assessment of GHG emissions from biogas production.

3.2 Optimal biogas subsidy with suboptimal taxes

If the two taxes τ and t are fixed at suboptimal levels, the optimal subsidy to biogas will depart from its Pigouvian level. The optimal subsidy will not only integrate the effect of biogas on emissions but also encompass indirect effects on natural gas and food production.

Proposition 1 *The optimal subsidy to biogas, given taxes on food τ and on natural gas t, is*

$$s^{SB} = -\delta \frac{\partial e}{\partial Y_b} - \Big(\delta \frac{\partial e}{\partial X} - \tau\Big)\rho_X + (\delta e_f - t)|\rho_f| \tag{11}$$

in which ρ_X and ρ_f are given by equations (9).

Proof 1 *The effect of the subsidy on Welfare is, from equation (4) and injecting the market equilibrium equations (6)*

$$\frac{dW}{ds} = \frac{\partial W}{\partial Y_b}\frac{\partial Y_b}{\partial s} + \frac{\partial W}{\partial X}\frac{\partial X}{\partial s} + \frac{\partial W}{\partial Y_f}\frac{\partial Y_f}{\partial s} \tag{12}$$

$$= -\Big(s + \delta \frac{\partial e}{\partial Y_b}\Big)\frac{\partial Y_b}{\partial s} + \Big(\tau - \delta \frac{\partial e}{\partial X}\Big)\frac{\partial X}{\partial s} + (t - \delta e_f)\frac{\partial Y_f}{\partial s} \tag{13}$$

Then, injecting equations (7) and (8) give expression (13).

Indeed, if food and natural gas are taxed at the Pigouvian level, their adjustment does not intervene in the expression of the optimal subsidy. Otherwise, the optimal second-best subsidy should encompass the adjustment of these quantities times the unpriced environmental damage.

Biogas might be subsidized even if it increases total emissions at the margin.

Corollary 1 *A subsidy to biogas increases total emissions if and only if food and biogas are complement ($\rho_X > 0$) and*

$$\frac{\partial e}{\partial X}\rho_X > |\frac{\partial e}{\partial Y_b}| + e_f|\rho_f|$$

Even if this is the case, it can be optimal to subsidize biogas if

$$\rho_X < \frac{1}{\delta \frac{\partial e}{\partial X} - \tau}\left[\delta|\frac{\partial e}{\partial Y_b}| + (\delta e_f - t)|\rho_f|\right]$$

For a small rebound effect, emissions are indeed decreasing with the quantity of biogas, and it should be subsidized. For an intermediary rebound effect, emissions increase, but biogas should still be subsidized, and for a large rebound effect, biogas should not be subsidized.

Regarding the influence of the SCC, the quantity of biogas increases with the SCC if and only if total emissions are decreasing with biogas.

Corollary 2 *Biogas production decreases with respect to the SCC if and only if a subsidy to biogas increases total emissions, that is, eq. (10) is positive.*

This result holds whether GHG emissions from food production and natural gas combustion are optimally priced or not.

The Proof is in Appendix A.

The model can be fully solved with quadratic gross surplus, cost functions, and linear emissions. Interestingly, in that case, the quantity of biogas produced in the second-best setting does not depend on the natural gas and food taxes. Welfare losses only depend on the gap between actual and Pigouvian taxes on food and natural gas. The complementarity between biogas production and food production does not intervene.

Proposition 2 *With a quadratic specification, with a linear emission function, and given taxes τ and t, at the second best optimum :*

- the quantity of biogas is the same as the one at first best; it does not depend on taxes;

- Welfare at the optimal subsidy is

$$W^{SB} = W^{FB} - \frac{1}{2}\frac{(\delta e_X - \tau)^2}{\partial^2 C/\partial X^2 - U''} - \frac{1}{2}\frac{(\delta e_f - t)^2}{C_f'' - V''}$$

The Proof is in the Appendix B.

In the general setting, the suboptimal regulation of food emissions has two opposing effects on the optimal quantity of biogas. On the one hand, more food production implies a lower cost of supplying biofuel, which pushes for larger biogas production. On the other hand, the second-best optimal quantity of biogas is reduced to account for its effect on the food produced, which explains a lower second-best subsidy compared to the first-best. In the quadratic setting, the two effects cancel each other out, and the optimal quantity of biogas does not depend on the tax on food. Indeed, this result rests on the assumption of a constant cross derivative of the cost function which implies that the effect of biogas production on food production, captured by ρ_X is constant. As we shall see in the numerical illustration, with the specification considered, the rebound effect increases with the quantity of biogas, and so, the distance between the second-best and first-best widens when the SCC increases.

It is essential to keep in mind that the result holds for linear environmental damage. For a given SCC, total emissions are higher at the second best, with suboptimal taxes, than at the first-best. If total emissions are fixed, the quantity of biogas would indeed be higher in a second-best situation than in the first-best one to compensate for too large quantities of food and natural gas. The welfare losses would then be higher and depend on the complementarity between food and biogas production.

4 The dairy market, introducing farmers heterogeneity

In this section, we open the black box of the production side by explicitly modeling heterogeneous individual farmers and looking at the allocation of production among them. Let us consider a continuum of heterogenous dairy farmers characterized by methanization cost $\theta \in [\underline{\theta}, \bar{\theta}]$, distributed according to the cumulative function $F(\theta)$ with density $f(\theta) = F'(\theta)$. Otherwise, producers are identical with a production cost of milk $c(x)$, positive, increasing, and strictly convex. For each unit of milk produced, α kWh of gas can be produced at a cost θ per kWh (or $\alpha\theta$ per unit of food). Individual production of milk and (bio)methane are $x(\theta)$ and $y(\theta)$, with $0 \leq y(\theta) \leq \alpha x(\theta)$ so total quantity X and Y_b are:

$$X = \int_{\underline{\theta}}^{\bar{\theta}} x(\theta) f(\theta) d\theta \text{ and } Y_b = \int_{\underline{\theta}}^{\bar{\theta}} y(\theta) f(\theta) d\theta \tag{14}$$

Emissions arise mainly from the production of milk (mostly methane), from manure, depending on its management, if it is not methanized and from the combustion of natural gas (CO_2).[6] Finally, we assume

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[6] Regarding the use of digestate and its effects on carbon rate in the soils, we consider that its use is equivalent to the use of manure. The combustion of biogas is considered carbon neutral since it is compensated by the removal of CO_2 from cow feed growth. For the same reason part of the farm emissions are not accounted for because they are compensated by the CO_2 removed by plant growth.

a linear relationship between emissions and quantities produced. Let us denote emissions rates: e_2 tCO$_2$ per liter of milk without methanization; e_1 tCO2 per liter of milk with methanization; e_f tCO$_2$ per kWh of natural gas. We assume $e_2 > e_1$ and emissions from dairy farmers are therefore

$$e(X, Y_b) = e_2 X - \frac{e_2 - e_1}{\alpha} Y_b \tag{15}$$

Methanization reduces the emission per litter of milk by $e_2 - e_1$ because of CH$_4$ not emitted, and thus each kWh of biomethane reduces emissions by $(e_2 - e_1)/\alpha$.

The profit of a farmer of type θ is

$$\pi_\theta(p_x, p_y, x, y) = (p_x - \tau)x + (p_y + s)y - c(x) - \theta y \tag{16}$$

The aggregated cost function $C(X, Y_b)$ is defined as

$$C(X, Y_b) = \min_{(x(\theta), y(\theta))_\theta} \left\{ \int_{\underline{\theta}}^{\bar{\theta}} \big[c(x(\theta)) + \theta y(\theta)\big] f(\theta) d\theta \;\Big|\; y(\theta) \leq \alpha x(\theta), \text{ and } (14) \right\} \tag{17}$$

At the optimum and at market equilibriums considered the allocation of total milk production and biogas among farmers will coincide with the cost minimizing ones, and the total cost will thus be $C(X, Y_b)$ defined above.

Furthermore, in all situations considered, there will be a threshold methanization cost $\tilde{\theta}$ such that farmers with a lower cost adopt methanization and others do not, and farmers that adopt methanization do so as much as possible ($y(\theta) = \alpha x(\theta)$). The aggregate cost and Welfare can be then be written distinguishing farms with and without methanization :

$$\begin{aligned} W =& U(X) + V(Y_f + Y_b) - C_f(Y_f) - \delta e_f Y_f \\ &- \int_{\underline{\theta}}^{\tilde{\theta}} \big[c(x) + \alpha\theta x + \delta e_1 x\big] f(\theta) d\theta \qquad \text{with methanizer} \\ &- \int_{\tilde{\theta}}^{\bar{\theta}} [c(x) + \delta e_2 x] f(\theta) d\theta \qquad \text{without} \end{aligned} \tag{18}$$

4.1 Optimal allocation

The optimal allocation does not consist only in the aggregated quantities as described by equations (5) but also in their allocation among farmers $(x^*(\theta), y^*(\theta))$ and a threshold $\tilde{\theta}^*$. These quantities maximize the welfare function (18) subject to $0 \leq y(\theta) \leq \alpha x(\theta)$, and thus satisfy the first order conditions:

$$U'(X^*) = p_x^* \text{ and } V'(Y_f^* + Y_b^*) = p_y^* \tag{19}$$

$$p_y^* = C_f'(Y_f^*) + \delta e_f \tag{20}$$

$$p_x^* = c'(x^*(\theta)) + \delta e_2 \qquad \text{for } \theta \geq p_y^* + \delta(e_2 - e_1)/\alpha \tag{21}$$

$$p_x^* + \alpha p_y^* = c'(x^*(\theta)) + \alpha\theta + \delta e_1 \qquad \text{for } \theta < p_y^* + \delta(e_2 - e_1)/\alpha \tag{22}$$

Methanization connect the two markets since some farmers produce both milk and biogas. Farmers only differ, in our framework, with respect to their methanization cost. Farmers with large methanization cost should not use methanization and these farmers all produce the same quantity $x^* = c'^{-1}(p_x^* - \delta e_1)$. Farmers with low methanization cost should use methanization for all the manure and their milk production is larger the lower their methanization cost.

The optimal allocation can also be described in two steps: by first characterizing the aggregate cost function given by equation (17), and then make use of the optimality conditions (5).

For any quantities (X, Y_b), with $Y_b < \alpha X$, there is a threshold $\tilde{\theta}(X, Y_b)$ such that farmers with a cost below the threshold adopt methanization while others do not. Then, the aggregate cost function partial derivatives are

$$\frac{\partial C}{\partial Y} = \tilde{\theta}(X, Y_b) \text{ and } \frac{\partial C}{\partial X} = c'(x(\theta)) \text{ for all } \theta \geq \tilde{\theta}. \tag{23}$$

The marginal cost of biogas production is equal to the methanization cost of the marginal farmer who uses methanization. And the marginal cost of milk production is equal to the marginal cost of farmers that do not use methanization. Indeed, at the optimum

$$\frac{\partial C}{\partial Y} = \tilde{\theta}(X^*, Y_b^*) = \tilde{\theta}^* = p_y^* + \delta \frac{e_2 - e_1}{\alpha}. \tag{24}$$

The rebound effect (9) is linked to the second-order derivatives of the cost function, which are formally derived in Appendix C.

4.2 Market Equilibrium

With taxes τ and t on milk and gas respectively, and a subsidy s on biogas, the market equilibrium is described by equations (6) at the aggregate level. At the farmer level, the threshold methanisation cost is $\tilde{\theta} = p_y + s$ and individual productions satisfy:

$$(p_x - \tau) + \alpha(p_y + s) = c'\big(x(\theta)\big) + \theta\alpha \qquad \text{if } \theta \leq \tilde{\theta} = p_y + s \tag{25}$$

$$p_x - \tau = c'\big(x(\theta)\big) \qquad \text{otherwise.} \tag{26}$$

Farmers with low methanisation cost produce more than others because of the additional revenue on the gas market. An increase of the biogas subsidy not only increases total quantities of milk and biogas but also their allocation among farmers, as described in the following proposition.

Proposition 3 *As the subsidy to bioenergy increases:*

- *Both prices, of energy and of milk, decreases;*
- *The total production of milk increases: farmers who use methanization increases their production while others reduce their production;*
- *More biogas is produced: more farmers adopt methanization ($\tilde{\theta}$ increases);*
- *The quantity of natural gas decreases.*

The amplitude of the rebound effect depends on whether the increase in biogas production comes from an increase in milk production by farmers using methanisation or an increase in the adoption of methanisation (increase of $\tilde{\theta}$). The decrease of the milk price and the associated reduction of milk production from farmers not using methanisation mitigates the rebound effect.

5 Numerical illustration

The purpose of this section is to provide a simplified numerical illustration that offers insight into the approximate magnitudes of the mechanisms analyzed in the theoretical model. This illustration is intended to clarify and support the theoretical findings rather than to represent a fully realistic simulation. Therefore, the parameters and assumptions used here are stylized, focusing on conceptual clarity over empirical accuracy.

5.1 Calibration

Based on the model described in Section 4, this numerical illustration allows to compare produced quantities between different scenarios. We calibrate our model with French quantities of the dairy and the natural gas sectors. For the production of biogas, we only consider the quantities which are injected in the grid. As described in Section 4, farmers are differentiated according to their biomethane production cost (θ)v which is assumed uniformly distributed over $[\underline{\theta}, \bar{\theta}]$. The cost function is such that the supply function is isoelastic:

$$c(x) = \frac{\epsilon}{\epsilon + 1}.c_x.x^{\frac{\epsilon+1}{\epsilon}} \tag{27}$$

in which ϵ is the price elasticity of milk production and cx is a constant. The calibration of c_x is detailed in Appendix D together with the calibration of the demand functions. The natural gas production cost is assumed linear which limits the interaction with the gas market. The total consumption of gas is solely determined by the variable cost of natural gas plus the tax on gas.

It should be noted that the emissions from dairy production do not account for the land needed to produce the food of livestock, whether grass or cereal. According to the calibration of **??** it would nearly double the carbon footprint per liter.

Table 1 gives the values of the parameters, sources are given in Appendix E. Three features regarding emissions are worth stressing: First, methanisation reduces farm emissions by 10%. Second, this reduction of emissions on the farm from methanisation is close to the reduction obtained by substituting natural gas:

$$\frac{e_2 - e_1}{\alpha} = 0.235 \text{ kgCO2 (at the farm)/ kWh of biogas } \simeq e_f = 0.227 \text{ kgCO2/kWh of natural gas} .$$

Third, total emissions per kWh of biogas, of a farm that methanises, are nearly ten times those of a kWh of natural gas ($e_1/\alpha \simeq 10e_f$). If adding a kWh of biogas requires to increase dairy production, then total emissions increase ten times.

Parameter	Value
P_{x0}	0.45 €/kg of milk
X_0	$2.33\,10^{10}$ kg of milk
milk demand elasticity	-0.6
milk supply elasticity (ϵ)	0.6
α	0.425 kWh/kg milk
e_2	1 kgCO2/kg milk
e_1	0.9 kgCO2/kg milk
$\underline{\theta}$	0.04 €/kWh
$\bar{\theta}$	0.14 €/kWh
$P_{y0} = c_f$	0.04 €/kWh
Y_0	530 TWh
gas demand elasticity	-0.1
e_f	0.227 kgCO2/kWh
Social Cost of Carbon δ	100 €/tCO2

Table 1: Values of the parameters for the calibration

We simulates three different scenarios :

- Business-As-Usual (BAU) : there is no policy in place $\tau = 0$, $t = 0$ and $s = 0$.
- First-Best (FB) : all the instruments are at the pigouvian level, as described by Lemma 1 :

$$\tau = \delta e_2,\ t = \delta e_f,\ s = \delta \frac{e_2 - e_1}{\alpha} \tag{28}$$

- Second-Best (SB) : there is no tax on dairy and a suboptimal tax on natural gas at half the Pigouvian value which is close to the carbon component currently set in France:

$$\tau = 0,\ t = \frac{1}{2}\delta e_f.$$

  the subsidy will be set optimaly to maximize Welfare.

In subsection 5.4, in which we consider higher values of the SCC, we consider another second-best situation in which dairy emissions are not taxed but natural gas emissions are taxed at the Pigouvian level. We do so to focus on the distortion in the dairy sector.

5.2 The effect of the biogas subsidy

Before comparing the three scenarios, let us have a look at the influence of the subsidy on relevant quantities in the second-best situation with $t = 0$ and $\tau = 1/2\delta e_f$. It will illustrate that the subsidy can induce a rise of total emissions. Figure 1 shows the evolution of total GHG emissions with respect to the biogas subsidy. As can be seen total emissions first decline and then rise with respect ot he biogas subsidy.

There are two regimes in our numerical model depending on whether all farmers engage in methanization ($\tilde{\theta} = \bar{\theta}$) or not ($\tilde{\theta} < \bar{\theta}$). In Figure 1 the two regimes are separated by the dashed vertical line. As the biogas subsidy increases, the share of farmers that adopt methanisation increases, along with total dairy production and biogas production. There are two sources of biogas expansion: the rise in the share of farmers that use methanisation and the rise in dairy production from farmers that use methanisation. Once all farmers adopt methanisation, the former source of expansion is exhausted, and the increase in methanisation can no longer compensate for the rise in dairy emissions. In Figure 1 the evolution of total emissions is decomposed in three: gross dairy emissions, which increase, methanisation reduces farm emissions (light grey), and the substitution of natural gas with biogas further reduces emissions (grey). As mentionned the two last components have similar size. Total emissions decrease as long as all farmers do not use methanisation and once they do, total emissions increase with the biogas subsidy .

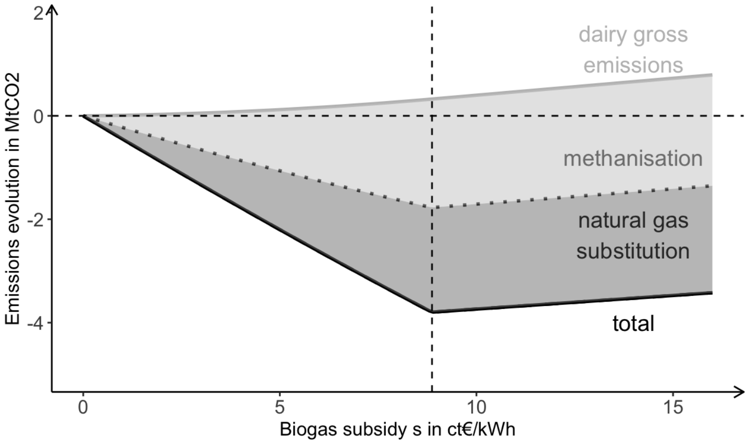

Figure 1: Evolution of total emissions with respect to the biogas subsidy. Variations are decomposed into: diary gross emissions, reduction in the farm from methanisation and reduction from natural gas substitution.

5.3 Comparison of Scenario

The results of the different scenarios are given in Table 2. Dairy emissions are denoted E_d there are gross dairy emissions minus the reduction from methanisation:

$$E_d = e_2 X - \frac{e_2 - e_1}{\alpha} Y_b.$$

Variable	BAU	FB	SB
τ, t	$0, 0$	$\delta e_2, \delta e_f$	$0, \frac{1}{2}\delta e_f$
W (€)	$1.069.10^{11}$	$1.074.10^{11}$	$1.073.10^{11}$
W -W^{BAU}	0	$5.2.10^{8}$	$3.6.10^{8}$
Prices			
s (cts€/kWh)	0	2.353	3.205
P_y (cts€/kWh)	4.000	6.270	5.135
$P_y + s$ (cts€/kWh)	4.000	8.623	8.340
P_x (cts€/kg of milk)	45.00	49.83	44.80
Quantities			
X (kg of milk)	$2.330\,10^{10}$	$2.180\,10^{10}$	$2.336\,10^{10}$
Yb (TWh)	0	4.32	4.34
Yf (TWh)	530	496	511
Y (TWh)	530	500	515
Emissions			
E$_d$ (tCO2)	$2.33\,10^{7}$	$2.078\,10^{7}$	$2.234\,10^{7}$
E$_f$ (tCO2)	$1.203\,10^{8}$	$1.125\,10^{8}$	$1.159\,10^{8}$
E (tCO2)	$1.436\,10^{8}$	$1.333\,10^{8}$	$1.383\,10^{8}$

Table 2: Comparison of the three scenarios

Prices and quantities. In the dairy market, the quantity produced is larger in SB than in BAU, while it is lower in FB than in BAU, the consumer price of milk varies accordingly. The quantity of biogas produced in SB is close to the first-best level, in line with Proposition 2, although slightly larger. Even though the total production of biomethane is similar in FB and SB, its allocation is drastically different. Figure 2 shows how total production is allocated among farmers. All farmers produce less dairy in FB than in BAU and in SB. In SB farmers who methanize produce more and others produce slightly less than in BAU. There are more farmers who produce biogas in FB (46%) than SB (35%) but they produce

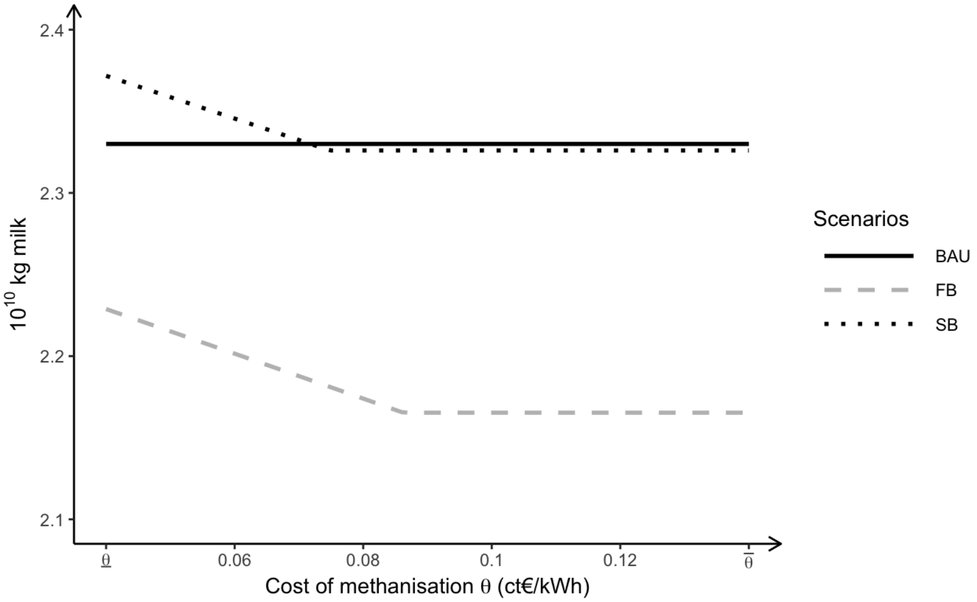

Figure 2: Allocation of total production among farmers in the BAU, FB and SB scenarios

The second best optimal subsidy to biogas is larger than the first best one, the difference is the sum of two terms related to the substitution of natural gas and the indirect subsidization of dairy production. The rebound effect is derived from the numerical value obtained and the formula (13):

$$
\begin{aligned}
s^{SB} &= \delta\frac{e_2 - e_1}{\alpha} + (\delta e_f - t) - \delta e_2|\rho_X| && (29)\\
3.21 &= 2.35 + 1.14 - 10 \times 0.029 && (30)
\end{aligned}
$$

Indirect effects account for a large share of the optimal subsidy. Approximately 35% of the optimal subsidy is justified by the subsitution with natural gas and the optimal subsidy should be reduced by 9% because of its indirect effect on milk production. In our SB scenario the tax on gas is half its Pigouvian value. If it were at the Pigouvian level of δe_f, the optimal subsidy would be reduced to 2.07 ct€per kWh, the quantity of dairy would be unchanged.

Welfare and its allocation. In absolute terms welfare gains look rather modest, even at the first best, it is essentially because of the size of the gas market. The second best achieves 68% of the welfare gain obtain at the first best:

$$\frac{W^{SB} - W^{BAU}}{W^{FB} - W^{BAU}} = 68\%.$$

It is possible to quantify that nearly half the difference is due to the underpricing of natural gas emissions, and the other half to the underpricing of dairy emissions.[7] Table 3 shows the allocation of welfare gains and losses among sectors and categories of agents. Net surpluses in the dairy sector and gas sectors are denoted W_x and W_y respectively, and total tax revenue is denoted R. In each sector the net surplus is the sum of consumer net surplus and producer profit minus emissions cost. The profit of natural gas producers is null ($p_y = c_f + t$). We decompose Welfare as the sum $W = W_x + W_y + R$ in which:

$$
\begin{aligned}
W_x =& [U(X) - p_x X] + [(p_x - \tau)X + (p_y + s)Y_b - C(X, Y_b)] - \delta[e_2 X - (e_2 - e_1)Y_b/\alpha]\\
W_y =& V(Y_f + Y_b) - (c_f + t)(Y_f + Y_b) - \delta e_f Y_f\\
R =& \tau X + tY_f - sY_b
\end{aligned}
$$

The last column of Table 3 indicates the ratio between the gains achieve in SB and in FB. Not only the total gain differ but also its allocation. In the dairy market both consumers and producers are better off in SB than in BAU whereas they are worst off in FB than in BAU (hence the negative numbers in the last column). For all other categories, SB and FB move in the same direction relative to BAU, but SB achieves approximately half of what FB achieves.

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[7] To evaluate those contributions we computed the Welfare obtained with an optimal tax on gas, no tax on dairy emissions and an optimal subsidy ($\tau = \delta e_f$, $t = 0$, and $s^{SB} = 2.05$c€/kWh). It increases the gains by 17%, achieving 85% of the first best welfare gains, the last 15% can be attributed to the the absence of a tax on dairy emissions.

	Scenario			
	BAU	FB	SB	$\frac{\text{SB-BAU}}{\text{FB-BAU}}$
Dairy sector				
Consumers	8.74	7.65	8.78	-4%
Producers	6.55	5.48	6.60	-4.5%
Emissions	-2.33	-2.08	-2.23	+38%
Total = W_x	12.96	11.05	13.15	-10‰
Gas sector				
Consumers	106.92	94.31	100.06	+50%
Emissions	-12.03	-11.25	-11.59	+56%
Total= W_y	93.96	83.06	88.47	+50%
Taxpayers				
R	0	13.33	5.66	+42%
Total				
W	106.93	107.45	107.28	+68%

Table 3: Allocation of Welfare (in 10^9 €) to the different categories of agents in BAU, FB and SB.

Emissions and allocation of efforts. In both sectors, emissions are lower in FB than in SB and lower in SB than in BAU. Even though dairy production is larger in SB than in BAU, the deployment of methanisation explains that emissions are lower. The reduction of emissions in the two scenarios can be decomposed in three components: reduction of the consumption of gas, reduction (or increase) of dairy production and increase of methanisation. The latter encompasses both the reduction of farm emissions and substitution with natural gas.

$$E - E^{BAU} = e_f(Y - Y^{BAU}) + e_2(X - X^{BAU}) - (\frac{e_2 - e_1}{\alpha} + e_f)Y_b \tag{31}$$

	FB		SB	
	Total	Share	Total	Share
ΔE	-10.3	100%	-5.36	100%
$e_f \Delta Y$	-6.83	66%	-3.41	64%
$e_2 \Delta X$	-1.5	14.5%	$+0.06$	-1%
$(\frac{e_2-e_1}{\alpha} + e_f)Yb$	-2	19.5%	-2	37%

Table 4: Abatment (in $MtCO_2$) in the two scenario SB and FB, and its allocation among the three channels.

In each scenario, approximatively 65 % of the abatment is due to the decrease of the gas demand. In FB the remaining effort is evenly shared among dairy production and methanisation. In SB, the remaining effort solely comes from methanisation, and dairy production slightly increases.

5.4 The influence of the Social Cost of Carbon

In the above comparison the SCC was fixed at 100€per tCO_2. Along an energetic transition the SCC is supposed to progressively increase until carbon neutrality is reached. We explore here the effect of increasing the SCC up to 500 €/tCO$_2$,[8] with an eye on the rebound effect. At 100€/tCO$_2$, in our simulation, only a subset of farmers adopt methanisation and the rebound effect remains limited, and the correction implied to the biogas subsidy is relatively small. With larger SCC all farmers use methanisation and the rebound effect is large: any additional kWh of biogas requires additional production of dairy and induces a rise of total emissions. In such a case the rebound effect calls for a large correction of the biogas subsidy, as long as dairy is untaxed. Consequently, as the SCC rises the difference between the first-best and a second-best situation in which dairy is untaxed widdens.

In the following, in order to focuss on inefficiencies in the dairy market, the emissions on the natural gas market are taxed at their Pigouvian level in all cases considered ($t = \delta e_f$) and we compare a first-best situation in which dairy emissions are taxed at the Pigouvian level ($\tau = \delta e_2$) with a second-best situation in which they are not taxed ($\tau = 0$). In all the following simulations the price of natural gas is $p_y = c_f + \delta e_f$.

Figure 3 illustrates the determination of the optimal subsidy when dairy emissions are properly taxed (FB) and when they are not (SB). Figure 3 depicts the evolution of Welfare with respect to the biogas subsidy for three different values of the SCC (100, 200 and 300 € per tCO$_2$), the maximums are indicated with dots. The vertical lines separate the two regimes: for small values of the subsidy only a subset of farmers adopt methanisation, and for large values they all do. The threshold subsidy is $s = \bar{\theta} - (c_f + \delta e_f)$, the last term being the price of natural gas. Welfare is not differentiable at the threshold subsidy. As can be seen on Figure 3, whether dairy emissions are taxed or not, for large values of the SCC it is optimal that all farmers use methanisation. When dairy emissions are efficiently taxed, the optimal subsidy is equal to the SCC times the reduction of emissions per biogas kWh (Lemma 1): $s^{FB} = \delta(e_2 - e_1)/\alpha$, which can be larger than the threshold subsidy. When dairy emissions are not taxed, Welfare decreases with respect to the subsidy above the threshold, once all farmers use methanisation, and for large value of the SCC, the optimal second-best subsidy is equal to the threshold subsidy ($\bar{\theta} - (c_f + \delta e_f)$).

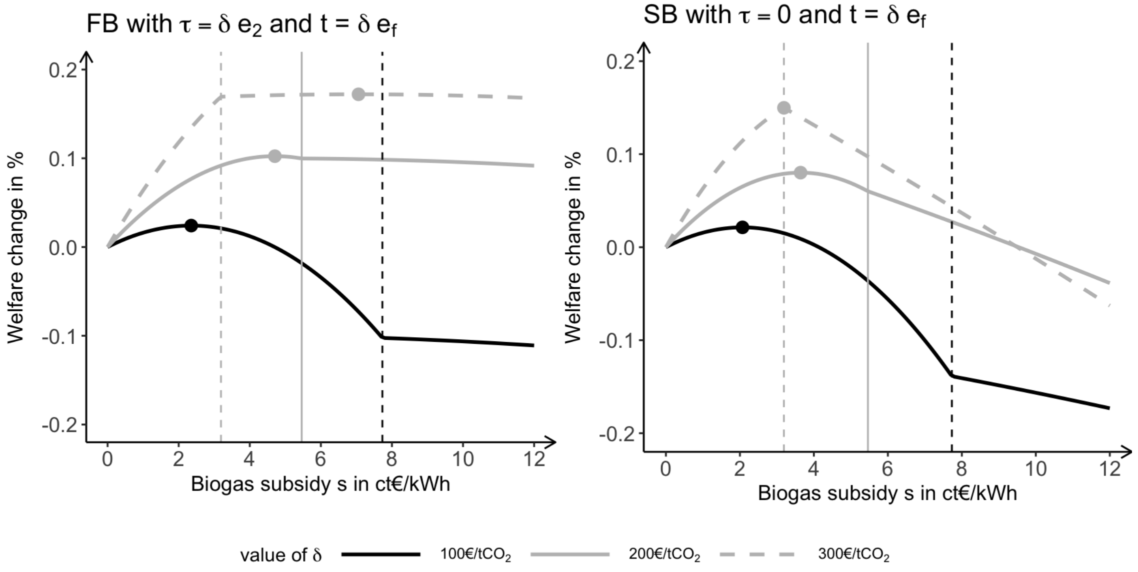

Figure 3: Evolution of Welfare with respect to the biogas subsidy with optimal taxation of dairy emissions (left) and without taxation of dairy emissions (right), with three different values of the SCC. The dots indicate to the maximums.

It is feasible to recover the rebound effect at the three maximums from the optimal subsidy found:[9]

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[8] In France, according to Quinet et al. (2019) the carbon price trajectory consistent with the 2050 carbon neutrality goal projects an increase in the SCC to as much as 750€/tCO2 by 2050.

[9] One can either recover it from the difference between the second-best and first-best subsidies or directly compute it numerically. However, at the threshold subsidy the rebound effect jumps and the implicit rebound recovered from the subsidies

δ €/tCO$_2$	ρ_X kg milk/MWh
100	29
200	53
300	129

It should be noted that at the threshold subsidy the rebound effect is discontinuous and jumps upward, the implicit rebound effect recovered from the subsidies falls between the left and right values. As can be seen the rebound effect, at the optimal subsidy, increases with respect to the SCC.

Figure 4 depicts the evolution of the optimal subsidy (A), the quantity of biogas (B), dairy production (C) and net dairy emissions (D) with respect to the SCC. The first-best is depicted in gray and the second-best in black. In both scenarios, for small values of the SCC only a subset of farmers adopt methanisation, and for large values they all do. The switch between the two regimes happens at the break of the slopes of the quantities depicted, as can be seen it happens for larger values of the SCC when dairy emissions are not taxed. At the first-best optimum, dairy production decreases with respect to the SCC while biogas production first increase until methanisation is used by all farmers, it then decreases together with dairy production and dairy emissions. In the second-best considered, when dairy emissions are not taxed, dairy production increases with the SCC while methanisation expands among farmers which ensures that dairy net emissions decrease. But once all farmers use methanisation, all three quantities plateau. The optimal biogas subsidy increases linearly with the SCC in the first-best scenario, while in the second-best scenario it first increases and then decreases once all farmers use methanisation. The difference between the two optimal subsidies and biogas quantities, in the first-best and second-best scenarions, windens as the SCC grows. Indeed, eventually biogas should not be subsidized if dairy is not taxed, revenue generated on the gas market, with a properly taxed natural gas, are sufficient to ensure full adoption of methanisation among dairy farmers.

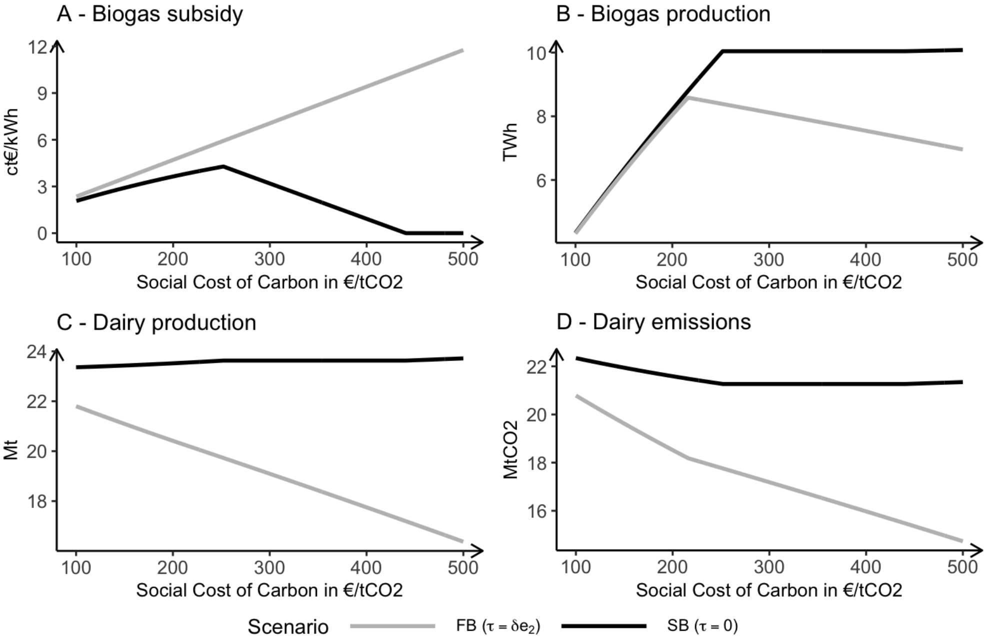

Figure 4: Evolution of the optimal biogas subsidy, biogas production, dairy production and dairy emissions with the SCC δ. In both scenario natural gas emissions are taxed at the Pigouvian level $t = \delta e_f$.

To conclude, let us look at the welfare losses associated with a myopic policy that consists in setting

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falls between the left and right values.

a Pigouvian subsidy on biogas while dairy emissions are not taxed. It corresponds to a situation in which the regulator does not take into account the rebound effect. Figure 5 depicts the evolution of welfare gains associated with the optimal second-best subsidy and a myopic subsidy. Not only the difference increases with the SCC but also the myopic policy eventually reduces Welfare. For value of the SCC larger than 350€/tCO$_2$ ignoring the rebound effect in a situation in which dairy emissions are not taxed reduces Welfare, while a small subsidy would increase Welfare.

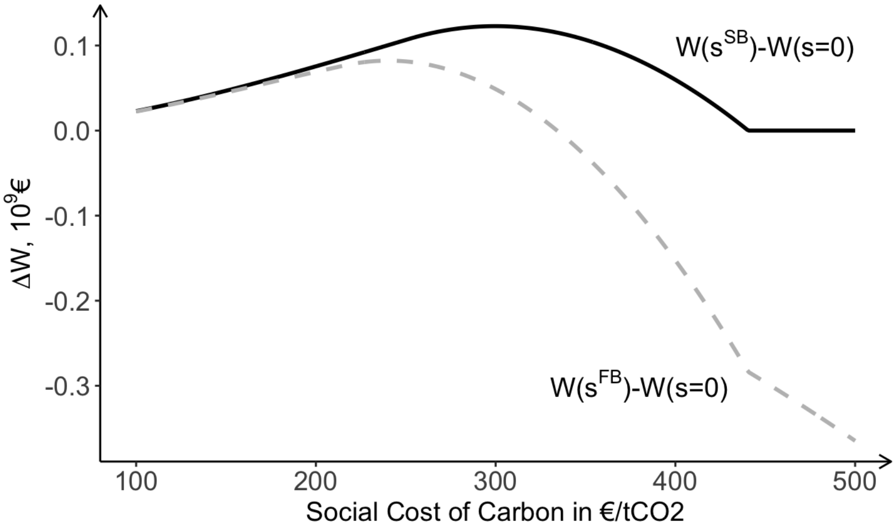

Figure 5: Evolution with the SCC of the welfare gains from two subsidies when dairy emissions are not taxed ($\tau = 0$): the optimal second-best subsidy s^{SB} (eq. (13)), and the first-best subsidy which does not account for the rebound effect.

6 Discussion

The model was kept as simple as possible in order to focus on the main trade-off at stake. Here, several limitations and associated path for future research are discussed:

First, a subsidy to bioenergy was considered while in reality other instruments could be used and most notably a blend mandate, as will be the case in France. The design of blend mandates as been discussed in a number of paper, notably for biofuels and the design of the Low Carbon Fuel Standard (De Gorter and Just 2009; Holland et al. 2009; Lemoine 2017, e.g.). In our context, a blend mandate is equivalent to a combination of a tax on natural gas and a subsidy to biogas which can dominate a subsidy if natural gas is imperfectly priced. However, it does not address the mispricing of emissions in the agricultural sector, and the rebound effect would still be present. As long as dairy emissions are not taxed, the design of a blend mandate should take into account emissions associated to the rebound effect.

Second, we considered GHG emissions directly associated with agricultural activities, and ignored both other externalities from farming practices and the carbon opportunity cost associated with land use change. Both would induce a higher external cost from farming and reinforce the need to carefully take into account the rebound effect when designing supports policies. Other externalities from dairy production include runoffs, air pollution, and biodiversity losses (see Funke et al. 2022, for a quantification). The carbon opportunity cost comes from the foregone opportunity to reforest, or aforest, the land used for farming. In a recent article, Gérard et al. (2024) analyze how it influences the efficiencies of second-best policies to regulate livestock. Their calibration suggest that the carbon opportunity cost is comparable to direct emissions from livestock.

Third, we only consider biogas generated by livestock manure and not by crop. In practice, both are mixed in the digesters to produce biogas. Given the criticism to the use of dedicated crops, the amount mixed is regulated, and subsidy to biogas in France depend on the share of manure. However it is difficult to monitor and verify practices. It would be interesting to add maize production to the model and analyze the interaction with the dairy market, maize could be used to both feed livestock or directly produce biogas, and both residues and manure could also be used to produce biogas. On top of these interactions, the design of subsidies, or blend mandates, should also take into account information asymetry and monitoring costs. Lankoski et al. (2010) is an inspiring work to analyze such issues.

7 Conclusion

Bioenergies from various feedstocks are subsidized in a number of countries in order to reduce GHG emissions by substituting fossil energy. Some pathways use a co-product of an agricultural good: crop residues, manure, or intermediate crops. Subsidies to these pathways may trigger an increase in the production of polluting activities, which would mitigates the environmental benefit awaited. We studied the design of subsidies to biogas when agricultural emissions are not regulated. First, we analyzed a general analytical model and drew some general conclusions. Second, we calibrate a specified version of the model with data from the French dairy sector and run numerical simulations.

In the general model, farmers produce both food and bioenergy, and the latter is a substitute for natural gas, which is polluting. Food and bioenergy are complementary in production: the marginal cost of bioenergy is decreasing with the quantity of food produced. That complementarity is at the root of a rebound effect: food production increases with the production of bioenergy. The first-best regulation consists of taxing emissions and subsidizing bioenergy if its production reduces farm emissions, which is the case for biogas produced from manure. If natural gas and agricultural emissions are not taxed at the Pigouvian level, the second-best optimal subsidy to bioenergy should integrate adjustment of food and natural gas production weighted by their unpriced emissions. The larger the rebound effect, the lower the optimal subsidy. With a quadratic specification, we show that the quantity of biogas is the same in the second-best and first-best situations. However, this quadratic specification can only be valid as a local approximation and ignores realistic constraints on biogas production.

We further explore a specification with heterogeneous farmers that we calibrate with the French dairy sector. We analyze the design of biogas subsidy produced from manure when dairy emissions are not taxed (second-best) and compare it to the optimum with properly taxed dairy emissions (first-best). Because of the rebound effect dairy production is larger in the second-best than in a laissez-faire situation without any regulation. The optimal subsidy should incorporate the rebound effect. For a SCC of 100€/tCO2, it amounts to a reduction of 10% compared to the Pigouvian level. The second-best quantity of biogas produced is larger than the first-best one. For larger values of the SCC, the gap between the first-best and second-best widens, the rebound effect increases as more and more farmers adopt methanization, and the second-best subsidy eventually decreases with respect to the SCC while biogas production plateaus. We further show that neglecting the rebound effect could have larger welfare costs when the SCC is large.

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Appendix

A Proof of Corollary 2

The Corollary derives from the following general result: For an economic benefit $B(q,e)$ (positive, increasing and concave) with q a quantity produced and $e(q)$ GHG emissions as a function of this quantity. Welfare is

$$B(q,e(q))-\delta e(q)$$

and the optimal q is increasing with respect to δ if and only if $e(q)$ is decreasing with respect to q.

To apply this to our setting: the good considered is biogas (so $q=Y_b$) and the two functions are

$$\begin{aligned} B(Y_b,e) &= U(X)+V(Y_f+Y_b)-C(X,Y_b)-C_f(Y_f)-\delta \\ e(Y_b) &= e_fY_f+e(X,Y_b) \end{aligned}$$

in which X and Y_f are functions of Y_b implicitly defined by market equilibrium equations (6a) and (6c).

B Proof of Proposition 2

The proof consists in showing that the optimal second best Y_b satisfies the same equation whatever the level of taxes τ and t. It is so because the effect of Y_b on X and Y_f does not depend on taxes in a quadratic setting.

If all functions are quadratic, Welfare is a quadratic function of X, Y_f and Y_b. Let us denote C_{XX} the second order derivative of the cost $C(.,.)$ with respect to X. For a given pair of taxes t and τ we write X and Y_f as function of Y_b and the tax: $X^e(\tau,Y_b)$ and $Y_f^e(t,Y_b)$, these functions are the solutions of equations (6a) and (6c) respectively. They are linear in both arguments. Their derivative with respect to Y_b being $-\rho_X$ and $-\rho_f$ respectively (with ρ_i given by (9)) and with respect to taxes:

$$\frac{\partial X^e}{\partial \tau}=-\frac{1}{C_{XX}-U''} \text{ and } \frac{\partial Y_f^e}{\partial t}=-\frac{1}{C_f''-V''} \tag{32}$$

With quadratic surplus and cost functions, together with linear emissions, Welfare can be written as

$$W\Big(X^e(\tau,Y_b),Y_f^e(t,Y_b),Y_b\Big)=W\Big(X^e(\delta e_X,Y_b),Y_f^e(\delta e_X,Y_b),Y_b\Big) \tag{33}$$

$$-\frac{1}{2}\frac{\big[X^e(t,Y_b)-X^e(\delta e_X,Y_b)\big]^2}{C_{XX}-U''} \tag{34}$$

$$-\frac{1}{2}\frac{\big[Y_f^e(t,Y_b)-Y_f^e(\delta e_X,Y_b)\big]^2}{C_f''-V''} \tag{35}$$

The second and third lines are equal to

$$\frac{1}{2}\frac{(\tau-\delta e_X)^2}{C_{XX}-U''} \text{ and } \frac{1}{2}\frac{(t-\delta e_f)^2}{C_f''-V''},$$

they only depend on the distance between taxes and their optimal levels, they do not depend on Y_b. Therefore, for any τ and t, as long as X and Y_f are non negative, the optimal Y_b is the unique solution of

$$\max_{Y_b} W\Big(X^e(\delta e_X,Y_b),Y_f^e(\delta e_X,Y_b),Y_b\Big).$$

C Characterisation of the aggregated cost function

Let us write the cost minimization problems for $Y < \alpha X$. The Lagrangian is, we drop the argument θ of individual production, to alleviate notations

$$\mathcal{L} = \int_{\underline{\theta}}^{\bar{\theta}} [c(x) + \theta y + \phi_\theta(y - \alpha x) - \psi_\theta y] f(\theta) d\theta \tag{36}$$

$$+ \lambda\Big[X - \int_{\underline{\theta}}^{\bar{\theta}} x f(\theta) d\theta\Big] + \mu\Big[Y_b - \int_{\underline{\theta}}^{\bar{\theta}} y f(\theta) d\theta\Big] \tag{37}$$

In which ϕ_θ is the Lagrange multiplier of $y(\theta) \leq \alpha x(\theta)$, ψ_θ is the Lagrange multiplier of $y(\theta) \geq 0$, λ the Lagrange multiplier of the constraint on aggregate milk production, and μ the one of the aggregate biogas production.

First order conditions

$$c'(x(\theta)) = \alpha\phi_\theta + \lambda \text{ and } \theta + \phi_\theta = \psi_\theta + \mu,\ \forall\theta \tag{38}$$

The comparison between θ and μ determine whether $y = 0$ ($\psi_\theta \geq 0$ and $\phi_\theta = 0$) or $y = \alpha x$ ($\psi_\theta = 0$ and $\phi_\theta \geq 0$). The former arise for $\theta > \mu$ and the latter for $\theta < \mu$. The threshold is then $\tilde{\theta} = \mu$ and

$$c'(x(\theta)) + \alpha\theta = \lambda + \alpha\mu \qquad \text{for } \theta \leq \mu \tag{39}$$

$$c'(x(\theta)) = \lambda \qquad \text{for } \theta \geq \mu \tag{40}$$

Note that $x(\theta)$ is continuous, there is no discontiuity at $\tilde{\theta}$, whereas $y(\theta)$ jumps down to zero at $\tilde{\theta}$. All farmers with type above $\tilde{\theta}$ produce the same quantity $c'^{-1}(\lambda)$.

The partial derivatives of C with respect to X and Y_b are λ and μ respectively. In order to retrieve the cross derivatives, take the full derivative of the two constraints with respect to X on aggregated production in equations (14):

$$1 = \int_{\underline{\theta}}^{\tilde{\theta}} \frac{\partial x}{\partial X} f(\theta) d\theta + \int_{\tilde{\theta}}^{\bar{\theta}} \frac{\partial x}{\partial X} f(\theta) d\theta \tag{41}$$

$$0 = \alpha \int_{\underline{\theta}}^{\tilde{\theta}} \frac{\partial x}{\partial X} f(\theta) d\theta + \alpha x f(\tilde{\theta}) \frac{\partial \tilde{\theta}}{\partial X} \tag{42}$$

Let us denote the derivatives of λ and μ with indexes : λ_X and μ_X. The first is the second-order derivative of the cost function with respect to X, and the second is the cross-derivative of the cost function. From the first order conditions the derivatives of $x(\theta)$ is

$$\frac{\partial x}{\partial X} = \frac{1}{c''} \begin{cases} \lambda_X + \alpha\mu_X & \text{for } \theta < \tilde{\theta} \\ \lambda_X & \text{for } \theta \geq \tilde{\theta} \end{cases} \tag{43}$$

so the equation above writes

$$1 = \int_{\underline{\theta}}^{\tilde{\theta}} \frac{1}{c''} f(\theta) d\theta \, (\lambda_X + \alpha\mu_X) + \int_{\tilde{\theta}}^{\bar{\theta}} \frac{1}{c''} f(\theta) d\theta \lambda_X \tag{44}$$

$$0 = \alpha \int_{\underline{\theta}}^{\tilde{\theta}} \frac{1}{c''} f(\theta) d\theta (\lambda_X + \alpha\mu_X) + \alpha x f(\tilde{\theta}) \mu_X \tag{45}$$

Let us denote by A, B and C the main factors:

$$A = \int_{\underline{\theta}}^{\tilde{\theta}} \frac{1}{c''} f(\theta) d\theta, \ B = \int_{\tilde{\theta}}^{\bar{\theta}} \frac{1}{c''} f(\theta) d\theta; \ C = x(\tilde{\theta}) f(\tilde{\theta})$$

The system of equations is

$$\begin{bmatrix} (A+B) & \alpha A \\ A & \alpha A + C \end{bmatrix} \begin{bmatrix} \lambda_X \\ \mu_X \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

and, denoting Δ the determinant of the matrix, the rebound effect is:

$$\rho_X = \frac{\mu_X}{\lambda_X - U''} = \frac{A}{\alpha A + C - U''\Delta} \tag{46}$$

Indeed, if $Y = \alpha X$, the rebound effect is equal to $1/\alpha$, which is the maximum it can be.

D Calibration of the cost and demand functions

In order to calibrate the parameter c_x of the cost function, we used the elasticity of milk production ϵ, P_{x0} and X_0 detailed in Table 1. Thus,

$$c_x = \frac{P_{x0}}{X_0^{\frac{1}{\epsilon}}} \tag{47}$$

The surplus functions are :

$$U(X) = a_x X - \frac{b_x}{2} X^2 \text{ and } V(Y) = a_y Y - \frac{b_y}{2} Y^2 \tag{48}$$

The calibration of the different parameters is obtained thanks to the definition of elasticity. We get :

$$a_x = P_{x0}(1 - \frac{1}{\epsilon_{xc}}) \text{ and } b_x = -\frac{P_{x0}}{\epsilon_{xc} X_0} \tag{49}$$

$$a_y = c_f(1 - \frac{1}{\epsilon_{yc}}) \text{ and } b_y = -\frac{cf}{\epsilon_{yc} Y_0} \tag{50}$$

E Parameters used for the numerical illustration

Parameter	Value	Meaning	Source
P_{x0}	$0,45$ €/kg of milk	BAU milk price on 12/20/2023	
X_0	$2,33.10^{10}$ kg of milk	Initial quantity	FranceAgriMer (2022)
Y_0	530 TWh	Total quantity of gas consumed in France	Ministère de la Transition énergétique (2021)
α	$0,425$ kWh/kg milk	The milk-to-energy ratio	Esnouf et al. (2021) Degueurce et al. (2016)
c_f	$0,040$ €/kWh	The marginal cost of production of natural gas per kWh	
$\underline{\theta}$	$0,040$ €/kWh	Minimum marginal cost of production of biomethane	ADEME and APCA (2022)
$\bar{\theta}$	$0,14$ €/kWh	Maximum marginal cost of production of biomethane	ADEME and APCA (2022)
ϵ	$0,6$	Price elasticity of milk supply	Bozic et al. (2012) and Colman et al. (2005)
ϵ_{xc}	$-0,6$	Price elasticity of milk consumption	Andreyeva et al. (2010)
ϵ_{yc}	-0.1	Price elasticity of gas consumption	Liu (2004)
e_1	$0,9$ kgCO2/kg milk	GHG emisions on a farm with methanization	Esnouf et al. (2021), Poeschl et al. (2012a) Dollé et al. (2011) and Gac et al. (2010)
e_2	1 kgCO2/kg milk	GHG emisions on a farm without methanization	Dollé et al. (2011) and Gac et al. (2010)
e_f	$0,227$ kgCO2/kWh	GHG emissions per kWh of natural gas	ADEME, Base Carbone
δ	100 €/tCO2	The social cost of carbon	

Table 5: Values of the parameters for the calibration