

Working Paper

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Carbon taxation in a global production network

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Abstract

Herein we study carbon taxation considering the structure of the global production network. With this purpose we characterize how the implementation of a carbon tax in one country-sector can generate sizable fluctuations on global emissions and welfare through its impact on the structure of production. We then apply this theoretical characterization to accommodate the structure of a multi-regional input-output database. This framework allows us to identify the country-sectors that should be taxed to reach the strongest potential for emission reduction (or welfare maximization) if no coordinated policy is possible. Interestingly, this choice not only depends on emission intensity but also on to which extent the sector is central in the global production network as well as on the pass-through effect on public or private spending. Additionally, we find that synergies between taxes applied to different country-sectors have a strong impact in emission reductions, calling for greater harmonization in carbon taxation around the world. We then use our model to simulate the impact of the European Carbon Border Adjustment Mechanism (CBAM) finding that, when looked into sector by sector, it reduces EU competitiveness loss due to carbon pricing but, when generalized to all EU sectors, the impact through the value chain ends up provoking a stronger contraction in the EU than without the CBAM.

Keywords: Carbon taxation, networks, global production, environmental policy, CBAM. (JEL Codes: B22,P18,P23)

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1. Introduction

Damages caused by green-house-gas (GHG) emissions are global but carbon taxes and prices are applied locally in a uncoordinated way. Herein, we study the impact that these local taxation policies may have in the globally integrated production network, and through it, on global emissions and welfare. We model a production network of heterogeneous producers who interact through the global input-output value chain. They are based in different regions, each with its own type of consumer and government spending, carbon taxation policy and labor market. In our case, a region is either a country, like the US or China, or a group of countries, like the European Union. We then calibrate the model with a multi-regional input-output database called Exiobase3.

We show that sectors facing a direct carbon tax reevaluate their usage of inputs to achieve cleaner production and that in an integrated production chain, the initial shift in costs and input usage has relevant upstream and downstream effects on global production and emissions. Moreover, the change in production structure will shift household income across regions, altering the structure of household demand for products. Similarly, the carbon tax increases local governments' income, changing its demand which in turn also impacts the production network, emissions and welfare. Carbon taxes may therefore have sizable impacts on the production network depending on which country/sector is taxed and on its degree of centrality in the network. This also determines the impact in terms of emission reduction and welfare. The interaction between the previous effects may lead to surprising results: global emissions decrease sharply when very interconnected sectors are taxed (e.g. extraction of crude oil in the Middle East) but, when the tax is applied to poorly interconnected sectors (e.g. electricity produced with coal in China), even if very polluting, emissions increase instead.¹

Additionally, if we consider the interaction between carbon taxation on two distinct regions/sectors, we find that impacts on emission reduction through the network considerably amplify, calling for a world-wide carbon tax. The use of Exiobase3 allows us to individually quantify the impact of carbon taxes on different sectors and regions in the world. It also allows us to simulate the impact of the European Carbon Border Adjustment Mechanism (CBAM) that will enter into force in October 2023. In this regard we find that, when looked into sector by sector, the CBAM reduces the loss of European competitiveness due to carbon pricing but, when the CBAM is simultaneously applied to all polluting sectors exporting to the EU, the contraction synergies through the value chain end up provoking a bigger decline inside the EU than without the CBAM.

The article is structured as follows. In Section 2 we review the related literature. In Section 3 we present the theoretical model describing the behavior of agents in each country and underlying our theoretical contribution to the analysis of synergies between carbon policies. In Section 4 we present the data we use to simulate the world network of interactions: Exiobase 3. In Section 5 we compare the emissions reduction and welfare

¹We show this is due to the increase in the Chinese government's revenue that in turn increases demand for other polluting goods.

impacts of alternative carbon taxes. In particular, we compare the impact of taxing only most polluting sectors versus several sectors based on their use of polluting inputs and connectedness. In Section 6 we use our framework to discuss the impact of the CBAM in European polluting sectors. Finally, in Section 7 we conclude.

2. Literature Review

Our work is first of all related to the literature on production networks. Networks can help us understand how idiosyncratic micro-economic shocks may lead to sizeable aggregate fluctuations. In a context where the economy is formed by millions of agents, if it is possible to write aggregate quantities as a weighted average of individual-level data, the law of large numbers implies that idiosyncratic shocks to individual units should average to zero with near-certainty (Hulten, 1978).² After the 2008's global financial crisis, numerous papers have challenged Hulten's findings. This is the case of Gabaix (2015), who emphasizes the properties of the distribution of sales at the firm level and Acemoglu et al. (2012), that using a Cobb-Douglas model in the line of Long and Plosser (1983) show that, even with the theorem of Hulten in place, individual weights may be significantly higher than zero. Particularly, Acemoglu et al. (2012) use a network structure for defining the equilibrium sales of firms. In this setup, sectors that have a central position in the network have a higher weight than peripheral ones. Interestingly, Acemoglu et al. (2012) show that taking into account the interconnected production structures, it is not only possible to determine the aggregate effects, but it is also possible to characterize the co-movements between sectors and firms along a process that is non-symmetrical. Moreover, several papers have shown that the presence of frictions can cause Hulten's theorem to fail (e.g. Baqaee, 2018, Bigio and La'O, 2016, Baqaee and Farhi, 2019). Particularly, Baqaee and Farhi (2019) show that Hulten's theorem does not hold beyond the first order effects of the shock. Methodologically, this last paper is close to ours since we provide a reduced form characterization of the second-order effects of carbon taxation, which allow us to study the effect that the interaction between taxes imposed in two different regions/sectors has on the production network and global emissions. The closest contribution in terms of the application is King et al.(2019) since they use Acemoglu et al. (2012) to characterize the first-order effect of sectoral carbon taxation in a production network located entirely in a single region (closed economy). Since they only consider first-order effects in a model without government heterogeneity, they find that a sector targeted policy is more effective for reducing emissions than an economy-wide carbon tax. Our results challenge this finding.³

²Hulten's (1978) theorem states that, if the equilibrium is efficient, the first-order impact on aggregate output of a factor productivity shock in a single sector is equal to the sector's share in aggregate output. Hulten's theorem gives a justification for overlooking the importance of micro-economic and network structures in the determination of aggregate fluctuations.

³Other papers have also used production networks to simulate the impact of carbon pricing on a single country. This is the case of Devulder and Lisack (2020) for France and Cavalcanti and Fernandes (2023) for Brazil.

Since we assess emissions reductions through the whole value chain due to carbon taxation, our paper is related to the literature focused on global emission accounting, trying to discern at which step of the Input-Output (IO) value chain should emissions be accounted for (and taxed) as well as its implications for the calculation of border carbon taxation, like the CBAM in Europe. It is worth clarifying that, the position of each country-sector in the global value chain is given by its usage of inputs (and also how its product is used as an input). The global Input-Output matrix shows that some sectors use (or are used as) inputs from numerous countries while some others are only interlinked locally with few other sectors.⁴ Nowadays, emissions are usually accounted for and taxed where polluting inputs are used (burned), although these inputs may be extracted somewhere else and be used to produce consumption goods that are also used elsewhere.⁵ Davis et al.(2015) use a multi-region IO model to characterize carbon inventories and argue that if a consistent and unavoidable price is imposed on CO_2 emissions somewhere through the supply chain, then all of the parties along the chain seek to impose that price to generate revenue from taxes collected.⁶ Related to the previous, other papers use IO models to quantify the pass-through of carbon taxation. Perese (2010) uses an IO model to estimate the effect of a 20\$ tax per metric ton of CO_2 in the US economy. His results (first-order effects) show that energy commodities such as natural gas, electricity and gasoline will experience price increases of approximately 10%. The study also shows that the distribution of the policy effects across sectors depends on the relative price increases and the mix of commodities consumed in each sector. Also for the US, Ho (2008) identifies a set of domestic industries that suffer particularly from an economy-wide tax of $10/10^{10}$ of CO₂ without accompanying border adjustment taxes. Zhang (2019) evaluates the economy-wide effects of carbon taxation in

⁴This is a given in our setting while other authors have, instead, studied the reasons for such interlinkages (see for example Bolela et al, 2022 and the papers cited therein).

⁵This discussion has major implications for assigning the share of polluting responsibility to different countries or economic agents. For example, a significant portion of Chinese emissions is due to the production processes of goods exported to other countries. There has been considerable literature on the appropriateness of assigning responsibility to (and consequently taxing) the polluting producers or the consumers of polluting goods. A non-exhaustible list of relevant papers on the subject are Lenzen, 2007, Lenzen et al., 2007, Lenzen, 2008, Lenzen and Murray, 2010 and, more recently, Temursho and Miller, 2020. Herein only producers' emissions are taxed. Those taxes produce, firstly, a change in relative usage of inputs and consequently a change in relative demand for workforce in different countries where goods are produced, which in turn change the countries representative consumers' income and consumption capabilities. Secondly, the tax revenue for the country that imposed it allows the government of such country to increase its public expenditure. The induced changes in private consumption patterns and public expenditure due to the carbon tax in turn have an effect on countries' emissions. Even if this is the case, differently from the referred literature, herein we only consider the polluter as responsible and we then study how, when taxed, (public and private) consumption change, inducing further changes in emission.

⁶Davis et al.(2015) also suggest that the geographical concentration of carbon-based fuels and the existence of a small number of parties involved in extracting and refining those fuels indicates that upstream regulation at the wellhead, mine mouth, or refinery might minimize opportunities for leakage. O'Rourke (2014) argues that the difficulties in measuring and managing these impacts can be partially explained by challenges in translating data into relevant information for decision-makers inside corporations and government agencies. Herein we contribute in this regard, by making available a model that can be used to simulate alternative taxation strategies.

China. The authors build a price model based on an hybrid energy IO table (quantities in real and nominal terms). The results indicate that carbon taxation has a small negative impact on GDP but generates substantial emissions reductions.⁷

A recent literature has emerged using IO models to analyze the design of the CBAM. Bellora and Fontagné (2023) simulate an IO model with an endogenous carbon price. They show that the CBAM is effective in reducing carbon leakage but competitiveness losses are expected in export markets for downstream sectors that are not covered by the CBAM, as well as for European exporters of high-emitting industries, even in the presence of rebates. Instead, Ernst et al. (2023) find that a border adjustment tax mitigates (but does not prevent) carbon leakage, its impact on emissions is limited, and it mainly "protects" dirty domestic production sectors of tradeable goods. Even if in a very different framework, our results for high-emitting industries are in line with Bellora and Fontagné (2023) and at odds with Ernst et al. (2023).

Finally, our paper is related, even if more from afar, with papers focusing on the social and rent distribution effects of carbon pricing. Using a model of global consumer demand and production value chains parameterized using trade data, Sager (2019) finds that a global uniform carbon price is globally regressive while within-country effects may be moderately progressive. Grainger and Kolstad (2010) use the 2003 US Consumer Expenditure Survey and emissions data from an input-output model to estimate the impact of carbon pricing finding that targeting CO_2 from energy consumption is more regressive than targeting all emissions. Mathur and Morris (2012) show that a \$15 carbon tax imposed in 2010 on fossil fuels is regressive. Renner (2018) offers a detailed view of the potential welfare effects of carbon tax scenarios for Mexico, finding that the distributional effects are slightly progressive when focused on CO_2 as compared to other pollutants. In relation to this literature, herein we are examine the effects of carbon pricing on consumers in different countries. For example, our results show that consumers in China would suffer the most after the application of a global carbon tax. We also perform a welfare analysis of carbon taxation.

All in all, herein we extend the model in King et al. (2019) in several important ways. First, we study second-order effects \dot{a} la Barquee and Farhi (2019), which proves the importance of the international coordination of carbon policies. Second, we consider a multi-regional framework with heterogeneous agents in those regions. Third, we calibrate this multi-regional multi-sectoral model with an international database. Our model allows us to study interaction between carbon taxes applied in different sectors/countries as well as the importance of heterogeneity. To the best of our knowledge we are the first to model a global multi-regional network accounting for heterogeneity of governments applying carbon taxes as well as heterogeneous labor markets and consumers. The fact of also being the firsts to apply this multi-regional model using Exiobase (see Stadler et al., 2018 for a detailed

⁷This issue has also been studied using a dynamic computable general equilibrium model (CGEM) by Shi (2019). He explores the impacts of different carbon tax conditions on the energy usage of the construction sector (and the whole economy) in China. Results show that the appropriate carbon tax is 60 /t, which cannot only achieve the emission reduction target but also minimize the negative macroeconomic impact.

description of the database) allows us to make important contributions in terms of policy recommendations regarding specific sectors and countries, as well as on the importance of the interactions. In this context we are able to single out winners and losers when a policy like the CBAM is put in place and to even study the impact of some central sectors in carbon accounting. In particular, King et al.(2019) find that a targeted carbon tax to a very polluting sector may outperform a global carbon tax. Herein we show that the previous result depends on the fact that their model disregards synergies between carbon taxes applied in different sectors/countries. Those synergies may indeed be very important, calling for a global coordination in the implementation of carbon taxation.

3. Theoretical model

In this section, we introduce the main features of our general equilibrium model with different regions, each one with its independent carbon tax policy, which may tackle one or more sectors, its labor market, its government and its household consumption profile. In this sense, and differently from King et al.(2019), we introduce the role of government expenditures, which in the data differs significantly from household consumption.⁸ Moreover, we explore the second order effects for a better characterization of the network dynamics, which proves to open a new dimension for analysis, characterizing the interaction between policies applied to different sectors in different countries.

Let us consider R different regions, each one with its own government and representative household and its own labor market (with its own equilibrium wage). In each region there are q sectors, each one producing an internationally tradable good that can be purchased by other sectors as an input, by households, and by governments. All in all, we model n = R * q sectors.⁹

Firms

Each of the *n* region-sectors produces following a Cobb-Douglas with constant returns to scale that combines the usage of labour L_i bought inside the region and a bundle of the *n* goods bought from all around the world. X_{ij} is the intermediate usage of good *j* by sector *i*. Let us write it as follows:

$$X_{i} = A_{i} L_{i}^{w_{iL}} \prod_{j}^{n} X_{ij}^{w_{ij}}.$$
 (1)

⁸We will see that an euro of government expenditure provokes less emissions than a euro of household consumption for all the regions in our database with the exception of China. Government expenditure provoked emissions per euro are especially high in China, then India and finally Russia. In the case of China (and Russia to a lesser extend), high emissions' intensity of government expenditure can be traced back to a significant expenditure in emission-intensive sectors such as construction.

⁹Each sector i modeled hereafter describes a sector in a specific region (which can be a country or a group of countries). We have therefore n country-sectors. This means that, for example, the Iron-and-Steel sector in region a can have a very different input usage (and resulting emissions) than the Iron-and-Steel sector in region b.

For now, we set $A_i = 1$ although to explore the role of technology is an interesting path for future research.

Emissions released by sector *i* are a function of *i*'s input choice and of the emission's intensity of these inputs, given by the parameter β_{ij} and can be described as:

$$E_i = \psi_i + \sum_j^n \beta_{ij} X_{ij},\tag{2}$$

where ψ_i is a parameter that captures sector's *i* technological characteristics unrelated with the input bundle of choice. It is worth noting that sector *i* in region *r* may use a very different technology from the same sector in region *k*, meaning that w_{ij} may be very different across regions and the subindex *i* could be simply considered as an index for each of the n country-sectors modelled.

To better understand the general implication of this equation let us use the example of the Iron-and-Steel sector. Its emissions come from either burning coal, oil or gas in a regular furnace or from electricity in an electric arc furnace. Depending on the mix used by each country's iron-and-steel sector, they will emit more or less. Additionally, for a more general specification, the parameter ψ_i captures emissions unrelated to the mix of inputs. This could be related to overall efficiency, for example.

Sector i maximizes the following profit function (we omit sub-indexes r for simplicity but labor used is from its own region while input usage can be international):

$$\max_{X_{ij}} P_i[A_i L_i^{w_{iL}} \prod_j^n X_{ij}^{w_{ij}}] - S_{r_i} L_i - \sum_i^n P_j X_{ij} - \tau_i [\psi_i \prod_j^n X_{ij}^{\beta_{ij}}],$$
(3)

where P_i is the price for sector i, S_{r_i} is the wage in i's region and τ_i stands for the carbon tax applied to this sector in its region.

Then, profit maximization problem defines optimal input usage from the $FOC_{X_{ij}}$ as follows:

$$X_{ij} = \frac{w_{ij}P_iX_i - \tau_i\beta_{ij}E_i}{P_j}.$$
(4)

Consumer

At a given region r, the representative consumer's utility depends on the consumption bundle of n goods sold internationally.¹⁰ The representative consumer at region rmaximizes its utility subject to its budget constraint. The utility maximization problem is the following:

$$\max_{C_{ri}} \prod_{i}^{n} C_{ri}^{\gamma_{ri}} + \lambda_C [H_r - \sum_{i}^{n} P_i C_{ri}],$$
(5)

¹⁰Let us underline that γ_{ri} are different across regions. Consumers in different regions differ in their preference for goods coming from different origins. Precisely, American consumers may prefer beef to fish and, among beef, they may prefer American beef to imported beef.

where H_r is the household income in region r. The household inelastically supplies L_r units of Labour and receives a competitive wage S_r . Hence, $H_r = L_r S_r$. Solving the $FOC_{C_{ri}}$ defines optimal household consumption as:

$$C_{ri} = \gamma_{ri} \frac{H_r}{P_i}.$$
(6)

Government

Each government has the ability to impose a carbon tax to the q = n/R sectors producing under its jurisdiction and then spend the tax revenue on goods according to its own preferences. The government revenue Υ_r is:

$$\Upsilon_r = \sum_j^n \tau_j E_j \{ j \in \mathbf{r} \},\tag{7}$$

with \mathbf{r} being the set of sectors belonging to the jurisdiction of the government in region r. The maximization of the government's utility can be therefore expressed as:

$$\max_{G_{ri}} \prod_{i}^{n} G_{ri}^{\phi_{ri}} + \lambda_G [\Upsilon_r - \sum_{i}^{n} P_i G_{ri}], \tag{8}$$

and it defines optimal government expenditure as:

$$G_{ri} = \phi_{ri} \frac{\Upsilon_r}{P_i}.$$
(9)

It is worth noting that, as it was the case for consumers, ϕ_{ri} allows governments across countries to have very different expenditure patterns as well as to prefer buying from specific origins. This allows our model to capture home-bias and product differentiation.

3.1. Equilibrium in the goods and labor markets

The clearing market condition for good i is $X_i = C_i + G_i + \sum_{j=1}^{n} X_{ji}$. In the Appendix 9.1 we show that introducing the FOCs from the different agents into this market clearing condition gives:

$$P_i X_i = \sum_{j=1}^{n} \ell_{ij} \left[\sum_{r=1}^{R} \gamma_{rj} H_r + \sum_{m=1}^{n} (\sigma_{jm} - \beta_{mj}) \tau_m E_m \right], \tag{10}$$

where ℓ_{ij} is the *ijth* element of the Leontief matrix $L = [I - W^T]^{(-1)}$ where W is a matrix of dimensions NxN whose *i*, *jth* element is the elasticity parameter w_{ji} .

The Leontief matrix defines aggregate output on the intersectoral network of production. Its elements have the following alternative interpretations: (i) The i, th element of the matrix reflects how much good i must be produced in order to serve 1 nominal unit of final demand of good j. This coincides with the interpretation of the Leontief matrix in Input-Output analysis, although in our case the matrix is constructed from a set of n^2 elasticity parameters from the firms production functions; (ii) The i, jth element reflects the upstream transmission of shocks from the sectors i and j through direct connections and through intermediary agents or hubs; (*iii*) The sum $\sum_{j}^{n} \ell_{ij}$, is also known as the Bonacich centrality and it reflects the position of sector i in the network. Network centrality measures are vital tools for understanding networks since they provide information on the importance of a given node in a network structure. Bonacich (1987) proposes a measure of centrality as a function of the connections of the nodes in one's neighborhood. Particularly, the centrality of each node is determined by the centrality of the nodes it is connected to. In the literature on production networks, for example in Acemoglu et al.(2012), it characterizes the degree by which an idiosyncratic shock to sector i is able to generate a sizeable fluctuation of aggregate outcome. Herein, since our model is more complex than the one in Acemoglu et al. (2021), a more complex version of the Bonacich centrality emerges featuring combinations of the Leontief matrix and other elements of the model such as the structure of final demand with heterogeneous agents.

The first term in equation (10), *i.e.* $\sum_{j}^{n} \ell_{ij} \sum_{r}^{R} \gamma_{rj} H_r$, is actually a version of the Bonacich centrality where the elements of the Leontief matrix are weighted by the structure of worldwide final demand by households. Hence, in the absence of taxes, sales in equilibrium for sector *i* depend on the combination of the structure of final demand and on the structure of the intersectoral network of production (and the position of the sector *i* in it).¹¹

The second half of equation (10), *i.e.* $\sum_{j}^{n} \ell_{ij} \sum_{m}^{n} (\sigma_{jm} - \beta_{mj}) \tau_m E_m$, reflects how taxes on emissions shape sector sales. Again it is a weighted version of the Bonacich centrality featuring two distinct components. Firstly, $\sum_{m}^{n} \sigma_{jm} \tau_m E_m$ reflects the change in government expenditure generated from carbon tax revenue. Hence, $\sum_{j}^{n} \ell_{ij} \sum_{m}^{n} \sigma_{jm} \tau_m E_m$ reflects sector sales as a combination of production network centrality and the structure of final demand. Secondly, $\sum_{m}^{n} \beta_{mj} \tau_m E_m$ reflects the change of intermediary demand of good j when the sectors shocked by a tax reevaluate their input bundle in order to reduce their emissions. Then $\sum_{j}^{n} \ell_{ij} \sum_{m}^{n} \beta_{mj} \tau_m E_m$ reflects sector sales as a combination of production network centrality and the structure of intermediate demand.

If we consider $\tau = 0$, equation (10) can be rewritten as:

$$X_i^* = \frac{\sum_j^n \ell_{ij} \sum_r^R \gamma_{rj} H_r^*}{P_i^*},\tag{11}$$

and therefore, we can also formulate it as:

$$X_{ij}^{*} = \frac{w_{ij} \sum_{j}^{n} \ell_{ij} \sum_{r}^{R} \gamma_{rj} H_{r}^{*}}{P_{j}^{*}}.$$
(12)

Let us now analyze the equilibrium in the R different labour markets, considering the hourly wage in the United States as numeraire.

¹¹There is an important difference between our case and previous literature. For example in Acemoglu et.al. (2012), the utility of the representative consumer is simply $\prod_{i=1}^{n} C_{i}^{\frac{1}{n}}$ and therefore the elasticity parameter of the utility function is 1/n for all products. As a result, sales in equilibrium depend solely on the Bonacich centrality.

The optimal demand for labor in sector i of region r satisfies:

$$l_i = \frac{w_{il} P_i X_i}{S_{r_i}}.$$
(13)

The market clearing condition in this region is $\sum_{i=1}^{n} l_i = L_r \{i \in \mathbf{r}\}$, where L_r is constant in each region r.

We then obtain the labor market clearing conditions:

$$S_{r}^{*} = \frac{\sum_{i}^{n} w_{il} P_{i}^{*} X_{i}^{*} \{i \in \mathbf{r}\}}{L_{r}};$$
(14)

$$H_{r}^{*} = \sum_{i}^{n} w_{il} P_{i}^{*} X_{i}^{*} \{ i \in \mathbf{r} \}.$$
(15)

Following previous literature, the labor supply in each country is assumed to be constant, capturing the fact that the available labor force in each country will not significantly vary (by massive migration, for example) following a short term change in wages. Wages, instead, are set in a competitive way and play a significant role in the results: when carbon taxes generate a shift in production across the globe, this shift in production will be accompanied by a redistribution of household income through wage variations. In turn, this changes purchase power of the representative consumer in each country.

Similarly, let us now define equilibrium prices for good *i* when $\tau = 0$ as:

$$ln(P_i^*) = \sum_{j}^{n} \ell_{ij}^{WT} [j_j + w_{jl} ln(S_{r_j})],$$
(16)

where ℓ_{ij}^{WT} is the *ijth* element of the matrix $[I - W^T]^{(-1)}$ and captures the downstream transmission of shocks through the production network. Then, $\sum_{j}^{n} \ell_{ij}^{WT}$ characterizes the position of centrality of sector *i* as a purchaser of inputs.

Also, we define j_i such that $j_i = -w_{il}ln(w_{il}) - \sum_j^n w_{ij}ln(w_{ij})$, (see Appendix 9.1 for detail). As a result, the equilibrium price is a function of the input usage and wages.

3.2. Impact of carbon taxes on the production network and emissions

In what follows we show how the impact of a carbon tax imposed to a given sector spreads to other sectors and across regions.

First order policy effects

The first-order derivative of region-sector i's emissions in response to a carbon tax to region-sector k is defined through the change in the sector i's input bundle, deriving equation (2) where we assume for simplicity that $\psi_i = 1$:

$$\frac{\partial ln(E_i)}{\partial \tau_k} = \sum_{j}^{n} \beta_{ij} \frac{\partial ln(X_{ij})}{\partial \tau_k}.$$
(17)

The impact of a carbon tax to sector k on sector i's emissions is defined by the linkages through the production network that are present between those two sectors, independently of whether they are in the same region or not. Hence, changes in global emissions are defined in the model through adjustments in input usage for all sectors in the global production structure. In Appendix 9.1 we show that deriving the log-linarized version of (4) defines input substitution for a given sector. The following equation characterizes the change in the use of input j by producer i due to the carbon taxation in sector k:

$$\frac{\partial ln(X_{ij})}{\partial \tau_k} = \begin{cases} \frac{\partial ln(P_iX_i)}{\partial \tau_k} - \frac{\partial ln(P_j)}{\partial \tau_k} - \frac{\beta_{ij}E_i^*}{w_{ij}P_i^*X_i^*}, & \{i=k\}\\ \frac{\partial ln(P_iX_i)}{\partial \tau_k} - \frac{\partial ln(P_j)}{\partial \tau_k}. & \{i\neq k\} \end{cases}$$
(18)

The component $-\frac{\beta_{kj}E_k^*}{w_{kj}P_k^*X_k^*}$, characterizes the initial input substitution in the sector targeted directly by the tax. This derivative component will be 0 for all sectors not directly impacted by the policy (second line in 18). The sector taxed, i.e. $\{i = k\}$, reevaluates its input usage in order to optimally reduce its emissions.¹²

Once the sector subject to the tax has re-optimized its input bundle, the effects of this original adjustment will be transmitted through the global production chain and on demand. The impact in equation (10) that a tax on k has on i's sales (see Appendix 9.1 for the intermediate steps) is:

$$\frac{\partial P_i X_i}{\partial \tau_k} = \sum_{j}^{n} \ell_{ij} \left[\sum_{r}^{R} \gamma_{rj} \frac{\partial H_r}{\partial \tau_k} + (\sigma_{jk} - \beta_{kj}) E_k^* \right].$$
(19)

Equation (19) shows that the impact is a combination of the impact in the structure of the production network, which is given by ℓ_{ij} for all j and constructed from the NxNmatrix of elasticity parameters W, and the change in the three components of demand: (i) the first shows the change in final demand by consumers determined by change in household income H_r , since its size will depend on the consumer preference parameter γ_{rj} for each region r and product j; (ii) the second is the change in government's demand that depends on tax revenue E_k and the government's expenditure preference parameters σ_{rj} for all product j; and, (iii) the change in the intermediate demand by sector k that depends on the sector's emission elasticities β_{kj} for the different inputs j. The second component is only relevant for the government that introduces the tax.

Another way to look at it is to consider that the component $-\sum_{j}^{n} \ell_{ij}\beta_{kj}E_{k}^{*}$ in equation (19) defines the upstream change in sector sales generated by direct input substitution in the sector receiving the tax (sector k); the component $\sum_{j}^{n} \ell_{ij}\sigma_{jk}E_{k}^{*}$ identifies the change in sector sales that originates from the income that goes to governments in each region, which may differ significantly (differences in σ_{rj}), and finally, $\sum_{j}^{n} \ell_{ij} \sum_{r}^{R} \gamma_{rj} \frac{\partial H_{r}}{\partial \tau_{k}}$ characterizes the

¹²This policy impact in equation (18) shows input substitution in sector k as the product of the ratio of elasticities and emissions intensity in sector k: $\frac{\beta_{kj}}{w_{kj}} \times \frac{E_k}{P_k^* X_k^*}$, that can also be rewritten as $\frac{\partial E_k}{\partial X_{kj}} \frac{1}{P_j^*}$. This means that the input substitution of input j by k depends on how pollutant the input is and on the input's price.

effects of shifting household demand on sector sales, that may also differ across regions due to difference in preferences. It is worth noting that, if a government with high emission intensity expenditures introduces a tax, the intended effect on emissions may be offset by higher government expenditures in polluting goods. As we will show in the empirical exercise hereafter, this is the case for China.

Let us now study the impact of carbon taxation in sector k on input price in sector i. Deriving equation (16) we find:

$$\frac{\partial ln(P_i)}{\partial \tau_k} = \sum_{j}^{n} \ell_{ij}^{WT} w_{jl} \frac{\partial ln(S_{r_j})}{\partial \tau_k} + \ell_{ik}^{WT} \sum_{m}^{n} \beta_{km} \frac{E_k^*}{P_k^* X_k^*}.$$
(20)

The first component, $\sum_{j}^{n} \ell_{ij} w_{jl} \frac{\partial \ln(S_{r_j})}{\partial \tau_k}$, shows the impact that the tax on sector k has on the regional wages weighted by the sector's position in the production network. This is the case because, as before, L^{WT} is the Leontief matrix $L^{WT} = (I - W^{(T)})^{(-1)}$ whose centrality measure $\sum_{j}^{n} \ell^{WT}$ should be interpreted from an input purchaser (or cost past-though) perspective.

The second component, $\ell_{ik}^{WT} \sum_{m}^{n} \beta_{km} \frac{E_k}{P_k^* X_k^*}$, shows how the increased cost for input k spreads downstream through the production network to the price of input i.

Let us now move to the taxation's impact on regional salary and home expenditure. Departing from equation (14) and (15) respectively, at a given region r we find:

$$\frac{\partial S_r}{\partial t_k} = \frac{\sum_{j}^{n} w_{jl} \frac{\partial P_j X_j}{\partial t_k}}{L_r} \{ j \in \mathbf{r} \},$$
(21)

$$\frac{\partial H_r}{\partial t_k} = \sum_{j}^{n} w_{jl} \frac{\partial P_j X_j}{\partial t_k} \{ j \in \mathbf{r} \},$$
(22)

where we only consider the subset of sectors that are inside region r.

Solving the system characterized by equations (18), (19), (20) and (21) we obtain the impact of taxation $\frac{\partial ln(X_{ij})}{\partial t_k}$ in terms of parameters and variables in equilibrium (see details in Appendix 9.1). The subscripts $\{j, m, q, b, d\}$ define different sectors that interact within the dynamics of the model:

$$\frac{\partial ln(X_{ij})}{\partial t_{k}} = \begin{cases} \sum_{j}^{n} \Lambda_{ij} \sum_{m}^{n} \ell_{jm} (\sigma_{mk} - \beta_{km}) \frac{E_{k}^{*}}{P_{i}^{*} X_{i}^{*}} - \sum_{m}^{n} \ell_{jm}^{WT} w_{ml} \sum_{q}^{n} \frac{w_{ql}}{L_{rm}} \sum_{b}^{n} \Lambda_{qb} \\ \sum_{d}^{n} \ell_{bd} (\sigma_{dk} - \beta_{kd}) \frac{E_{k}^{*}}{S_{rm}^{*}} - \ell_{jk}^{WT} \sum_{m}^{n} \beta_{km} \frac{E_{k}^{*}}{P_{k}^{*} X_{k}^{*}} - \frac{\beta_{kj} E_{k}^{*}}{w_{kj} P_{k}^{*} X_{k}^{*}}, \qquad \{i = k\} \\ \frac{\sum_{j}^{n} \Lambda_{ij} \sum_{m}^{n} \ell_{jm} (\sigma_{mk} - \beta_{km}) \frac{E_{k}^{*}}{P_{i}^{*} X_{i}^{*}} - \sum_{m}^{n} \ell_{jm}^{WT} w_{ml} \sum_{q}^{n} \frac{w_{ql}}{L_{rm}} \sum_{b}^{n} \Lambda_{qb} \\ \sum_{d}^{n} \ell_{bd} (\sigma_{dk} - \beta_{kd}) \frac{E_{k}^{*}}{S_{rm}^{*}} - \ell_{jk}^{WT} \sum_{m}^{n} \beta_{km} \frac{E_{k}^{*}}{P_{k}^{*} X_{k}^{*}}, \qquad \{i \neq k\} \end{cases}$$

where Λ_{ij} is the *ij*th element of the NxN matrix $\Lambda = (I - L\Gamma \Xi W_L)^{(-1)}$ and therefore Λ_{ij} characterizes sales at *i* through private consumption embodied in the sales at *j*. All in all, equation (23) defines in terms of parameters and equilibrium values the different dynamics and policy effects described along this section.

Second order policy effects

We now characterize the second-order effects of carbon taxing. The importance of modelling these effects are twofold. First, we study second-order effect to be able to characterize the synergies that emerge when a carbon tax is applied at two distinct points of the production structure.¹³ Second, using a second order Taylor expansion it is possible to approximate the policy effects along the tax curve beyond $\tau = 0$.

The impact on emissions of taxing sector k and sector q due to a change in input usage is defined as:

$$\frac{\partial ln(E_i)}{\partial \tau_k \tau_q} = \sum_{j}^{n} \beta_{ij} \frac{\partial ln(X_{ij})}{\partial \tau_k \tau_q}.$$
(24)

To develop $\frac{\partial ln(X_{ij})}{\partial \tau_k \tau_a}$ we derive equation (23).¹⁴ The resulting expression is:

$$\frac{\partial ln(X_{ij})}{\partial \tau_{k}\tau_{q}} = \begin{cases} \sum_{j}^{n} \Lambda_{ij} \sum_{m}^{n} \ell_{jm} (\sigma_{mk} - \beta_{km}) \frac{\frac{\partial E_{k}}{\partial \tau_{q}} P_{i}^{*} X_{i}^{*} - \frac{\partial P_{i} X_{i}}{\partial \tau_{q}} E_{k}^{*}}{(P_{i}^{*} X_{i}^{*})^{2}} - \sum_{m}^{n} \ell_{jm}^{WT} w_{ml} \frac{\frac{\partial P_{k} X_{k}}{\partial \tau_{q}} S_{r_{m}} - \frac{\partial S_{r_{m}}}{\partial \tau_{q}} P_{k}^{*} X_{k}^{*}}{(P_{i}^{*} X_{i}^{*})^{2}} - \ell_{jk}^{WT} \sum_{m}^{n} \beta_{km} \frac{\frac{\partial E_{k}}{\partial \tau_{q}} P_{k}^{*} X_{k}^{*} - \frac{\partial P_{k} X_{k}}{\partial \tau_{q}} E_{k}^{*}}{(P_{k}^{*} X_{k}^{*})^{2}} - \frac{\beta_{kj}}{w_{kj}} \frac{\frac{\partial E_{k}}{\partial \tau_{q}} P_{k}^{*} X_{k}^{*} - \frac{\partial P_{k} X_{k}}{\partial \tau_{q}} E_{k}^{*}}{(P_{k}^{*} X_{k}^{*})^{2}} \left\{ i = k \right\} \\ \sum_{j}^{n} \Lambda_{ij} \sum_{m}^{n} \ell_{jm} (\sigma_{mk} - \beta_{km}) \frac{\frac{\partial E_{k}}{\partial \tau_{q}} P_{i}^{*} X_{i}^{*} - \frac{\partial P_{i} X_{i}}{\partial \tau_{q}} E_{k}^{*}}{(P_{i}^{*} X_{i}^{*})^{2}} - \sum_{m}^{n} \ell_{jm}^{WT} w_{ml} \frac{\frac{\partial P_{k} X_{k}}{\partial \tau_{q}} S_{r_{m}} - \frac{\partial S_{r_{m}}}{\partial \tau_{q}} P_{k}^{*} X_{k}^{*}}{P_{r_{m}}^{*} X_{k}^{*}} - \frac{\partial P_{k} X_{k}}{\partial \tau_{q}} P_{k}^{*} X_{k}^{*}} \right\} \\ - \ell_{jk}^{WT} \sum_{m}^{n} \beta_{km} \frac{\frac{\partial E_{k}}{\partial \tau_{q}} P_{k}^{*} X_{k}^{*} - \frac{\partial P_{k} X_{k}}{\partial \tau_{q}} E_{k}^{*}}}{(P_{k}^{*} X_{k}^{*})^{2}}} \left\{ i \neq k \right\}$$

$$(25)$$

Equation (25) shows the synergies between local policies. For example, let us focus on direct input substitution in sector k when imposing a tax on emissions in sector q, which is given by the term $-\frac{\beta_{kj}}{w_{kj}} \frac{\frac{\partial E_k}{\partial \tau_q} P_k^* X_k^* - \frac{\partial P_k X_k}{\partial \tau_q} E_k^*}{(P_k^* X_k^*)^2}$. If τ_q generates a decrease in sector k's sales emissions intensity, τ_k will generate a smaller impact on global emissions through the direct input substitution mechanism. This is the case because, when the emissions intensity of a sector *i*s low, it has little margin to further reduce emissions.

¹³Second-order effects accommodate changes in industry/government/consumer behavior following the introduction of carbon taxes in different region-sectors. This is the case because when, for example, the Chinese Iron-and-Steel industry is taxed, it will substitute its usage of inputs upstream (and the same will be done by those upstream sectors through the value chain). Chinese steel will also be less used downstream since it is more expensive now, and will be substituted downstream (and the same for those sectors that it downstream). Additionally, tax revenue will allow the Chinese government to spend more and all the changes in demand will change consumption patterns internationally. When studying second-order effects, we are considering the interaction between this first wave of emission changes due to taxing country-sector a and a wave that would be caused by taxing country-sector b.

¹⁴The derivation of equation (23) in order to get (25) is straightforward. We simply need to obtain the derivative of the following ratios in equilibrium: $\frac{E_k^*}{P_i^* X_i^*}$, $\frac{E_k^*}{P_k^* X_k^*}$ and $\frac{E_k^*}{S_{r_m}^*}$.

Let us now concentrate on the meaning of the component $-\ell_{jk}^{W^T} \sum_m^n \beta_{km} \frac{\frac{\partial E_k}{\partial \tau_q} P_k^* X_k^* - \frac{\partial P_k X_k}{\partial \tau_q} E_k^*}{(P_k^* X_k^*)^2}$ This expression defines the cost pass-through effects arising from the original substitution of inputs. Finally, the rest of equation (25) is characterized by the first derivative of either $\frac{E_k^*}{P_i^* X_i^*}$ or $\frac{E_k^*}{S_{\tau_m}^*}$ in which the same logic can be applied.

As a result, we can define the first and the second order impact on emissions of sector i under as follows (see details in Appendix 9.1):

$$E_i^{1st} = E_i^* + \sum_j^n \frac{\partial E_i}{\partial \tau_j} \tau_j, \qquad (26)$$

$$E_i^{2nd} = E_i^* + \sum_j^n \frac{\partial E_i}{\partial \tau_j} \tau_j + \sum_j^n \sum_m^n \frac{\partial^2 E_i}{\partial \tau_j \tau_m} \tau_j \tau_m.$$
(27)

Using the previous equations we can study to which extent it is important to fix carbon taxes simultaneously in several sectors/countries. If the size of the interaction between such taxation is important, the direct effect could be amplified, calling for greater harmonization among countries.

In the following section we offer an empirical representation of the different network dynamics that play a role in globally spreading the effects of carbon taxation by calibrating the model with data from the multiregional input-output database Exiobase3. This provides a description of the trade flows between the different sectors across the globe, together with their use of raw materials and other factors, as well as their release of emissions. In Exiobase3 there are 23 regions (R = 23) and 163 products (q = 163), meaning we will treat 3749 sectors (n = 3749), where UE-27 is treated as a single jurisdiction.

4. Empirical characterization of the world economy

Herein we use Exiobase3, that presents trade flows and emissions from 3749 sectors.¹⁵ Our calibration allows us to identify the sectors that should be strongly taxed to have the greatest impact on the network. We are also able to empirically identify synergies between taxes applied to different sectors. We calibrate the model assuming that in equilibrium $\tau = 0$. Unless stated otherwise the simulations are performed using the most recent data available, i.e. 2015 and the EU is considered as a single region. In Table 1 we present the notation for each region and in Figure 1 we show the first-order interconnection between sectors considered in our database.

¹⁵There are other multi-regional input-output databases e.g. WIOD and EORA. Exiobase3 provides the greatest granularity in the production structure, which helps to display at its fullest the network dynamics and production structure effects. It is worth noting that the discrepancies between the different databases have been shown to be relatively small. For example, Moran and Wood (2014) tests the robustness of several of the biggest global multi-regional input-output (including Exiobase3) and investigates how much each diverges from the multi-model mean. The paper finds that carbon footprint results for most major economies differ by < 10% among databases.

Code	Name
GB	Great Britain
US	United States
JP	Japan
CN	China
CA	Canada
KR	South Korea
\mathbf{BR}	Brazil
IN	India
MX	Mexico
RU	Russia
AU	Australia
CH	Switzerland
TR	Turkey
TW	Taiwan
NO	Norway
ID	Indonesia
ZA	South Africa
WA	Rest of Asia and the Pacific
WL	Rest of America
WE	Rest of Europe
WF	Rest of Africa
WM	Rest of Middle East

Table 1: Regions' names



Figure 1: 2015's interconnections among 100 most emitting sectors (if $w_{ij} > 1\%$). (colours are by country)

We calibrate the elasticity parameters w_{ij} , w_{iL} , ϕ_{ir} , γ_{ri} for all sectors i, j and r. The parameters can be calibrated via the FOCs expressions for the respective agent's utility/profit maximization problems. In particular, rearranging the $FOC_{X_{ij}}$ in equation (23) we get:

$$w_{ij} = \frac{P_j X_{ij} + \tau_i \beta_{ij} E_i}{P_i Y_i}.$$
(28)

When $\tau = 0$, equation (28) becomes $w_{ij} = \frac{X_{ij}P_j}{X_iP_i}$. Then, the input-output data can be used to calibrate this set of NxN elasticity parameters.

Similarly, the FOC_{G_i} and FOC_{C_i} imply, respectively:

$$\phi_{ri} = \frac{G_{ri}P_i}{\sum_i^n G_{ri}P_i},\tag{29}$$

$$\gamma_{ri} = \frac{P_i C_{ri}}{\sum_i^n P_i C_{ri}}.$$
(30)

Expressions (29) and (30) can be used to calibrate the 23 different regional sets of n elasticity parameters both for household's and government's final demand. Each regional set of parameters characterizes its own final demand structure.

In Figure 2 we show how government and consumer spending have very different CO_2 intensities across countries in our database. Government expenditures in Russia, India, China and in the Middle East are very carbon insensitive, while households in New Zeland are the ones with the highest carbon intensity among households.



Figure 2: TCO_2e emissions resulting from each euro spent

Similarly, results in Figure 3 show the unequal contribution of governments and consumers in different countries to GHG emissions. When considering the percentage of GHG that are attributed to each country we observe that the US, and to a lesser extent

the European Union, appear as sharing a big responsibility, particularly due to household consumption. Figures 2 and 3 underline the importance of considering the heterogeneity in demand between agents in different countries to choose optimal policy.



Figure 3: Share of global emissions

Since US salary is the model's numeraire, we calibrate it using the model identity $S_{US} = \frac{\sum_{i}^{n} P_i C_{USi}}{L_{US}}$ where L_{US} are worked hours. Additionally, all monetary units are expressed in euros.

We also assume that the emission intensities β_{ij} (of using an energy vector j in sector i) are equal to the emission intensities reported by the European Commission's Regulation (EU 601/2012).¹⁶ The remaining variables are endogenous. In particular, we consider the version of equation (2) without ψ_i , i.e. $E_i = \sum_{j}^{n} \beta_{ij} X_{ij}$, which in turn gives global emissions equal to $\Sigma E = \sum_{i}^{n} E_i$.

5. Results

5.1. Targeted carbon taxation

In Figure 4 we show the simulation of the first order policy effects, meaning that we tax a single sector in a single country and analyze the impact of this shock through the value chain. On the left panel, we show the impact described in equation (17) for each sector-region considered in the horizontal axis. We observe that the first-order impact on global emissions of a carbon tax in a given sector in a given country may be positive or negative

¹⁶We refer to the EU No 601/2012 of the 21 June 2012 on the monitoring and reporting of greenhouse gas emissions pursuant to Directive 2003/87/EC of the European Parliament and of the Council. We believe this is not a very restrictive assumption since emissions derived from burning an energy vector, for example oil in a thermal plant, depend on physical properties that are independent of location. In the Appendix 9.2 we perform a sensitivity analysis of our results to other emission intensities. Results do not change significantly.

and the magnitudes can be very different depending on the targeted sector-country. This counter-intuitive result of an increase in emissions after imposing a carbon tax on one sector is due to the direct and indirect impacts that the tax has on consumption per country (see e.g. the impact on utility in the right panel of Figure 4), government consumption per country as well as on the chain of input substitutions through the production network.



(a) Impact in global emissions after a tax to each sector (b) Impact in global household utility after a tax to each sector

Figure 4: First-order impact of imposing a tax of $1 \text{€}/\text{TCO}_2 e$ to sector k

In Table 2 and Table 3 we pick the sectors with the most important variations on the left panel of Figure 4 and present the total impact on global emissions of imposing a tax of 1 \bigcirc per TCO2e on those sectors (i.e. shows the value of $\frac{\partial ln(\Sigma E)}{\partial t_k}$). A tax of 1 \bigcirc per TCO2e applied to electricity production from coal in China generates an increase in global emissions of 0.005037861%. This surprising result is driven by the fact that the carbon tax raises China's government revenue. The emissions generated from the resulting Chinese government expenditure (0.0095% of global emissions) is bigger than the emissions reduction generated by the tax through the other dynamics of the model, mostly input substitution and demand reduction due to pass-through (i.e. the pass-through reduces in -0.00441% global emissions). This is the case because, as we observed in Figures 2 and Figure 3, the Chinese government expenditure (together with India and Russia) is a big contributor to global emissions.

A completely opposite case arises when we apply a carbon tax to the extraction of crude petroleum and services related in the Middle East. Such taxation generates a -0,011578412% reduction in global emissions. This result derives from the central position of this sector that is very interconnected in the global production network. Particularly the costs generated by the tax will pass-through to the entire global economy and generate relevant input-substitution that decrease emissions (i.e. the pass-through reduces in -0,00753% global emissions).

Country	Sector	Impact $(\%)$
WM	'Extraction of crude petroleum and services re-	-0.011578412
	lated to crude oil extraction, excluding surveying'	
ZA	'Manufacture of coke oven products'	-0.001766143
MX	'Mining of coal and lignite; extraction of peat (10) '	-0.000485274
EU	'Manufacture of other transport equipment (35)'	-0.000313037
WF	'Manufacture of electrical machinery and appara-	-0.000310752
	tus n.e.c. (31)'	
EU	'Manufacture of machinery and equipment n.e.c.	-0.000309891
	(29)'	
EU	'Manufacture of electrical machinery and appara-	-0.000307398
	tus n.e.c. (31)'	
EU	'Manufacture of fabricated metal products, except	-0.000306545
	machinery and equipment $(28)'$	
WF	'Manufacture of motor vehicles, trailers and semi-	-0.000303363
	trailers (34)'	
TR	'Mining of chemical and fertilizer minerals, pro-	-0.000301943
	duction of salt, other mining and quarrying n.e.c.'	
EU	'Manufacture of motor vehicles, trailers and semi-	-0.000298406
	trailers (34)'	
EU	'Manufacture of rubber and plastic products (25)'	-0.000288113
WΜ	'Manufacture of electrical machinery and appara-	-0.000287891
	tus n.e.c. (31)	0.0000005
WF	'Mining of copper ores and concentrates'	-0.000283856
WM	'Manufacture of office machinery and computers (30)'	-0.000283543

Table 2: Sector policy with a larger negative impact on global emissions

Country	Sector	Impact $(\%)$
CN	'Production of electricity by coal'	0.005037861
WM	'Production of electricity by gas'	0.001990541
CN	'Manufacture of basic iron and steel and of ferro-	0.001981553
	alloys and first products thereof'	
WA	'Production of electricity by coal'	0.001159023
WA	'Production of electricity by gas'	0.00107841
CN	'Chemicals nec'	0.000979745
WM	'Production of electricity by petroleum and other	0.000936564
	oil derivatives'	
CN	'Petroleum Refinery'	0.00081518
IN	'Manufacture of basic iron and steel and of ferro-	0.00076312
	alloys and first products thereof'	
CN	'Manufacture of other non-metallic mineral prod-	0.00067765
	ucts n.e.c.'	
IN	'Production of electricity by coal'	0.000563803
WA	'Education (80)'	0.000547012
WM	'Hotels and restaurants (55)'	0.000527015
WA	'Real estate activities (70)'	0.000411154
WA	'Manufacture of wearing apparel; dressing and	0.000328861
	dyeing of fur (18) '	

Table 3: Sector policy with a larger positive impact on global emissions

When considering only first-order effects as we do in this subsection (i.e. no interaction of taxes simultaneously imposed is considered), Figure 5 shows that a targeted carbon tax seems to outperform a global carbon tax. We call "targeted carbon tax" a taxation strategy in which we tax all sectors for which the taxation first-order impact is negative (i.e. $\frac{\partial ln(\Sigma E)}{\partial t_k} < 0$). The left panel in Figure 5 shows that such strategy is more or less equivalent to a global tax in terms of emission reduction¹⁷ per region but it has a weaker negative impact on utility, as shown in the right panel, which implies it has smaller negative impact on welfare.

In Figure 6 we compare the first-order impact on emissions per country of taxing the two sectors leading the positive and negative impacts in Table 3, i.e. a carbon tax to electricity produced by coal in China versus a carbon tax to crude oil extraction in the Middle East. We observe that the increase in emissions when taxing coal in China is strong

¹⁷After taxation, (polluting) input usage changes, producing a change in emissions. Even if we are considering a marginal change in tax (1 euro per ton of CO_2e), the impact is important due to the fact that, differently from what happens in real life in regions with carbon pricing mechanisms, almost all sectors are subject to the carbon tax, producing a generalized emission reduction.



Figure 5: First-order effects of a global vs. targeted carbon tax $(1 \notin /\text{TCO}_2 e)$.

but concentrated in China and in few Asian countries while the decrease in emissions due to taxing oil in the Middle East is smaller in each country but spreads globally.



Figure 6: First-order effects of a $1 \text{€}/\text{TCO}_2 e$ tax on two selected sectors.

All in all, the simulation of first order policy effects shows that, firstly, imposing a carbon tax to some of the top CO_2 emitting sectors in the world may actually generate an increase in global emissions instead of a decrease and, secondly, that a "targeted carbon tax" has a similar impact to a global one but welfare loss is smaller.

The previous results also underline that, if only emission intensities are considered for carbon taxation, impacts on global emissions could be counterproductive. This is because heterogeneity of private and public consumption across countries as well as the impact on the whole production network may be sizeable.

In the following section we show that the previous conclusions are modified when

considering second-order policy effects that characterize the synergies that emerge when a carbon tax is applied in two distinct sectors-countries at a time.

5.2. Interaction between carbon taxes around the world

In Figure 7 we show that a global carbon tax outperforms a "targeted carbon tax" in terms of emission reduction for a global tax that is higher than $12\mathfrak{C}$ the TCO₂e. This is the case because for a tax higher than $12\mathfrak{C}$, the synergies generated through the production network are more important than the punctual positive impact on emissions that some sectors may have (like electricity produced by coal in China). Table 4 and Table ?? show the synergies between sectors that have a higher negative and positive impact on emissions respectively.¹⁸



Figure 7: Second-order effects of a global vs. targeted carbon tax.

Let us study the most counter-intuitive example to make our point regarding the importance of synergies. Applying simultaneously a carbon tax to electricity generation from coal in China and to electricity generation from gas in the middle east (the two sectors with higher first order effects that increase global emissions when taxed) we find that there exist a synergy between these two carbon taxes that results in a reduction in global emissions instead of an increase. This is the case for a tax higher than $50 \\mbox{ the TCO}_2 e$, as we can see in the left panel of Figure 8. Even if the first order effect on global emissions $(\frac{\partial \Sigma E}{\partial t_k})$ was clearly positive, the indirect effect on other sectors dominates when the tax level is sufficiently high. Additionally, in the right panel of Figure 8 we observe that if two sectors with negative first-order impacts are taxed simultaneously, emissions decrease even for small tax values.

 $^{^{18}\}mathrm{See}$ Technical Appendix 9.3 for further details on this.

Sector q	Sector k	Impact
WA Sea and coastal water transport	CN Manufacture of basic iron and steel and of ferro-alloys and first products thereof	-2.58561E-06
CN Sea and coastal water transport	RU Manufacture of gas; distribution of gaseous fuels through mains	-1.68579E-06
WA Sea and coastal water transport	ID Manufacture of basic iron and steel and of ferro-alloys and first products thereof	-1.58801E-06
EU Sea and coastal water transport WA Sea and coastal water transport	CN Production of electricity by coal ID Manufacture of basic iron and steel and of ferro-alloys and first products thereof	-1.03515E-06 -8.47647E-07

Table 4: Sector policy synergies with a larger negative impact on global emissions

Table 5: Sector policy synergies with a larger positive impact on global emissions

Sector q	Sector k	Impact
WA Sea and coastal water transport	RU Manufacture of gas; distribution of gaseous fuels through mains	9.08824E-08
CN Manufacture of cement, lime and plaster	EU Steam and hot water supply	3.43485E-08
CN Sea and coastal water transport	ID Manufacture of rubber and plas- tic products (25)	3.16579E-08
EU Sea and coastal water transport	EU Construction Work	2.9182 E-08
RU Manufacture of gas; distribution of gaseous fuels through mains	CN Production of electricity by coal	2.52087E-08



(a) Electricity generation by coal in China and electricity gen-Middle East and manufacture of coke oven products in South 23^{Africa}

Figure 8: Second order approximation for the effects of a sector carbon tax on global emissions. Policy effects beyond $\tau = 0$.



Figure 9: Impact of a carbon tax to electricity generation by coal in China in the context of a carbon tax to the rest of the global economy.

5.3. Welfare analysis

To be able to focus on how the global value chain structure has a crucial role in emissions variation after carbon taxation, in the previous sections we have chosen to model an environmental tax that is not redistributed to consumers as well as consumers that only care about consumption. Herein we relax this last assumption by looking into the influence that a preference for environmental quality (lower emissions) may have on the choice of carbon taxes. With this purpose, let us now maximize a simple global welfare function that increases with household utility derived from goods consumption at the different regions $(U_r = \prod_i^n C_{ri}^{\gamma_{ri}})$ and decreases with global emissions (ΣE) i.e.:

$$\Omega = \sum_{r}^{R} \frac{P_r}{\Sigma P} f_r(U_r, \Sigma E), \qquad (31)$$

where we assume $\frac{\partial f_r(U_r, \Sigma E)}{\partial U_r} > 0$ and $\frac{\partial f_r(U_r, \Sigma E)}{\partial \Sigma E} < 0$ and where P_r and ΣP are regional and global population, respectively.

Given the welfare function in equation (31) and the marginal impacts discussed in the previous section $\left(\frac{\partial \Sigma E}{\partial t_k}, \frac{\partial^2 \Sigma E}{\partial t_k t_q}, \frac{\partial U_r}{\partial t_k}\right)$, we find a set of n tax levels that we call Tthat maximize the second order approximation of the welfare function as follows:

$$argMax\Omega^{2nd} = argMax\Omega^* + \frac{\partial\Omega}{\partial T}T + T'\frac{\partial^2\Omega}{\partial T^2}T.$$
(32)

The resulting optimal T is then:

$$T^{\prime*} = \frac{\partial\Omega}{\partial T} \left(\frac{\partial^2\Omega}{\partial T^2}\right)^{(-1)}.$$
(33)

For simplicity, let us use the following welfare function:

$$\Omega = \sum_{r}^{R} \frac{P_r}{\Sigma P} U_r^{(1-\rho)} \Sigma E^{(\rho)}, \qquad (34)$$

where ρ is the preference for the environment and where we use the linear specification of emissions as before.

Let us now present the optimal tax level for a subset of the 700 most polluting sectors (where optimal means welfare maximizing), limiting the tax between 0 and $150 \oplus$ per TCO₂e.¹⁹ The results show that the optimal tax per sector greatly depends on the value of ρ . Without considering the synergies between taxes imposed to distinct sectors, a targeted strategy is optimal but, depending on ρ , the optimal level of such targeted tax greatly varies between sectors. For a low environmental preference $\rho = 0.1$, the optimal tax is low for most sectors with just a few exceptions (see Figure 10a). Instead, for a high environmental preference $\rho = 0.75$ (see Figure 10b), the optimal tax sits at 150 \oplus per TCO2e for most sectors.



Figure 10: Optimal level of carbon taxation per sector

We now move to the comparison of a targeted tax (only taxing sectors that we have found to have negative first-order impact) versus a global tax, as a function of the environmental preference parameter ρ . In the right panel of Figure 11 we observe that for a high environmental preference $\rho = 0.75$, the impact of both type of taxes on global welfare is positive. Moreover, a global tax outperforms a targeted tax if higher than $31 \\mathcal{eq}$ per TCO₂e.

 $^{^{19}}$ If we wish to obtain numerical results for the optimal tax level for each of the sectors in our database, the exercise becomes challenging since we have to optimize 3749 variables while taking into account 3749^2 interaction terms.

This is the case because high taxes have important impacts in terms of emission reductions compensating impacts on consumption reduction.

In the left panel of Figure 11 we show that, for a low environmental preference $\rho = 0.25$, the impact on global welfare of a targeted carbon tax becomes negative for a tax level above 44 $\mbox{\ err}$ per TCO₂e. This means that the welfare loss due to diminished consumption is higher than welfare gain due to emission reduction for a targeted tax higher than 44 euros. Moreover, the two curves intersect for a negative welfare value at a tax equal to 66 $\mbox{\ err}$ per TCO₂e. This is because a global carbon tax outperforms a targeted tax in terms of less welfare loss for a tax level higher than 66 $\mbox{\ err}$ per TCO₂e a global tax generates welfare gains.



Figure 11: Second-order effects of a global vs. targeted carbon tax on global welfare.

Let us now study, from a welfare perspective, the impact of synergies between policies in detail. In Figure 12 we observe that a carbon tax to electricity generation by coal in China (left panel) impacts global welfare negatively. Emissions are not reduced at low level of taxes and this is not compensated by an increase in household's utility. However if $\rho = 0.75$ a sector-level carbon tax has a positive effect on welfare for a tax above 120 C per TCO₂e. This is the case because, with a high carbon tax and a strong environmental preference, the second order impact on emission reduction compensates in terms of welfare the consumption decrease. This shows the importance of environmental preferences to modulate the results we have discussed regarding targeted policies in the previous section. On the right panel of Figure 12 we show the impact on welfare of a carbon tax to the extraction of crude petroleum and related services in the Middle East. This generates a positive effect on global welfare for all tax levels and ρ values considered.



(a) Carbon tax to electricity generation by coal in China related in the Middle East

Figure 12: Second-order effects of a sector carbon tax on global welfare.

In the following section we pursue our analysis of optimal carbon taxation applied to a real-world example: the EU-CBAM.

6. Analysis of the European Union Carbon Border Adjustment Mechanism

In this section, we make use of our global network production model to assess the effects of the Carbon Border Adjustment Mechanism (CBAM). By requiring importers of certain goods to pay for the carbon emissions that were generated during the production of these goods outside of the EU, the CBAM aims to create a level playing field for EU industries while also encouraging non-EU countries to reduce their carbon footprint. On the 13th of December 2022, the European Council and the European Parliament reached a political agreement on the implementation of the new CBAM that will enter into force in its transitional phase as of October 1st, 2023 and, when fully phased in, capture more than 50% of the emissions in the sectors covered by the EU-ETS. In particular, the measure will apply to sectors under the EU-ETS that are considered to be subject to international competition e.g. steel, cement, aluminum and fertilizers. In what follows we study the impact of such policy.

6.1. Modeling the CBAM in our global production network

The implementation of a CBAM mechanism in the global network production model closely follows the characterization of the standard sector-based carbon tax policy discussed previously.

Concretely, under the CBAM, the sector/country i pays a carbon tax for the imported emissions through its immediate production chain providers outside of the EU. Typically the sector/country i does not pay for the emissions of all imported goods but for the emissions of a particular set of imported products that fall inside the scope of the policy.

In our model, the total amount of emissions subject to a carbon tax under the CBAM for the sector i (noted as \hat{E}_i^*) can be defined as follows:

$$\hat{E}_i^* = \sum_j^n \frac{X_{ij}^*}{X_j^*} E_j^* \{ j \in \mathbf{CBAM} \}$$

$$(35)$$

where $\frac{X_{ij}^*}{X_j^*}$ is the fraction of j's output consumed as an intermediary input by sector/country *i* while $j \in \{\text{CBAM}\}$ is the set of imported products whose emissions are subject to the CBAM. Therefore, the ratio $\frac{X_{ij}^*}{X_j^*}$ defines the share of j's emissions that are imputed to *i* under the CBAM tax regime while $\frac{E_j^*}{X_j^*}$ defines the emissions accounted per unit of imported product X_{ij} . With this in mind, the CBAM policy effects can be derived analogously to the traditional carbon tax policy with a linear emission function. In particular, the equilibrium-defined element $\frac{E_j^*}{X_j^*}$ is treated as a fixed component in the subsequent policy simulations and used analogously to the emissions intensity parameter β_j . With the CBAM in place, sector *i* maximizes the following profit function:

$$\max_{X_{ij}} P_i[A_i L_i^{w_{iL}} \prod_j^n X_{ij}^{w_{ij}}] - S_{r_i} L_i - \sum_i^n P_j X_{ij} - \tau_i \psi_i \sum_j^n [\beta_{ij} X_{ij}] - \sum_j^n \tau_j^{CBAM} \frac{E_j^*}{X_j^*} X_{ij} \{ j \in \mathbf{CBAM} \} \}$$
(36)

where τ_i stands for the carbon tax applicable to direct emissions, and τ_j^{CBAM} is the tax rate applicable to imported emissions from the set of foreign sector/countries $j \in \mathbf{CBAM}$.

6.2. Impact of the CBAM

Since we are particularly interested in understanding the mechanism and the transmission of effects in the production network, we choose to focus in this part on one of the industrial sectors covered by the CBAM that is most important in terms of its contribution to CO₂: 'Basic iron and steel and of ferro-alloys and first products'.

First, we consider a scenario in which: (i) a 100 per TCO₂e tax covers the emissions of the European production of 'Basic iron and steel and of ferro-alloys and first products'; and, (ii) the same 100 per TCO₂e tax covers emissions of imports by the EU of iron and steel as in the CBAM.

We calculate the production contraction in the EU steel sector as compared to what happens in the same sector in other exporting countries in scenario (i), when only EU producers of steel are taxed versus scenario (ii), when all steel consumed in Europe is taxed.

The results in Table 6 show that the carbon tax only covering the EU sector would generate a sizable production contraction in such sector while having, as expected, almost no impact in the same sector in other regions. Instead, when the carbon tax is complemented with the CBAM, the production contraction is generalized and now negatively affects production overseas. Interestingly, the European production contraction is less strong with the CBAM, which may be explained by the fact the CBAM avoids the loss of competitiveness of the EU producer and, by generating government resources, it increases public expenditure in Europe that, through other network effects, increases demand. In particular, a carbon tax to iron and steel produced in Europe decreases its production by 9.83% while having an almost null impact on the same sector outside of Europe. When such tax is accompanied by a CBAM, the decrease in Europe is lower (7.32%) and it impacts the same sector else-ware. We observe, for example, a strong decrease in Indian steel of 5.91%, explained by its interconnections with Europe. These effects can, in particular, occur inside a firm that produces the same good in two distinct countries.

Table 6: Change in production in the 'Basic iron and steel and of ferro-alloys and first products thereof' in different regions after a 100 EUR carbon tax to this sector inside the EU

Region	Change in production (%)	
	Carbon tax	CBAM
EU	-9.83%	-7.32%
US	-0.08%	-3.24%
China	-0.10%	-4.65%
India	-0.14%	-5.91%

Let us now consider the more realistic case in which all European and imported emissions in the EU are taxed. That is, we compare two scenarios: (a) a direct tax of $100 \\embed{model}$ per TCO₂e applied to all sectors in the European economy; and, (b) a $100 \\embed{model}$ per TCO₂e tax accompagnied by a CBAM that covers all imported emissions in the EU.

The results in Table 7 show that the implementation of a EU-wide carbon tax (scenario a) generates a significant reduction in output for the steel sector in Europe of 24.86%. This contraction is much higher than the contraction of 9.83% in Table 6, when only steel was taxed. This is due to the network feedbacks and the demand contraction produced by the generalized carbon tax as opposed to the sector-specific carbon tax. We also observe that, even if production overseas is not taxed, this policy generates a sizable overall contraction of output that ends up impacting non-European producers through the supply chain (particularly Indian steel with a contraction of 10.89%).

Instead, when the CBAM policy is implemented on top of the carbon tax (scenario b) we find that the reduction in output overseas is stronger but, surprisingly, the reduction in production inside the EU is even stronger than without the CBAM. This is due to the synergies generated through the contraction of demand in the value chain. They are stronger in terms of output than the competitiveness gain generated.

We also observe that the CBAM is not large enough to generate a reduction in output overseas of the same magnitude as the one provoked to EU producers: while EU producers have 100% of their production subject to a direct carbon tax, overseas producers will only pay the tax for a fraction $\frac{X_{ij}}{X_j}$ of its production. Network effects will spread the contraction in production outside the EU, but production network effects will also impact

Region	Change in production $(\%)$	
	Carbon tax	CBAM
EU	-24.86%	-25.68%
US	-6.28%	-13.59%
China	-8.34%	-16.08%
India	-10.89%	-19.23%

Table 7: Change in production in 'Basic iron and steel and of ferro-alloys and first products thereof' in different regions after a 100 EUR carbon tax to all European emissions

more European producers that will be generally closer in the network structure.

It is worth mentioning that, as explained by Zhang et al. (2017a, b), with increasing fragmentation of production, there is the problem of tracing (embedded) emissions through the global value chain as well as the question of which is the appropriate "location" of tax incidence when considering the related border crossing problem. Herein we only consider a carbon tax to a sector-country or a group of sectors in a country and study its impact on production, (private and public) consumption and global emissions, without really looking into how those emissions cross borders after the tax. In this regard we do not tax embedded emissions but only direct emissions from the sector. We have chosen this approach to be closer to the way carbon taxation and the CBAM work nowadays even if the question in Zhang et al. (2017 a,b) is extremely important to avoid multiple taxation, particularly when considering scope 2 and 3 emissions in international environmental policy.

7. Concluding remarks

Herein we present a multi-regional, multi-sector general equilibrium model that characterizes the impact of carbon taxation on emissions and welfare depending on the structure of the global production network. Moreover, the model accounts for the interaction of the production network with multiple heterogeneous households, governments and labor markets. The model is then empirically calibrated using a database of 23 regions and 163 sectors.

When imposing a tax on emissions in a particular sector in a particular country, the impact on global emissions is determined by the change in household demand and government spending as well as on the way each sector (included those not affected by the tax) reevaluate inputs usage. First, the sector that is hit by the tax reevaluates its inputs usage in order to reduce its emissions. This unleashes a transformation of the structure of the global production network. If sector i does not face a direct tax, the derivative of its inputs usage depends on the derivative of its sector sales (which are characterized by the structure of the production network as well as the transfers of income across government and households) and on the derivative of input prices.

In this context we are able to compare the impact in terms of emissions and welfare of targeted carbon taxes as opposed to a global carbon tax and to show when one option is preferable to the other depending on key parameters.

Our main findings are as follows. First, a targeted carbon tax to most polluting sectors can result in a first-order increase in emissions. This is because the resulting change in global emissions also depend on the impact of the tax on global demand through the production network (particularly on government spending and sector 's centrality). Secondly, synergies between carbon taxes may be sizable, contradicting the previous finding and calling for generalized carbon pricing world-wide.

Finally, we are also able to study the impact of the European Carbon Border Adjustment Mechanism (CBAM) showing its impact in terms of output contraction inside and outside of the EU. We find that the CBAM reduces the loss of European competitiveness but, when the CBAM is applied to numerous sectors simultaneously, the contraction synergies through the value chain end up provoking an important downturn that is bigger than the one that would have happened only with an EU tax (without the CBAM).

This paper's model and calibration can be used to simulate many alternative policies and their impact on global production, emissions and welfare.

8. References

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9. Appendix

9.1. Appendix to the theoretical model

In this section we detail the resolution to the theoretical model.

9.1.1. Equilibria Equilibrium sales

The clearing market condition for good *i* is $X_i = C_i + G_i + \sum_{j=1}^{n} X_{ji}$. Introducing the FOCs from the different agents into this expression we obtain:

$$P_i X_i = \sum_r^R \gamma_{ri} H_r + \sum_r^R \phi_{ri} \sum_j^n \tau_j E_j \{ j \in \mathbf{r} \} + \sum_j^n w_{ji} P_j X_j - \tau_j \beta_{ji} E_j$$
(37)

The previous can be then transformed into equation (10) in the main text, that is: $P_i X_i = \sum_{j}^{n} \ell_{ij} [\sum_{r}^{R} \gamma_{rj} H_r + \sum_{m}^{n} (\sigma_{jm} - \beta_{mj}) \tau_m E_m]$ where ℓ_{ij} is the *ijth* element o the Leontief matrix $L = [I - W^T]^{(-1)}$ and W is a

where ℓ_{ij} is the *ijth* element o the Leontief matrix $L = [I - W^I]^{(-1)}$ and W is a matrix of dimensions NxN whose *i*, *jth* element is the elasticity parameter w_{ji} .

To do so, let us express each of the therms of the equation (37) in matrix form:

(i) the first term $\sum_{r}^{R} \gamma_{ri} Hr$ can be expressed in matrix form as $\Gamma_{(NxR)} H_{(Rx1)}$

$$\Gamma H = \begin{pmatrix} \gamma_{11} & \gamma_{21} & \cdots & \gamma_{R1} \\ \gamma_{12} & \gamma_{22} & \cdots & \gamma_{R2} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{1n} & \gamma_{2n} & \cdots & \gamma_{Rn} \end{pmatrix} \begin{pmatrix} H_1 \\ \vdots \\ H_R \end{pmatrix}$$
(38)

(ii) the second term $\sum_{r}^{R} \phi_{ri} \sum_{j}^{n} \tau_{j} E_{j} \{j \in \mathbf{r}\}$ defines public expenditure per product as a function of the different sector emission levels and sector carbon taxes across all jurisdictions. In matrix form this is $\Sigma \tau E$ where Σ is the NxN matrix of parameters that can be expressed as:

$$\Sigma = \begin{pmatrix} \sigma_{1,1} & \sigma_{1,2} & \cdots & \sigma_{1,n} \\ \sigma_{2,1} & \sigma_{2,2} & \cdots & \sigma_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n,1} & \sigma_{n,2} & \cdots & \sigma_{n,n} \end{pmatrix} = \begin{pmatrix} \phi_{1,1} & \phi_{2,1} & \cdots & \phi_{R,1} \\ \phi_{1,2} & \phi_{2,2} & \cdots & \phi_{R,2} \\ \vdots & \vdots & \ddots & \vdots \\ \phi_{1,n} & \phi_{2,n} & \cdots & \phi_{R,n} \end{pmatrix} * \Xi$$
(39)

where we introduce a bridge or structural matrix Ξ that links each region with its respective sectors:

$$\Xi = \begin{pmatrix} 1 & \cdots & 1 & 0 & \cdots & 0 & \cdots & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 1 & \cdots & 1 & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & 0 & \cdots & 1 & \cdots & 1 \end{pmatrix}$$
(40)

and where τE is a Nx1 vector as follows

$$\tau E = \begin{pmatrix} \tau_1 E_1 \\ \vdots \\ \tau_N E_N \end{pmatrix} \tag{41}$$

With this notation we can observe that Σ 's i, jth element links government revenue raised by taxing emissions at sector j with public expenditure in good i.

(iii) the third term $\sum_{j=1}^{n} w_{ji} P_j X_j$ in matrix form is $W_{(N \times N)} P X_{(N \times 1)}$:

$$WPX = \begin{pmatrix} w_{1,1} & w_{2,1} & \cdots & w_{n,1} \\ w_{1,2} & w_{2,2} & \cdots & w_{n,2} \\ \vdots & \vdots & \ddots & \vdots \\ w_{1,n} & w_{2,n} & \cdots & w_{n,n} \end{pmatrix} \begin{pmatrix} P_1 X_1 \\ \vdots \\ P_N X_N \end{pmatrix}$$
(42)

(iv) the last term $\tau_j \beta_{ji} E_j$ in matrix form is $\beta \tau \to \beta$ with β being the NxN matrix

$$\beta = \begin{pmatrix} \beta_{1,1} & \beta_{2,1} & \cdots & \beta_{n,1} \\ \beta_{1,2} & \beta_{2,2} & \cdots & \beta_{n,2} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{1,n} & \beta_{2,n} & \cdots & \beta_{n,n} \end{pmatrix}$$
(43)

Then, using these matrix to rewrite equilibrium condition (37) we get $PX = \Gamma H + \Sigma \tau E + WPX - \beta \tau E$ that after rearranging becomes (10) in matrix form:

$$PX = (I - W)^{(-1)} [\Gamma H + (\Sigma - \beta)\tau E]$$
(44)

Equilibrium wages

Wage for the labour market at region r is defined in equation (13) as $l_i = \frac{w_{il}P_iX_i}{S_{r_i}}$ and the market clearing condition is $\sum_j^n l_j = L_r\{j \in \mathbf{r}\}$. Then, we obtain equation (14) $S_r^* = \frac{\sum_j^n w_{jl}P_j^*X_j^*\{j \in \mathbf{r}\}}{L_r}$, with \mathbf{r} being the set of sectors belonging to the jurisdiction of the government r. In matrix form, the matrix with use of labor per sector $W_L(NxN)$ and a matrix $\Delta_{(RxR)}$ of labour supply allotments for each country:

$$W_{L} = \begin{pmatrix} w_{1l} & 0 & \cdots & 0\\ 0 & w_{2l} & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & w_{nl} \end{pmatrix}; \Delta = \begin{pmatrix} \frac{1}{L_{1}} & 0 & \cdots & 0\\ 0 & \frac{1}{L_{2}} & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \frac{1}{L_{R}} \end{pmatrix}$$
(45)

The salary in matrix form is then:

$$S^* = \Delta \Xi W_L P^* X^* \tag{46}$$

where Ξ is the bridge matrix linking each sector to its respective region. Then aggregate regional household income is defined as $H_r = S_r L_r$ which in matrix form transforms into

$$H^* = \Xi W_L P^* X^* \tag{47}$$

that, for a given region r is

$$H_r = \sum_{i}^{n} w_{il} P_i X_i \{ i \in \mathbf{r} \}$$
(48)

Equilibrium prices

The derivation of equilibrium prices is similar to King et al.(2019). Let us first log-linearize the production function as follows

$$ln(X_{i}) = w_{il}ln(l_{i}) + \sum_{j}^{n} w_{ij}ln(X_{ij})$$
(49)

Now, we plug in the first order conditions in 4 and 13:

$$ln(X_i) = w_{il}[ln(w_{il}) + ln(P_iX_i) - ln(S_{r_i})] + \sum_{j=1}^{n} w_{ij}[ln(w_{ij}P_iX_i + \tau_i\beta_jE_i) - ln(P_j)]$$
(50)

Substracting $ln(P_iX_i)$ from both sides and then multiplying by -1 we get:

$$ln(P_i) = ln(P_iX_i) - w_{il}[ln(w_{il}) + ln(P_iX_i) - ln(S_{r_i})] - \sum_{j}^{n} w_{ij}[ln(w_{ij}P_iX_i + \tau_i\beta_jE_i) - ln(P_j)]$$
(51)

Since $w_{il} + \sum_{j}^{n} w_{ij} = 1$ due to the fact that we are using a Cobb Douglas with constant returns to scale, if we introduce $\tau = 0$ we get:

$$ln(P_i) = -w_{il}[ln(w_{il}) - ln(S_{r_i})] - \sum_{j}^{n} w_{ij}[ln(w_{ij}) - ln(P_j)]$$
(52)

Let us define a constant $j_i = -w_{il}ln(w_{il}) - \sum_{j=1}^{n} w_{ij}ln(w_{ij})$ so that we can rewrite the previous as follows

$$ln(P_i) = j_i + w_{il}ln(S_{r_i}) + \sum_j^n w_{ij}ln(P_j)$$
(53)

expressing equation (53) in matrix terms we get $ln(P) = j + W_L ln(S)) + W^T ln(P)$ with

$$ln(S) = \begin{pmatrix} ln(S_1) \\ \vdots \\ ln(S_R) \end{pmatrix}, ln(P) = \begin{pmatrix} ln(P_1) \\ \vdots \\ ln(P_N) \end{pmatrix}, j = \begin{pmatrix} j_1 \\ \vdots \\ j_N \end{pmatrix}$$
(54)

clearing the expression we obtain price in equilibrium

$$ln(P) = L^{T}(j + W_{L}ln(P))$$
(55)

where L^T is the matrix with elements $\ell_{ij}^{W^T}$ and where S_{US} is the numeraire of the global economy. Equation (55) therefore corresponds to equation (16) in the main text for sector $i. \ln(P_i^*) = \sum_{j}^{n} \ell_{ij}^{W^T} [j_j + w_{jl} \ln(S_{r_j})].$

9.1.2. Carbon taxation

Herein we show how the effects of applying a carbon tax to a given sector k spreads across sectors and regions.

Impact on emissions E_i

The impact of a tax on sector k on emissions in sector i is given by equation (17), that is

$$\frac{\partial ln(E_i)}{\partial \tau_k} = \sum_{j}^{n} \beta_{ij} \frac{\partial ln(X_{ij})}{\partial \tau_k}$$

Impact on Input Usage X_{ij}

To get equation (18) we derive the log linearization of equation (4).

$$\frac{\partial ln(X_{ij})}{\partial \tau_k} = \frac{\partial ln(w_{ij}P_iX_i - \beta_{ij}\tau_iE_i)}{\partial \tau_k} - \frac{\partial ln(P_j)}{\partial \tau_k}$$
(56)

being

i.e.:

$$\frac{\partial ln(w_{ij}P_iX_i - \beta_{ij}\tau_iE_i)}{\partial \tau_k} = \frac{1}{w_{ij}P_iX_i - \beta_{ij}\tau_iE_i}\frac{\partial w_{ij}P_iX_i - \beta_{ij}\tau_iE_i}{\partial \tau_k}$$
(57)

If $\tau = 0$, the previous equation simplifies to the following two alternatives:

$$\frac{w_{ij}\frac{\partial P_i X_i}{\partial \tau_k} - \beta_{ij}\frac{\partial \tau_m}{\partial \tau_k}\frac{\partial E_m}{\partial \tau_k}}{w_{ij}P_k^* X_k^* i - \beta_{ij}\tau_i E_i^*} = \begin{cases} \frac{\partial P_i X_i}{\partial \tau_k} - \frac{\beta_{kj}E_k^*}{w_{kj}P_k^* X_k^*}, & \{i = k\} \\ \\ \frac{\partial P_i X_i}{\partial \tau_k} \\ \frac{\partial P_i X_i}{\partial \tau_k}, & \{i \neq k\} \end{cases}$$
(58)

and therefore, combining (58) and (56) we obtain equation (18) in the main text,

$$\frac{\partial ln(X_{ij})}{\partial \tau_k} = \begin{cases} \frac{\partial ln(P_iX_i)}{\partial \tau_k} - \frac{\partial ln(P_j)}{\partial \tau_k} - \frac{\beta_{ij}E_i^*}{w_{ij}P_i^*X_i^*}, & \{i=k\}\\ \frac{\partial ln(P_iX_i)}{\partial \tau_k} - \frac{\partial ln(P_j)}{\partial \tau_k}, & \{i\neq k\} \end{cases}$$

Impact on sales $P_i X_i$

Deriving equation (10) we obtain:

$$\frac{\partial P_i X_i}{\partial \tau_k} = \sum_{j}^{n} \ell_{ij} \left[\sum_{r}^{R} \gamma_{rj} \frac{\partial H_r}{\partial \tau_k} + \sum_{m}^{n} (\sigma_{jm} - \beta_{mj}) \frac{\partial \tau_m}{\partial \tau_k} \frac{\partial E_m}{\partial \tau_k} \right]$$
(59)

that if $\tau = 0$, simplifies to

$$\frac{\partial P_i X_i}{\partial \tau_k} = \sum_{j}^{n} \ell_{ij} \left[\sum_{r}^{R} \gamma_{rj} \frac{\partial H_r}{\partial \tau_k} + (\sigma_{jk} - \beta_{kj}) E_k^* \right]$$
(60)

Now, substituting the derivative of (48) into equation (60) and deriving we find:

$$\frac{\partial P_i X_i}{\partial \tau_k} = \sum_{j}^{n} \ell_{ij} \left[\sum_{r}^{R} \gamma_{rj} \sum_{m}^{n} w_{ml} \frac{\partial P_m X_m}{\partial t_k} \{ m \in \mathbf{r} \} + (\sigma_{jk} - \beta_{kj}) E_k^* \right]$$
(61)

clearing the expression by means of its matrix elements and factorizing we find $\frac{\partial P_i X_i}{\partial \tau_k}$ can be expressed as

$$\frac{\partial PX}{\partial \tau_k} = L\Gamma \hat{\Xi} W_L \frac{\partial PX}{\partial \tau_k} + L(\Sigma - \beta) E_K^*$$
(62)

where E_K^* is the vector of dimensions Nx1

$$E_K^* = \begin{pmatrix} 0\\ \vdots\\ E_k^*\\ \vdots\\ 0 \end{pmatrix}$$
(63)

isolating $\frac{\partial PX}{\partial \tau_k}$ we obtain

$$\frac{\partial PX}{\partial \tau_k} = \Lambda L(\Sigma - \beta) E_K^* \tag{64}$$

which can be rewritten for a given sector i as

$$\frac{\partial P_i X_i}{\partial \tau_k} = \sum_j^n \Lambda_{ij} \sum_m^n \ell_{jm} (\sigma_{mk} - \beta_{km}) E_k^*$$
(65)

with

$$\frac{\partial ln(P_iXi)}{\partial \tau_k} = \frac{1}{P_i^*X_i^*} \frac{\partial P_iX_i}{\partial \tau_k}$$
(66)

where Λ_{rk} is the rkth element of the NxN matrix $\Lambda = (I - L\Gamma \hat{\Xi} W_L)^{(-1)}$ and $\hat{\Xi}$ is a modified version of the structural matrix $\hat{\Xi}$ where all the elements of the row corresponding to the numeraire are equal to 0.

Impact on input price P_i

The derivative equation 51 is

$$\frac{\partial ln(P_i)}{\partial \tau_k} = \frac{\partial ln(P_iX_i)}{\partial \tau_k} - w_{il} \left[\frac{\partial ln(P_iX_i)}{\partial \tau_k} - \frac{\partial ln(S_{r_i})}{\partial \tau_k}\right] - \sum_j^n w_{ij} \left[\frac{\partial ln(w_{ij}P_iX_i - \beta_{ij}\tau_iE_i)}{\partial \tau_k} - \frac{\partial ln(P_j)}{\partial \tau_k}\right]$$
(67)

If $\tau = 0$ and following the steps defined in 57 and 58

$$\frac{\partial ln(P_i)}{\partial \tau_k} = \begin{cases} \frac{\partial ln(R_i)}{\partial \tau_k} - w_{il} \left[\frac{\partial ln(R_i)}{\partial \tau_k} - \frac{\partial ln(S_{\tau_i})}{\partial \tau_k} \right] - \sum_j^n w_{ij} \left[\frac{\partial ln(P_iX_i)}{\partial \tau_k} - \frac{\partial ln(P_j)}{\partial \tau_k} - \frac{\beta_{kj} E_k^*}{w_{kj} P_k^* X_k^*} \right], \quad \{i = k\} \end{cases}$$

$$\tau_{k} = - \left\{ \frac{\partial ln(R_{i})}{\partial \tau_{k}} - w_{il} \left[\frac{\partial ln(R_{i})}{\partial \tau_{k}} - \frac{\partial ln(S_{\tau_{i}})}{\partial \tau_{k}} \right] - \sum_{j}^{n} w_{ij} \left[\frac{\partial ln(P_{i}X_{i})}{\partial \tau_{k}} - \frac{\partial ln(P_{j})}{\partial \tau_{k}} \right], \qquad \{i \neq k\}$$

$$\tag{68}$$

and since $\sum w_{ij} + w_{iL} = 1$ then

$$\frac{\partial ln(P_i)}{\partial \tau_k} = w_{il} \frac{\partial ln(S_{r_i})}{\partial \tau_k} + \sum_j^n w_{ij} \left[\frac{\partial ln(P_j)}{\partial \tau_k} + \frac{\beta_{kj} E_k^*}{w_{kj} P_k^* X_k^*} \{i = k\} \right]$$
(69)

$$\frac{\partial ln(P_i)}{\partial \tau_k} = \begin{cases} w_{il} \frac{\partial ln(S_{r_i})}{\partial \tau_k} + \sum_j^n w_{ij} \left[\frac{\partial ln(P_j)}{\partial \tau_k} + \frac{\beta_{kj} E_k^*}{w_{kj} P_k^* X_k^*} \right], & \{i = k\} \\ \\ w_{il} \frac{\partial ln(S_{r_i})}{\partial \tau_k} + \sum_j^n w_{ij} \left[\frac{\partial ln(P_j)}{\partial \tau_k} \right], & \{i \neq k\} \end{cases}$$
(70)

I transform expression 70 into matrix form $\frac{\partial ln(P)}{\partial \tau_k} = W^T \frac{\partial ln(P)}{\partial \tau_k} + \Xi^T W_L \frac{\partial ln(S)}{\partial \tau_k} +$ and clearing for $\frac{\partial ln(P)}{\partial \tau_k}$ we obtain

$$\frac{\partial \ln(P_i)}{\partial \tau_k} = \sum_{j=1}^n \ell_{ij}^{WT} w_{jl} \frac{\partial \ln(S_{r_j})}{\partial \tau_k} + \ell_{ik}^{WT} \sum_{m=1}^n \beta_{km} \frac{E_k^*}{P_k^* X_k^*}$$
(71)

Impact in salary S_r

Deriving equation 14 $S_r^* = \frac{\sum_j^n w_{jl} R_j^* \{j \in \mathbf{r}\}}{L_r}$ we obtain equation 21 $\frac{\partial S_r}{\partial \tau_k} = \frac{\sum_m^n w_{ml}}{L_r} \frac{\partial P_m X_j}{\partial \tau_k} \{m \in \mathbf{r}\}$. Then, introducing now the derivative of sector sales and introducing now the derivative of sector sales 65 in terms of parameters and equilibrium values

$$\frac{\partial S_r}{\partial \tau_k} = \sum_{j}^{n} \frac{w_{jl}}{L_{r_j}} \sum_{m}^{n} \Lambda_{jm} \sum_{q}^{n} \ell_{mq} (\sigma_{qk} - \beta_{kq}) \frac{E_k}{P_k X_k} \{ j \in \mathbf{r} \}$$
(72)

The subscripts $\{j, m, q\}$ are required to account for the different layers of interaction for the all the different combinations of inputs and products. Also, since $H_r = S_r L_r$:

$$\frac{\partial S_r}{\partial \tau_k} = \sum_j^n w_{jl} \sum_m^n \Lambda_{jm} \sum_q^n \ell_{mq} (\sigma_{qk} - \beta_{kq}) \frac{E_k}{P_k X_k} \{ j \in \mathbf{r} \}$$
(73)

$$\frac{\partial ln(S_r)}{\partial \tau_k} = \frac{1}{S_r} \frac{\partial S_r}{\partial \tau_k} \tag{74}$$

with $\frac{\partial S_{US}}{\partial \tau_k} = 0$ Solution to the system of derivatives

Now we show how to obtain the equation 23 that characterizes $\frac{\partial ln(X_{ij})}{\partial \tau_k}$ in terms of parameters and equilibrium values. We begin from equation 18 and plug in the derivative of sector sales 19 leading to

$$\frac{\partial ln(X_{ij})}{\partial \tau_k} = \begin{cases} \sum_{j}^{n} \ell_{ij} \left[\sum_{r}^{R} \gamma_{rj} \frac{\partial H_r}{\partial \tau_k} + (\sigma_{jk} - \beta_{kj}) E_k^* \right] - \frac{\partial ln(P_j)}{\partial \tau_k} - \frac{\beta_{kj} E_k^*}{w_{kj} P_k^* X_k^*}, & \{i = k\} \\ \\ \sum_{j}^{n} \ell_{ij} \left[\sum_{r}^{R} \gamma_{rj} \frac{\partial H_r}{\partial \tau_k} + (\sigma_{jk} - \beta_{kj}) E_k^* \right] - \frac{\partial ln(P_j)}{\partial \tau_k}, & \{i \neq k\} \end{cases}$$
(75)

followed by the price derivative 20 which gives

$$\frac{\partial ln(X_{ij})}{\partial \tau_k} = \begin{cases} \sum_{j}^{n} \ell_{ij} [\sum_{r}^{R} \gamma_{rj} \frac{\partial H_r}{\partial \tau_k} + (\sigma_{jk} - \beta_{kj}) E_k^*] - \sum_{m}^{n} \ell_{jm} [\sum_{r}^{R} \gamma_{rj} \frac{\partial H_r}{\partial \tau_k} + (\sigma_{mk} - \beta_{km}) E_k^*] - \frac{\beta_{kj} E_k^*}{w_{kj} P_k^* X_k^*}, & \{i = k\} \\ \sum_{j}^{n} \ell_{ij} [\sum_{r}^{R} \gamma_{rj} \frac{\partial H_r}{\partial \tau_k} + (\sigma_{jk} - \beta_{kj}) E_k^*] - (\tau_{jk} - \beta_{km}) E_k^*] - (\tau_{jk} - \beta_{km}) E_k^*], & \{i \neq k\} \end{cases}$$
(76)

Finally we introduce the derivative of household income 22 to directly obtain 23. Additional subscripts q,b,d must be introduced in order to account for all the dimension of interaction between sectors.

9.2. Sensitivity analysis: Emission intensity parameters (β_{ij})

In Figure 13 we consider a (β_{ij}) that is 50% higher than the values considered in the main text. By comparing the impact in global emissions of a tax on sector k in this case with the impact we have discussed in the main text (i.e. in the left panel of Figure 4), we observe that results do not change significantly.



Figure 13: Impact on global emissions after a tax to each sector. Emission intensity parameters (β_{ij}) is 50% higher than original values.

9.3. Technical Appendix to Figure 7

The following Figure explains why a global carbon tax increases global emissions when the level of the tax is small.



Figure 14: Impact of a 1e/Ton carbon tax on sector's emissions

In the left subfigure, we display the first-order impact on emissions across various sectors following the imposition of a global carbon tax of 1 euro per ton of CO2. We observe that most sectors experience a negative and small change in emissions, while a few sectors see a positive and comparatively large increase. This increase in emissions in a few sectors more than offsets the reductions in the other sectors (i.e., the magnitude of emissions reductions in most sectors does not compensate for the emission increases in the few sectors where they rise).

Why do some sectors pollute more after the imposition of an emissions tax? This phenomenon is akin to what we discussed in Table 2. The sectors in question produce more due to increased demand from the government, which now has more resources available from the tax revenue. If the government, now wealthier, consumes polluting goods, the increased demand for these goods leads to higher emissions. This occurs even though efforts may be made to substitute inputs with less polluting options; the relatively small size of the tax means that the direct demand effect outweighs the incentives for such substitutions.

It's important to note that emissions increase less than they would without a global

tax. This is evident when comparing the right subfigure with the left: isolating the impact of public expenditure on emissions in the right subfigure (without considering the input-output substitutions across firms due to the carbon tax), we see that emissions would increase even more as a result of increased government expenditure. The presence of a global carbon tax mitigates the polluting impact of public expenditure, thereby reducing the overall increase in emissions.